ASSIGNMENT 6 Solutions

PROBLEM 1 (25 points)

For the elbow duct shown in Figure 1 below, SAE 30 oil (γ = 8720N/m3) at 20°C enters section **1** at 350N/s, and exits at section **2**. Assuming steady incompressible flow, compute the force, and its direction, of the oil on the elbow due to momentum change only (no pressure change or friction effects).

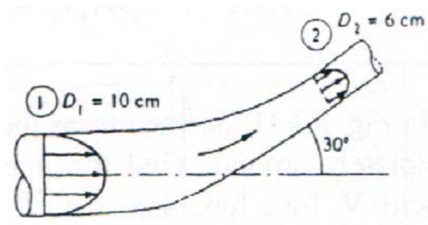

Figure 1

Solution:

$$\gamma = 8790 N/m^3$$

$$thus\ Q = \frac{350}{8720} = 0.0401\frac{m^3}{s}$$

$$Q = 0.0401\frac{m^3}{s} = \pi(0.05)^2 V_{av,1} = \pi(0.03)^2 V_{av,2}$$

$$V_{av,1} = 5.11 m/s$$

$$V_{av,2} = 14.2 m/s$$

$$F_x = \beta_2 \rho A_2 V_{av,2}^2 \cos\theta - \beta_1 \rho A_1 V_{av,1}^2; \quad F_y = \beta_2 \rho A_2 V_{av,2}^2 \sin\theta$$

$$(a)\ Neglecting\ Momentum - Flux\ Correction\ Factors\ (\beta = 1)$$

$$F_x = (890)\pi(0.03)^2(14.2)^2\cos 30° - (890)\pi(0.05)^2(5.11)^2 = 439 - 183 \approx \mathbf{256\ N}$$

$$F_y = (890)(\pi)(0.03)^2(14.2)^2\sin(30) = \mathbf{254 N}$$

PROBLEM 2 (25 points)

Water at 20°C flows through a 5-cm-diamter pipe which has a 180° vertical bend, as shown in Figure 2. The total length of the pipe between flanges 1 and 2 is 75cm. When the weight flow rate is 230N/s, p_1=165 kPa, and p_2= 134kPa. Neglecting pipe weight, determine the total force which the flanges must withstand for this flow.

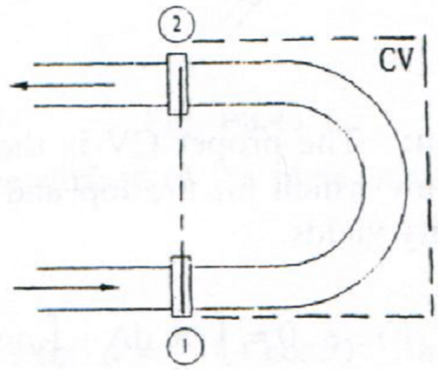

Figure 2

Solution: Let the CV cut through the flanges and surround the pipe bend. The mass flow rate is (230 N/s)/(9.81 m/s^2) = 23.45 kg/s. The volume flow rate is Q = 230/9790 = 0.0235 m^3/s. Then the pipe inlet and exit velocities are the same magnitude:

$$V_1 = V_2 = V = Q/A = \frac{0.0235\ \mathrm{m^3/s}}{(\pi/4)(0.05\ \mathrm{m})^2} \approx 12.0\ \frac{\mathrm{m}}{\mathrm{s}}$$

Subtract p_a everywhere, so only p_1 and p_2 are non-zero. The horizontal force balance is:

$$\sum F_x = F_{x,flange} + (p_1 - p_a)A_1 + (p_2 - p_a)A_2 = \dot{m}_2 u_2 - \dot{m}_1 u_1$$

$$= F_{x,fl} + (64000)\frac{\pi}{4}(0.05)^2 + (33000)\frac{\pi}{4}(0.05)^2 = (23.45)(-12.0 - 12.0\ \mathrm{m/s})$$

$$\text{or:}\quad F_{x,flange} = -126 - 65 - 561 \approx \mathbf{-750\ N} \quad \textit{Ans.}$$

PROBLEM 3 (20 points)
In Figure 3 a perfectly balanced 700-N weight and level platform are supported by a steady water jet. What is the proper jet velocity?

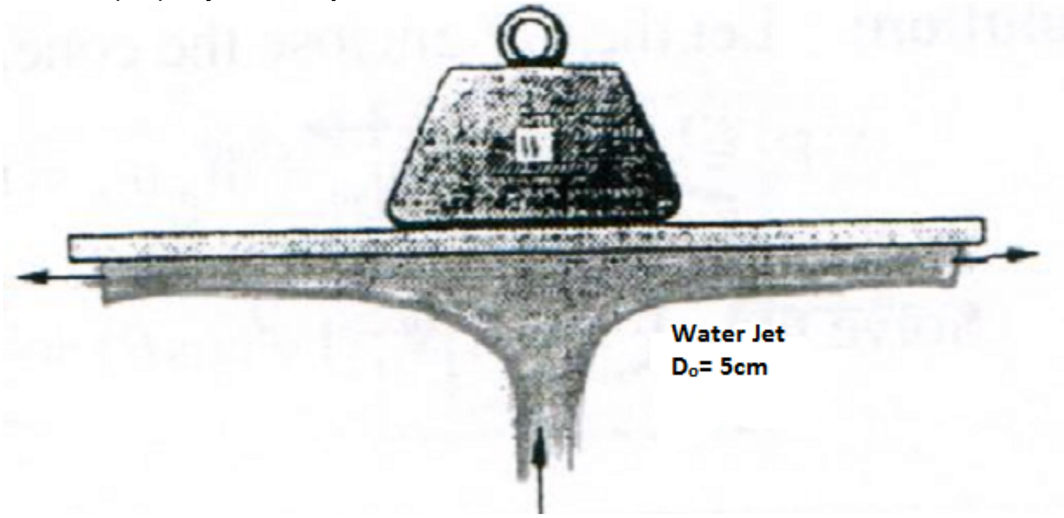

Figure 3

Solution:
For a CV around surrounding the weight, platform, and jet, vertical forces yield:

$$\sum F_y = -W = \dot{m}_{left} V_{left} + \dot{m}_{right} V_{right} - \dot{m}_o V_o = 0 + 0 - (\rho_o A_o V_o) V_o$$

Thus $W = 700 \text{ N} = (998)\frac{\pi}{4}(0.05)^2 V_o^2$, solve for $V_o = V_{jet} = \mathbf{18.9\ \frac{m}{s}}$ *Ans.*

PROBLEM 4 (30 points)

A liquid jet of density r and area A strikes a block and splits into two jets, as shown in Fig. 4. All three jets have the same velocity V. The upper jet exits at angle θ and area αA, the lower jet turns down at 90° and area (1 – α)A. (a) Derive a formula for the forces (F_x,F_y) required to support the block against momentum changes. (b) Show that F_y=0 only if α ≥ 0.5. (c) Find the values of α and θ for which both F_x and F_y are zero.

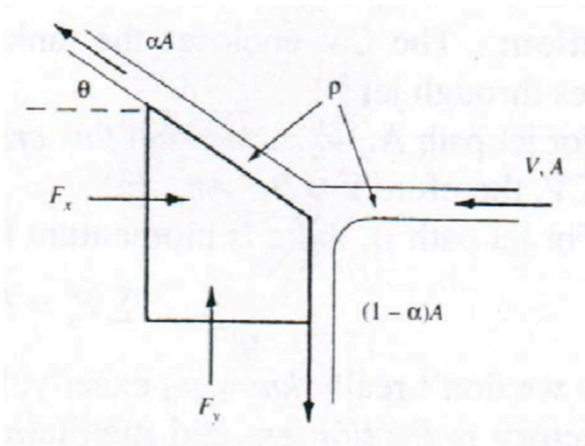

Figure 4

Solution: (a) Set up the *x*- and *y*-momentum relations:

$$\sum F_x = F_x = \alpha \dot{m}(-V\cos\theta) - \dot{m}(-V) \quad \textit{where } \dot{m} = \rho A V \textit{ of the inlet jet}$$

$$\sum F_y = F_y = \alpha \dot{m} V \sin\theta + (1-\alpha)\dot{m}(-V)$$

Clean this up for the final result:

$$\mathbf{F_x = \dot{m}V(1 - \alpha\cos\theta)}$$

$$\mathbf{F_y = \dot{m}V(\alpha\sin\theta + \alpha - 1)} \quad \textit{Ans.} \text{ (a)}$$

(b) Examining F_y above, we see that it can be zero only when,

$$\sin\theta = \frac{1-\alpha}{\alpha}$$

But this makes no sense if $\alpha < 0.5$, hence $\mathbf{F_y = 0}$ **only if** $\boldsymbol{\alpha \geq 0.5}$. *Ans.* (b)

(c) Examining F_x we see that it can be zero only in cosθ = 1/α, which makes no sense unless α = 1, θ =0°. This situation also makes F_x=0 above (sinθ=0). Therefore the only scenario for which both forces are zero is the trivial case for which all the flow goes horizontally across a flat block:

$$F_x = F_y = 0 \; only\, if: \alpha = 1, \theta = 0°$$