ASSIGNMENT 7 -Solutions

PROBLEM 1 (15 points)
In Figure 1 the pipe entrance is sharp-edged. If the flow rate is 0.004m^3/s, what power, in W, is extracted by the turbine? (Cast iron pipe: L=125m, D = 5cm)

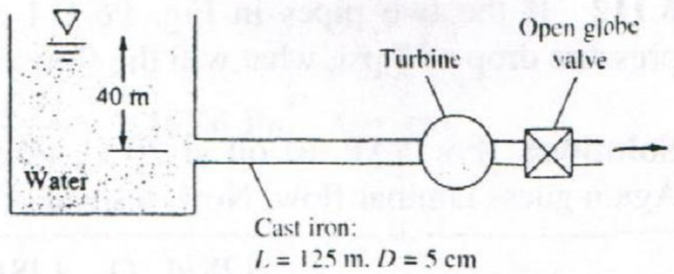

Figure 1

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For cast iron, $\varepsilon \approx$ 0.26 mm, hence ε/d = 0.26/50 $\approx$ 0.0052. The minor loss coefficients are Entrance: K $\approx$ 0.5; 5-cm($\approx$2″) open globe valve: K $\approx$ 6.9

The flow rate is known, hence we can compute V, Re, and f:

$$V = \frac{Q}{A} = \frac{0.004}{(\pi/4)(0.05)^2} = 2.04\ \frac{m}{s}, \quad Re = \frac{998(2.04)(0.05)}{0.001} \approx 102000, \quad f \approx 0.0316$$

The turbine head equals the elevation difference minus losses and exit velocity head:

$$h_t = \Delta z - h_f - \sum h_m - \frac{V^2}{2g} = 40 - \frac{(2.04)^2}{2(9.81)}\left[(0.0316)\left(\frac{125}{0.05}\right) + 0.5 + 6.9 + 1\right] \approx 21.5\ m$$

$$\text{Power} = \rho g Q h_t = (998)(9.81)(0.004)(21.5) \approx \mathbf{840\ W} \quad \textit{Ans.}$$

PROBLEM 2 (15 points)

SAE 30 oil at 20°C flows in the 3-cm-diameter pipe in Figure 2, which slopes at 37°. For the pressure measurements shown, determine (a) whether the flow is up or down and (b) the flow rate in m^3/h.

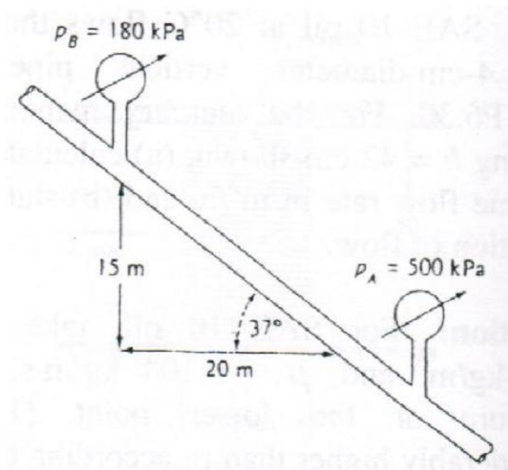

Figure 2

(SAE 30 Oil: ρ=891 kg/m^3 and μ=0.29 kg/m·s)

Solution: For SAE 30 oil, take $\rho = 891$ kg/m^3 and $\mu = 0.29$ kg/m·s. Evaluate the hydraulic grade lines:

$$HGL_B = \frac{p_B}{\rho g} + z_B = \frac{180000}{891(9.81)} + 15 = 35.6 \text{ m}; \quad HGL_A = \frac{500000}{891(9.81)} + 0 = 57.2 \text{ m}$$

$$\text{Since} \quad HGL_A > HGL_B \text{ the } \textbf{flow is } \boldsymbol{up} \quad \textit{Ans.} \text{(a)}$$

The head loss is the difference between hydraulic grade levels:

$$h_f = 57.2 - 35.6 = 21.6 \text{ m} = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.29)(25)Q}{\pi(891)(9.81)(0.03)^4}$$

$$\text{Solve for} \quad Q = 0.000518 \text{ m}^3/\text{s} \approx \mathbf{1.86\ m^3/h} \quad \textit{Ans.} \text{ (b)}$$

$$\text{Finally, check} \quad Re = 4\rho Q/(\pi\mu d) \approx 68 \text{ (OK, laminar flow).}$$

PROBLEM 3 (20 points)
In Problem 2, suppose it is desired to add a pump between A and B to drive the oil *upward* from A to B at a rate of 3 kg/s. At 100% efficiency, what pump power is required?

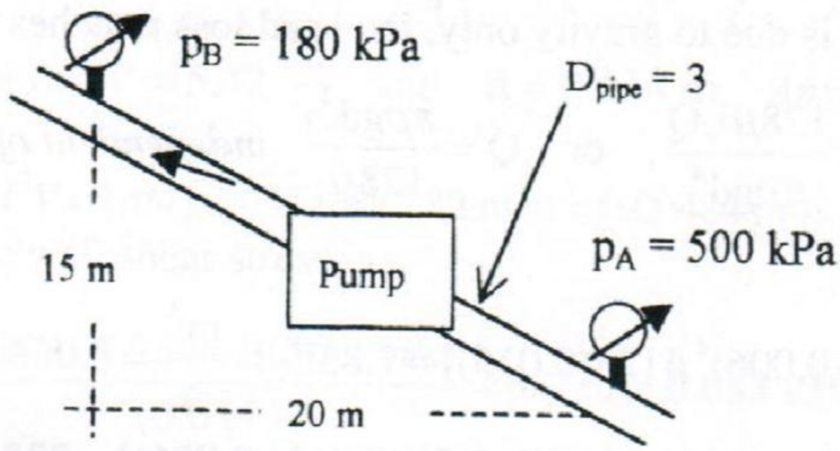

Figure 3

Solution: For SAE 30 oil at 20°C, $\rho = 891$ kg/m^3 and $\mu = 0.29$ kg/m·s. With mass flow known, we can evaluate the pipe velocity:

$$V = \frac{\dot{m}}{\rho A} = \frac{3\ kg/s}{891\pi(0.015)^2} = 4.76\ \frac{m}{s},$$

$$\text{Check } Re_d = \frac{891(4.76)(0.03)}{0.29} = \mathbf{439}\ (OK,\ laminar)$$

Apply the steady flow energy equation between A and B:

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_f - h_p, \quad or: \quad \frac{500000}{891(9.81)} = \frac{180000}{891(9.81)} + 15 + h_f - h_p$$

$$\text{where } h_f = \frac{32\mu LV}{\rho g d^2} = \frac{32(0.29)(25)(4.76)}{891(9.81)(0.03)^2} = 140.5\ m. \quad \text{Solve for } h_{pump} = 118.9\ m$$

The pump power is then given by

$$\textbf{Power} = \rho g Q h_p = \dot{m} g h_p = \left(3\ \frac{kg}{s}\right)\left(9.81\ \frac{m}{s^2}\right)(118.9\ m) = \mathbf{3500\ watts} \quad Ans.$$

PROBLEM 4 (15 points)

Water at 20°C is to be pumped through 2000ft of pipe from reservoir 1 to 2 at a rate of 3 ft^3/s, as shown in Figure 4. If the pipe is cast iron of diameter 6in and the pump is 75% efficient, what horsepower pump is needed?

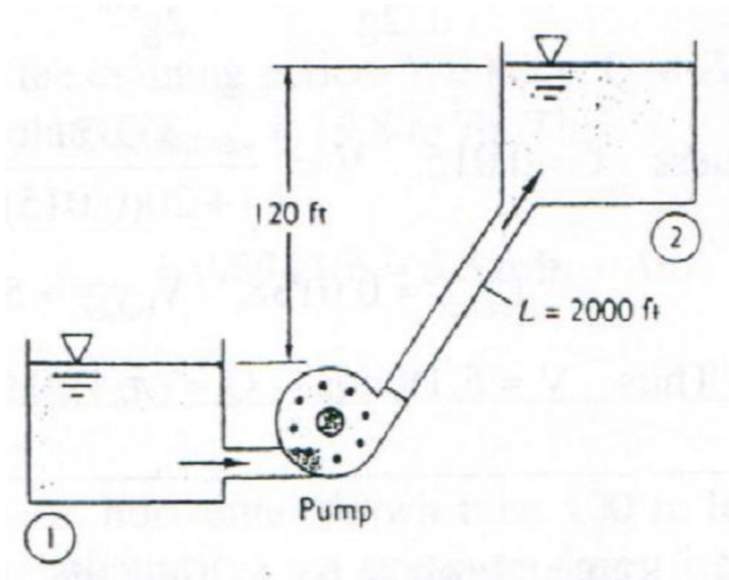

Figure 4

Solution: For water at 20°C, take ρ = 1.94 slug/ft³ and μ = 2.09E−5 slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft, or ε/d = 0.00085/(6/12) ≈ 0.0017. Compute V, Re, and *f*:

$$V = \frac{Q}{A} = \frac{3}{(\pi/4)(6/12)^2} = 15.3\ \frac{ft}{s};$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.94(15.3)(6/12)}{2.09E-5} \approx 709000 \quad \varepsilon/d = 0.0017, \quad f_{Moody} \approx 0.0227$$

The energy equation, with $p_1 = p_2$ and $V_1 \approx V_2 \approx 0$, yields an expression for pump head:

$$h_{pump} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 120\ ft + 0.0227\left(\frac{2000}{6/12}\right)\frac{(15.3)^2}{2(32.2)} = 120 + 330 \approx 450\ ft$$

$$\text{Power:}\quad P = \frac{\rho g Q h_p}{\eta} = \frac{1.94(32.2)(3.0)(450)}{0.75} = 112200 \div 550 \approx \mathbf{204\ hp} \quad \textit{Ans.}$$

PROBLEM 5 (20 points)

The small turbine in Figure 5 extracts 400W of power from the water flow. Both pipes are wrought iron. Compute the flow rate *Q* m^3/h. Sketch the EGL and HGL accurately.

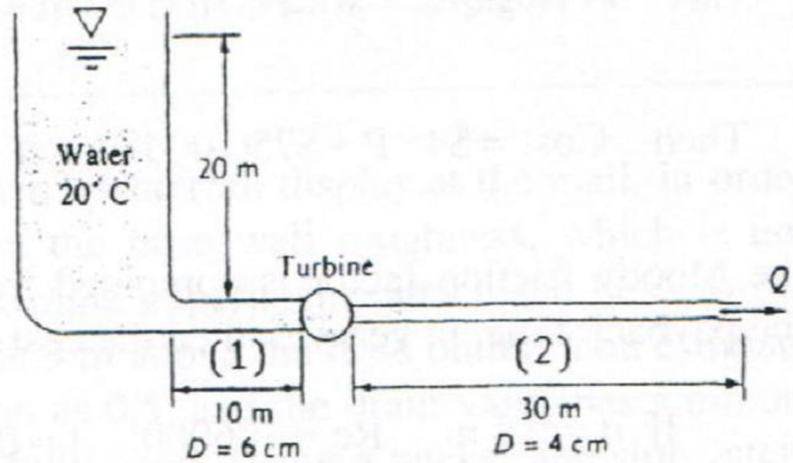

Figure 5

Solution: For water, take $\rho = 998\ kg/m^3$ and $\mu = 0.001\ kg/m{\cdot}s$. For wrought iron, take $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d_1 = 0.046/60 \approx 0.000767$ and $\varepsilon/d_2 = 0.046/40 \approx 0.00115$. The energy equation, with $V_1 \approx 0$ and $p_1 = p_2$, gives

$$z_1 - z_2 = 20\ m = \frac{V_2^2}{2g} + h_{f2} + h_{f1} + h_{turbine},\quad h_{f1} = f_1\frac{L_1}{d_1}\frac{V_1^2}{2g}\quad \text{and}\quad h_{f2} = f_2\frac{L_2}{d_2}\frac{V_2^2}{2g}$$

$$\text{Also,}\quad h_{turbine} = \frac{P}{\rho g Q} = \frac{400\ W}{998(9.81)Q}\quad \text{and}\quad Q = \frac{\pi}{4}d_1^2 V_1 = \frac{\pi}{4}d_2^2 V_2$$

The only unknown is Q, which we may determine by iteration after an initial guess:

$$h_{turb} = \frac{400}{998(9.81)Q} = 20 - \frac{8f_1L_1Q^2}{\pi^2 g d_1^5} - \frac{8f_2L_2Q^2}{\pi^2 g d_2^5} - \frac{8Q^2}{\pi^2 g d_2^4}$$

$$\text{Guess}\quad Q = 0.003\ \frac{m^3}{s},\quad \text{then}\ Re_1 = \frac{4\rho Q}{\pi\mu d_1} = 63500,\quad f_{1,Moody} \approx 0.0226,$$

$$Re_2 = 95300,\quad f_2 \approx 0.0228.$$

But, for this guess, h_{turb}(left hand side) ≈ 13.62 m, h_{turb}(right hand side) ≈ 14.53 m (wrong). Other guesses converge to $h_{turb} \approx 9.9$ meters. For $Q \approx 0.00413\ m^3/s \approx$ **15 m^3/h**. *Ans.*

PROBLEM 6 (15 points)

The system in Figure 6 consists of 1200m of 5cm cast-iron pipe, two 45° and four 90° flanged long-radius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. If the elevation at point 1 is 400m, what gage pressure is required at point 1 to deliver 0.005 m^3/s of water at 20°C into the reservoir?

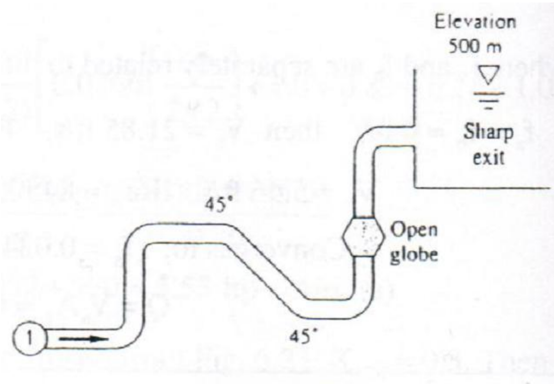

Figure 6

Solution: For water at 20°C, take ρ = 998 kg/m³ and μ = 0.001 kg/m·s. For cast iron, take $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.0052$. With the flow rate known, we can compute V, Re:

$$V = \frac{Q}{A} = \frac{0.005}{(\pi/4)(0.05)^2} = 2.55\ \frac{m}{s};\quad Re = \frac{998(2.55)(0.05)}{0.001} \approx 127000,\quad f_{Moody} \approx 0.0315$$

The minor losses may be listed as follows:

45° long-radius elbow: K ≈ 0.2; 90° long-radius elbow: K ≈ 0.3

Open flanged globe valve: K ≈ 8.5; submerged exit: K ≈ 1.0

Then the energy equation between (1) and (2—the reservoir surface) yields

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = 0 + 0 + z_2 + h_f + \sum h_m,$$

$$\text{or:}\quad p_1/(\rho g) = 500 - 400 + \frac{(2.55)^2}{2(9.81)}\left[0.0315\left(\frac{1200}{0.05}\right) + 0.5 + 2(0.2) + 4(0.3) + 8.5 + 1 - 1\right]$$

$$= 100 + 253 = 353\ \text{m.}\quad \text{or:}\quad \mathbf{p_1} = (998)(9.81)(353) \approx \mathbf{3.46\ MPa}\quad \textit{Ans.}$$