1. Express each of the following sums in summation notation and then compute where possible. Let X take the values $x_1 = -2, x_2 = -1, x_3 = 0, x_4 = 1, x_5 = 2$ and Y take the values $y_1 = -1, y_2 = -0.5, y_3 = 0, y_4 = 1, y_5 = 1.5$.

   a) $x_1 + x_2 + x_3 + x_4 + x_5 = \sum_{i=1}^{5} x_i = \mathbf{0}$
   b) $y_2 + y_3 + y_4 = \sum_{i=2}^{4} y_i = \mathbf{0.5}$
   c) $(x_1 - 2y_1) + (x_2 - 2y_2) + (x_3 - 2y_3) + (x_4 - 2y_4) + (x_5 - 2y_5) = \sum_{i=1}^{5}(x_i - 2y_i) = \sum_{i=1}^{5} x_i - 2\sum_{i=1}^{5} y_i = 0 - 2 * 1 = \mathbf{2}$
   d) $(x_1 * y_1) + (x_2 * y_2) + (x_3 * y_3) = \sum_{i=1}^{3}(x_i * y_i) = \mathbf{2.5}$
   e) $(3x_2 * y_1) + (3x_3 * y_2) + (3x_4 * y_3) = \sum_{i=2}^{4}(3x_i * y_{i-1}) = 3\sum_{i=2}^{4}(x_i * y_{i-1}) = \mathbf{3}$
   f) $({x_1}^3 * y_3) + ({x_2}^3 * y_4) + ({x_3}^3 * y_5) = \sum_{i=1}^{3}({x_i}^3 * y_{i+2}) = \mathbf{-1}$

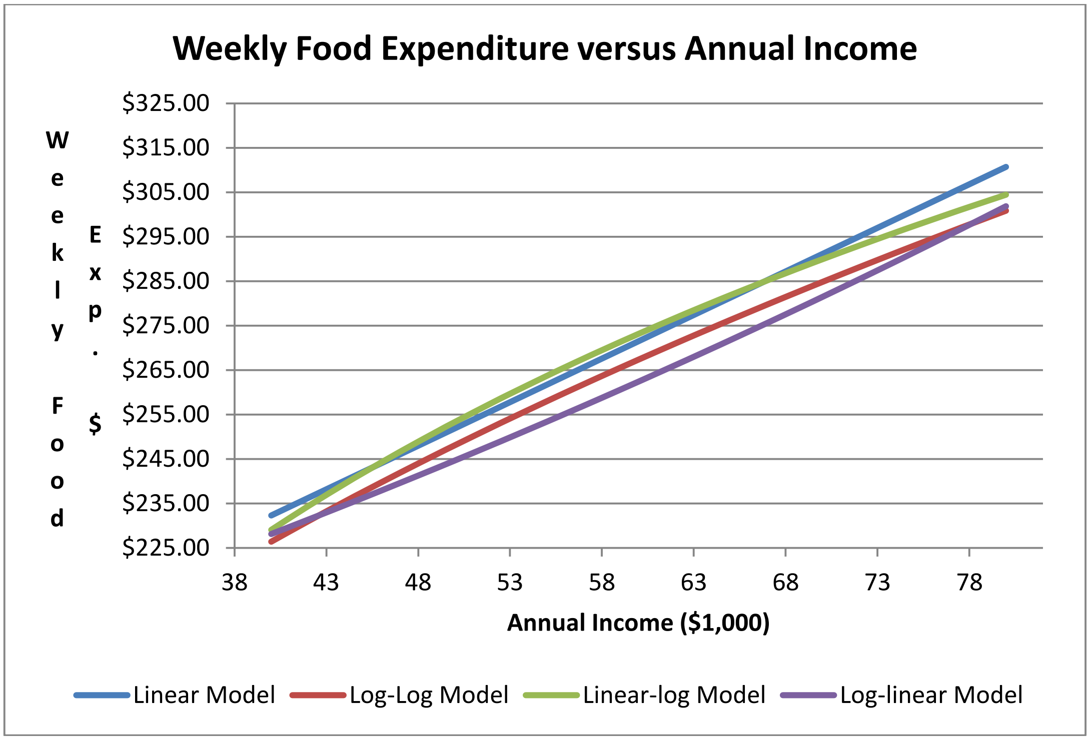
2. Show that $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$

$$\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n}(x_i y_i - \bar{x}\, y_i - x_i \bar{y} + \bar{x}\bar{y})$$
$$= \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} \bar{x}\, y_i - \sum_{i=1}^{n} x_i\, \bar{y} + \sum_{i=1}^{n} \bar{x}\bar{y}$$
$$= \sum_{i=1}^{n} x_i y_i - \bar{x}\frac{n}{n}\sum_{i=1}^{n} y_i - \bar{y}\frac{n}{n}\sum_{i=1}^{n} x_i + n\bar{x}\bar{y}$$
$$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\frac{\sum_{i=1}^{n} y_i}{n} - n\bar{y}\frac{\sum_{i=1}^{n} x_i}{n} + n\bar{x}\bar{y}$$
$$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} - n\bar{y}\bar{x} + n\bar{x}\bar{y} = \sum_{i=1}^{n} x_i\, y_i - n\bar{x}\bar{y}$$

3. An economist for the federal government is attempting to produce a better measure of poverty than is currently in use. To help acquire information, she recorded the annual household income, in thousands of dollars, and the amount of money spent on food during one week for a random sample of households. Below are four equations that were estimated by the economist with the data where *x* is the annual income (in \$1,000) and *y* is the weekly food expenditure for the household.

$$y = 153.90 + 1.96x$$
$$\ln(y) = 3.91 + 0.41\ln(x)$$
$$y = -172.34 + 108.81\ln(x)$$
$$\ln(y) = 5.15 + 0.007x$$

   a) Plot each of the functions for *x* = 40 to *x* = 80.



b) Calculate the slope of each function at x = 60. State the interpretation of the slope.

$y = 153.90 + 1.96x$ and the slope is 1.96, constant, at all values of x. For each extra $1,000 in annual income there is an average increase in weekly food expenditure of $1.96.

$\ln(y) = 3.91 + 0.41\ln(x)$ The slope is 0.41*(y/x), so for x = 60, 0.41 * ($267.38/60) = $1.83. A 1% increase in annual income leads to a 0.41% increase in the average weekly food expenditure. When the annual income is $60,000 an increase of $1,000 will be associated with an average increase in weekly food expenditure of $1.83.

$y = -172.34 + 108.81\ln(x)$. A 1% increase in annual income leads to a 108.81/100 =$1.08 increase in the average weekly food expenditure. The slope is 108.81*(1/x), so for x = 60, 108.81*(1/60) = $1.81. When the annual income is $60,000 an increase of $1,000 will be associated with an average increase in weekly food expenditure of $1.81.

$\ln(y) = 5.15 + 0.007x$. A $1000 increase in annual income leads to an average of 0.007*100% = 0.7% increase in the average weekly food expenditure. The slope is 0.007*y, so for x = 60, 0.007 * $262.43 = $1.84. When the annual income is $60,000 an increase of $1,000 will be associated with an average increase in weekly food expenditure of $1.84.

c) Calculate the elasticity of each function at *x* = 60 and give its interpretation.

$y = 153.90 + 1.96x$. The elasticity is 1.96*(*x*/*y*) = 1.96*(60/271.50) = 0.43, inelastic. When the average annual income is $60,000, weekly food expenditure increases by 0.43% when the annual income increases by 1%.

$\ln(y) = 3.91 + 0.41\ln(x)$ This is the constant elasticity model, where a 1% increase in the average annual income is associated with a 0.41% increase in the household weekly food expenditure. This is also inelastic.

$y = -172.34 + 108.81\ln(x)$. The elasticity is 108.81*(1/*y*), so for *x* = 60, 108.81*(1/273.17) = 0.398. When the average annual income is $60,000, weekly food expenditure increases by 0.39% when the annual income increases by 1%. This is also inelastic.

$\ln(y) = 5.15 + 0.007x$. The elasticity is 0.007*x*, so for *x* = 60, 0.007*60 = 0.42. When the average annual income is $60,000, weekly food expenditure increases by 0.42% when the annual income increases by 1%. This is also inelastic.

4. Let *X* be a discrete random variable that is the value shown on a single roll of a fair die.

a) What is the probability the *X* = 3 or *X* = 4 or *X* = 5?

$$f(X = 3 \ or\ X = 4 \ or\ X = 5) = f(X = 3) + f(X = 4) + f(X = 5) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

<u>The probability the *X* = 3 or *X* = 4 or *X* = 5 is ½ **or 0.5.**</u>

b) What is the expected value of *X*? What is the expected value of X^2?

$$E(X) = \sum_{i=1}^{6} x_i f(x_i) = \left(1 * \frac{1}{6}\right) + \left(2 * \frac{1}{6}\right) + \left(3 * \frac{1}{6}\right) + \left(4 * \frac{1}{6}\right) + \left(5 * \frac{1}{6}\right) + \left(6 * \frac{1}{6}\right) = \frac{21}{6}$$

$$= 3.5$$

<u>**The expected value of *X* is 3.5.**</u>

$$E(X^2) = \sum_{i=1}^{6} x_i^2 f(x_i) = \left(1^2 * \frac{1}{6}\right) + \left(2^2 * \frac{1}{6}\right) + \left(3^2 * \frac{1}{6}\right) + \left(4^2 * \frac{1}{6}\right) + \left(5^2 * \frac{1}{6}\right) + \left(6^2 * \frac{1}{6}\right)$$

$$= \frac{91}{6} = 15.167$$

<u>**The expected value of X^2 is 15.167.**</u>

c) Find the variance of *X*.

$$var(X) = \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2 = 15.167 - (3.5^2) = 2.9167.$$

<u>**The variance of *X* is 2.92.**</u>

d) Find the expected value and variance of W if W = g(X) = 4X – 2.

$E(W) = E(4X - 2) = 4E(X) - 2 = 4 * 3.5 - 2 = 12$

<u>**The expected value of *W* is 12.**</u>

$var(W) = var(4X - 2) = 4^2 var(X) = 16 * 2.9167 = 46.667.$

The variance of *W* is 46.67.

e) Now, assume the die has been weighted so that P(X=1) = 1/6, P(X=2) = 1/3, and the P(X=3) = P(X=4) = P(X=5) = P(X=6) = 1/8. Find the answers to part b) and c) under the new assumption.

$$E(X) = \sum_{i=1}^{6} x_i\, f(x_i) = \left(1 * \frac{1}{6}\right) + \left(2 * \frac{1}{3}\right) + \left(3 * \frac{1}{8}\right) + \left(4 * \frac{1}{8}\right) + \left(5 * \frac{1}{8}\right) + \left(6 * \frac{1}{8}\right) = \frac{37}{12}$$
$$= 3.083$$

The expected value of *X* is 3.1.

$$E(X^2) = \sum_{i=1}^{6} {x_i}^2\, f(x_i) = \left(1^2 * \frac{1}{6}\right) + \left(2^2 * \frac{1}{3}\right) + \left(3^2 * \frac{1}{8}\right) + \left(4^2 * \frac{1}{8}\right) + \left(5^2 * \frac{1}{8}\right) + \left(6^2 * \frac{1}{8}\right)$$
$$= 12.25$$

The expected value of X^2 is 12.25.

$var(X) = \sigma^2 = E(X-\mu)^2 = E(X^2) - \mu^2 = 12.25 - (3.083^2) = 2.743.$

The variance of *X* is 2.74.

5. After watching a number of children playing games in a video arcade, a statistics practitioner estimated the following probability distribution of *X*, the number of games per visit.

x	1	2	3	4	5	6	7
f(x)	.05	.10	.15	.25	.15	.15	.15

a) What is the probability that a child will play at most three games?
P(X ≤ 3) = P(X = 1) + P(X = 2) + P(X = 3) = 0.05 + 0.10 + 0.15 = 0.3
The probability that a child will play at most three games is 0.3.

b) What is the mean and variance of the number of games played?

$$E(X) = \sum_{i=1}^{7} x_i\, f(x_i)$$
$$= (1 * 0.05) + (2 * 0.10) + (3 * 0.15) + (4 * 0.25) + (5 * 0.15) + (6 * 0.15) + (7 * 0.15) = 0.05 + 0.2 + 0.45 + 1 + 0.75 + 0.9 + 1.05 = 4.4$$

$$E(X^2) = \sum_{i=1}^{7} {x_i}^2\, f(x_i)$$
$$= (1^2 * 0.05) + (2^2 * 0.10) + (3^2 * 0.15) + (4^2 * 0.25) + (5^2 * 0.15) + (6^2 * 0.15) + (7^2 * 0.15)$$
$$= 0.05 + 0.4 + 1.35 + 4 + 3.75 + 5.4 + 7.35 = 22.3$$

$var(X) = \sigma^2 = E(X-\mu)^2 = E(X^2) - \mu^2 = 22.3 - 4.4^2 = 2.9399 \approx 2.94$

The mean of the number of games played is 4.4 and the variance is 2.94 games squared.

c) Suppose that each game costs $1 to play. Use the laws of expected value and variance to determine the expected value and variance of the revenue (the amount of money) the arcade receives per child.

Let R be the revenue the arcade receives for each child. $\boldsymbol{R = \$1 * X}$

$E(R) = E(\$1X) = \$1E(X) = \$1 * 4.4 = \$4.4$

$var(R) = var(1X) = 1^2 var(X) = 1 * 2.94 = \$2.94.$

The expected value of the revenue is $4.40 and the variance is $2.94 dollars squared.

d) Determine the probability distribution of the revenue the arcade receives per child.

x	$1	$2	$3	$4	$5	$6	$7
f(x)	.05	.10	.15	.25	.15	.15	.15

e) Use the probability distribution you created in part d) to calculate the mean and variance of the revenue the arcade receives per child.

$$E(X) = \sum_{i=1}^{7} x_i f(x_i)$$

$$= (\$1 * 0.05) + (\$2 * 0.10) + (\$3 * 0.15) + (\$4 * 0.25) + (\$5 * 0.15) + (\$6 * 0.15) + (\$7 * 0.15)$$

$$= 0.05 + 0.2 + 0.45 + 1 + 0.75 + 0.9 + 1.05 = \$4.4$$

$$E(X^2) = \sum_{i=1}^{7} {x_i}^2 f(x_i)$$

$$= (\$1^2 * 0.05) + (\$2^2 * 0.10) + (\$3^2 * 0.15) + (\$4^2 * 0.25) + (\$5^2 * 0.15) + (\$6^2 * 0.15) + (\$7^2 * 0.15)$$

$$= 0.05 + 0.4 + 1.35 + 4 + 3.75 + 5.4 + 7.35 = \$22.3$$

$var(X) = \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2 = 22.3 - 4.4^2 = \$2.9399 \approx \$2.94$

The mean of the revenue per child is $4.4 and the variance is 2.94 dollars squared.

f) Are your answers in part c) & part e) identical? **YES**

6. To gauge the relationship between education and unemployment an economist turned to the US Census, from which the following table was produced:

Education	**Employed**	**Unemployed**
Not a high school graduate	0.0975	0.0080
High school graduate	0.3108	0.0128
Some college, no degree	0.1785	0.0062
Associate’s degree	0.0849	0.0023
Bachelor’s degree	0.1959	0.0041
Advanced degree	0.0975	0.0015

a. Determine the probability that a randomly selected individual is employed.

P(employed) = .0975 + .3108 + .1785 + .0849 + .1959 + .0975 = 0.9651

The probability that a randomly selected individual is employed is 0.9651.

b. What is the probability of a randomly selected individual having an associate's degree and being unemployed?
P(Associate's degree and Unemployed) = 0.0023
The probability of a randomly selected individual having an associate's degree and being unemployed

c. What is the probability that a person who has only High school graduation is unemployed?
P(unemployed | High school grad.) =

$$\frac{P(Highschool\,grad.\ and\ unemployed)}{P(Highschool\,grad)} = \frac{.0128}{.3108 + .0128} = .03955 \cong 0.0396$$

The probability that a person with only a High school graduation is unemployed is 0.0396.

d. Are education and employment status independent? Prove your answer and explain it.
There are two tests for independence.

That is, does *f* (X=a|Y=b) = *f* (a and b)/ *f* (b) = *f* (a)?
P(unemployed | High school grad.) = $0.396 \neq 0.0349 = P(unemployed)$ *so* **NOT independent**.

OR This can also be expressed as *f*(a and b) = *f*(a)**f*(b) for all X and for all Y.
$P(Associate's\ degree\ and\ Unemployed) = 0.0023 \neq 0.0030 = 0.0872 * 0.0349 = P(unemployed) * P(Associate's\ degree)$ *so* **NOT independent**.

Education and employment status are **NOT independent.**

Data from the Office on Smoking and Health, Centers for Disease Control and Prevention, indicate that 40% of adults who did not finish high school, 34% of high school graduates, 24% of adults who completed some college (no degree), and 14% of graduates (Degree holders) smoke.

e. Suppose that one individual is selected at random and it is discovered that the individual does not smoke. What is the probability that the individual does not have a degree? (Use the probabilities from the table above to solve this question)

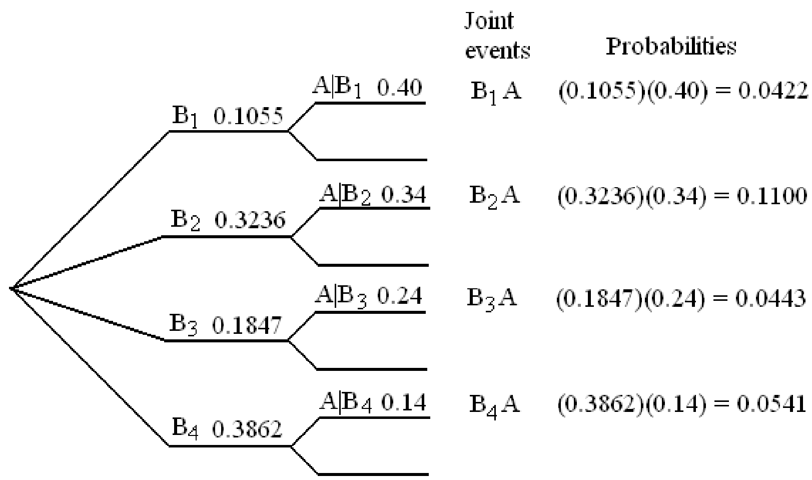
HINT: Bayes' Law Formula

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \ldots + P(A_k)P(B \mid A_k)}$$

Define events: A = smoke, A^C *= does not smoke,* B_1 *= did not finish high school,* B_2 *= high school graduate,* B_3 *= some college, no degree,* B_4 *= completed a degree*

$P(A \mid B_1) = 0.40$ so $P(A^C \mid B_1) = 0.60$, $P(A \mid B_2) = 0.34$ so $P(A^C \mid B_2) = 0.66$, $P(A \mid B_3) = 0.24$ so $P(A^C \mid B_3) = 0.76$, $P(A \mid B_4) = 0.14$ so $P(A^C \mid B_4) = 0.86$.

From Exercise above: $P(B_1) = 0.1055$, $P(B_2) = 0.3236$, $P(B_3) = 0.1847$, $P(B_4) = 0.3862$



$P(B_4^C \mid A^C) = P(B_4^C \text{ and } A^C) / P(A^C)$

$P(B_4^C \text{ and } A^C) = P(B_1 \text{ and } A^C) + P(B_2 \text{ and } A^C) + P(B_3 \text{ and } A^C) =$
$[0.1055*0.6]+[0.3236*0.66]+[0.1847*0.76] = 0.0633 + 0.213576 + 0.140372 = 0.4172$

$P(A^C) = 1 – (.0422 + .1100 + .0443 + .0541) = 1 - 0.2506 = 0.7494$
$P(B_4^C \mid A^C) = 0.4172 / 0.7494 = 0.5568$

Given an individual who does not smoke, the probability that the individual does not have a degree is 0.5568.

7. An investor holds a portfolio consisting of two stocks. He puts 30% of his money in Stock C and 70% into stock F. Stock C has an expected return of R_C = 10% and a standard deviation of σ_C = 10%. Stock F has an expected return of R_F = 15% and a standard deviation of σ_F = 20%. The portfolio return is $P = 0.3 R_C + 0.7 R_F$.

   a) What is the expected return on the portfolio?
   $P = 0.3 R_C + 0.7 R_F$ $E(P) = E(0.3 R_C + 0.7 R_F) = 0.3 E(R_C) + 0.7 E(R_F)$
   $= (0.3* 10\%) + (0.7*15\%) = 13.5\%$
   The expected return on the portfolio is 13.5%.

b) Compute the standard deviation of the portfolio if the two stocks' returns are perfectly positively correlated.

$var(P) = 0.3^2 var(R_C) + 0.7^2 var(R_F) + 2(0.3)(0.7)cov(R_C, R_F)$

$\rho = 1 = \frac{cov(R_C,R_F)}{\sigma_{R_C} * \sigma_{R_F}},\ cov(R_C, R_F) = \sigma_{R_C} * \sigma_{R_F} = 10 * 20 = 200$

$var(P) = 0.3^2 var(R_C) + 0.7^2 var(R_F) + 2(0.3)(0.7)cov(R_C, R_F)$

$= (0.3^2 * 10^2) + (0.7^2 * 20^2) + 2(0.3)(0.7)200 = 289$

$\sigma_P = \sqrt{289} = 17$

<u>The standard deviation of the portfolio is 17%</u>.

c) Compute the standard deviation of the portfolio if the two stocks' returns have a correlation of 0.25.

$var(P) = 0.3^2 var(R_C) + 0.7^2 var(R_F) + 2(0.3)(0.7)cov(R_C, R_F)$

$\rho = 0.5 = \frac{cov(R_C,R_F)}{\sigma_{R_C} * \sigma_{R_F}},\ cov(R_C, R_F) = \sigma_{R_C} * \sigma_{R_F} * .5 = 10 * 20 * .5 = 100$

$var(P) = 0.3^2 var(R_C) + 0.7^2 var(R_F) + 2(0.3)(0.7)cov(R_C, R_F)$

$= (0.3^2 * 10^2) + (0.7^2 * 20^2) + 2(0.3)(0.7)100 = 247$

$\sigma_P = \sqrt{247} = 15.72$

<u>The standard deviation of the portfolio is 15.72%</u>.

d) Compute the standard deviation of the portfolio if the two stocks' returns are uncorrelated.

$var(P) = 0.3^2 var(R_C) + 0.7^2 var(R_F) + 2(0.3)(0.7)cov(R_C, R_F)$

$= (0.3^2 * 10^2) + (0.7^2 * 20^2) + 2(0.3)(0.7)0 = 205$

$\sigma_P = \sqrt{205} = 14.32$

<u>The standard deviation of the portfolio is 14.32%</u>.

8. Let Y_1, Y_2, and Y_3, be independent, identically distributed random variables from a population with a mean μ and a variance σ^2. Consider a different estimator of μ:

$$W = \frac{5}{12} Y_1 + \frac{1}{3} Y_2 + \frac{1}{4} Y_3.$$

This is an example of a weighted average of the Y_i.

a) Show that *W* is a linear estimator.

$W = \sum_{i=1}^{3} a_i Y_i$ *where* a_i *are constants for* $i = 1,2,3$.

b) Is *W* an unbiased estimator of μ? Show that it is – or it isn't (E(W) = ?).

$E(W) = E(\frac{5}{12} Y_1 + \frac{1}{3} Y_2 + \frac{1}{4} Y_3) = \frac{5}{12} E(Y_1) + \frac{1}{3} E(Y_2) + \frac{1}{4} E(Y_3)$

$= \frac{5}{12}\mu + \frac{1}{3}\mu + \frac{1}{4}\mu = \mu$

<u>*W* is an unbiased estimator of μ.</u>

c) Find the variance of *W* and compare it to the variance of the sample mean $\bar{Y}$.

$$var(W) = var\left(\frac{5}{12}\,Y_1 + \frac{1}{3}\,Y_2 + \frac{1}{4}\,Y_3\right)$$
$$= \frac{5}{12}^2\, var(Y_1) + \frac{1}{3}^2\, var(Y_2) + \frac{1}{4}^2\, var(Y_3)$$
$$= \frac{25}{144}\sigma^2 + \frac{1}{9}\sigma^2 + \frac{1}{16}\sigma^2 = \frac{50}{144}\sigma^2 = 0.3472\sigma^2$$
$var(\bar{Y}) = \frac{\sigma^2}{N} = \frac{\sigma^2}{3} = 0.3333\sigma^2$ $\quad var(\bar{Y}) = 0.33\sigma^2 < 0.35\sigma^2 = var(W)$

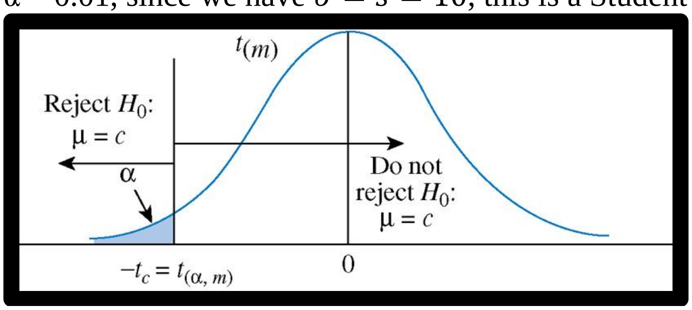
The variance of *W* is larger than the variance of the sample mean $\bar{Y}$.

d) Is *W* as good an estimator as $\bar{Y}$? Explain your answer. **No *W* is not as good an estimator as $\bar{Y}$, since the variance of *W* is larger than the variance of $\bar{Y}$.**

9. A sample of 25 observations is taken and $\bar{X} = 60$ and $\hat{\sigma} = s = 10$.

   a) Test the hypothesis that the mean is less than 56 at a 1% significance level. Be sure to state the null and alternative hypotheses, give the test statistic and its distribution, indicate the rejection region, including a sketch, state your conclusion, and calculate the p-value for the test.

$H_0 : \mu = 56$
$H_1 : \mu < 56$
α = 0.01, since we have $\hat{\sigma} = s = 10$, this is a Student *t*-distribution with df = n-1 = 24.



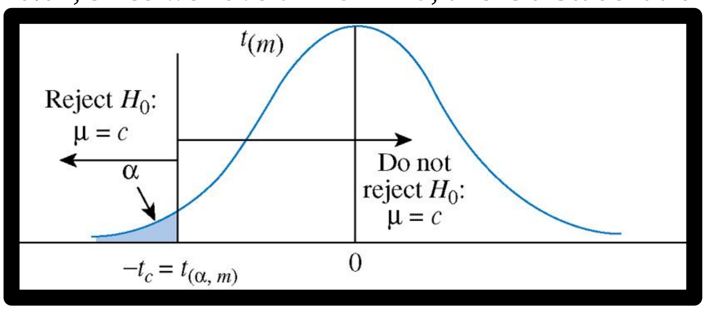
Test statistic: $t = \frac{\bar{X}-\mu}{\hat{\sigma}/\sqrt{n}} = \frac{60-56}{10/5} = \frac{4}{2} = 2$

Rejection region: (from Table 2, Appendix E, $t_{0.99,24} = -2.492$) (Rejection region: $t \leq t_{0.01,24} = -2.492$ from Excel)
Since -2.492 < 2, do not reject the null hypothesis.
Conclusion: There is no evidence in this sample to support the conclusion that the mean is less than 56, the evidence supports the mean equal to 56.

p-value: Using the table at df = 24, 1.711 < 2 < 2.064, so 0.05 > $P(t_{24} \geq 2) \geq 0.025$. The $P(t_{24} \geq 2) = 0.0285$ (from Excel) The $P(t_{24} \geq 2) = 0.0285 > 0.01$, so do not reject the null hypothesis.

b) Repeat the test in part a) with $\hat{\sigma} = s = 15$.

$H_0 : \mu = 56$
$H_1 : \mu < 56$
α = 0.01, since we have $\hat{\sigma} = s = 15$, this is a Student *t*-distribution with df = n-1 = 24.



Test statistic: $t = \frac{\bar{X}-\mu}{\hat{\sigma}/\sqrt{n}} = \frac{60-56}{15/5} = \frac{4}{3} = 1.33$

Rejection region: (from Table 2, Appendix E, $t_{0.99,24} = -2.492$) (Rejection region: $t \leq t_{0.01,24} = -2.492$ from Excel)
Since -2.492 < 1.33, do not reject the null hypothesis.
Conclusion: There is no evidence in this sample to support the conclusion that the mean is less than 56, the evidence supports the mean equal to 56.

p-value: Using the table at df = 24, 1.318 < 1.33 < 1.711, so 0.1 > $P(t_{24} \geq 1.33) \geq$ 0.05.
The $P(t_{24} \geq 1.33) = 0.0975$ (from Excel) The $P(t_{24} \geq 1.33) = 0.0975 > 0.01$, so do not reject the null hypothesis.

c) Create the 95% confidence interval for the mean when $\hat{\sigma} = s = 15$.
$\bar{X} \pm t_{(df,\alpha/2)} \frac{\hat{\sigma}}{\sqrt{n}} = 60 \pm t_{(24,0.05/2)} \frac{15}{\sqrt{25}} = 60 \pm$ (2.064*3) = 60 ± 6.192 = **[53.808, 66.192] Using the $t_{0.975,24} = 2.064$ from Table 2, Appendix E**
With 95% confidence the true mean is equal to or between 53.808 and 66.192.

d) What is the p-value of your test statistic in part a) and part b)?
See above in part a) and part b) for the calculation. **The p-value for part a) is 0.0285 and for part b) is 0.0975.**

e) What happens to your test statistic and the p-value when the variance gets larger?
The test statistic gets smaller as the variance gets larger. The standard deviation, the square root of variance, is in the denominator of the test statistic. When the test statistic gets smaller the p-value gets larger. There is an inverse relationship between the change in the test statistic and the change in the p-value.

MM 25/9/14