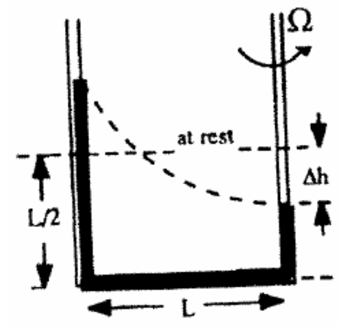
ASSIGNMENT 5 SOLUTIONS (DUE Friday, March 18th 2011)

PROBLEM 1 (15 points)
The U-tube shown is rotated about the right leg at 95 r/min. Find the level *h* in the left leg if L = 18cm and D=5mm.



Solution:

$$\Delta h = \frac{\Omega^2 R^2}{2g} = \frac{(9.95)^2(0.18)^2}{2(9.81)} = 0.163m$$

$$thus\ h_{left\,leg} = 9 + 16.3 = \mathbf{25.3cm}$$

PROBLEM 2 (15 points)
For what uniform rotation rate in r/min about axis C will the U-tube fluid in the figure below take the position shown? The fluid is mercury at 20°C.

Solution: Let h_o be the height of the free surface at the centerline. Then, from Eq. (2.64),

$$z_B = h_o + \frac{\Omega^2 R_B^2}{2g}; \quad z_A = h_o + \frac{\Omega^2 R_A^2}{2g}; \quad R_B = 0.05 \text{ m} \quad \text{and} \quad R_A = 0.1 \text{ m}$$

$$\text{Subtract:} \quad z_A - z_B = 0.08 \text{ m} = \frac{\Omega^2}{2(9.81)}[(0.1)^2 - (0.05)^2],$$

$$\text{solve} \quad \Omega = 14.5 \frac{\text{rad}}{\text{s}} = \mathbf{138 \frac{r}{min}} \quad \textit{Ans.}$$

The fact that the fluid is mercury does not enter into this "kinematic" calculation.

PROBLEM 3 (20 points)
Three pipes steadily deliver water at 20°C to a large exit pipe shown in the figure below. The velocity V_2 = 5 m/s, and the exit flow rate $Q_4 = 120\ m^3/h$. Find (a) V_1, (b) V_3, and (c) V_4 if it is known that increasing Q_3 by 20% would increase Q_4 by 10%.

Solution: (a) For steady flow we have $Q_1 + Q_2 + Q_3 = Q_4$, or

$$V_1A_1 + V_2A_2 + V_3A_3 = V_4A_4 \qquad (1)$$

Since $0.2Q_3 = 0.1Q_4$, and $Q_4 = (120\ m^3/h)(h/3600\ s) = 0.0333\ m^3/s$,

$$V_3 = \frac{Q_4}{2A_3} = \frac{(0.0333\ m^3/s)}{\frac{\pi}{2}(0.06^2)} = \mathbf{5.89\ m/s} \quad \textit{Ans. (b)}$$

Substituting into (1),

$$V_1\left(\frac{\pi}{4}\right)(0.04^2) + (5)\left(\frac{\pi}{4}\right)(0.05^2) + (5.89)\left(\frac{\pi}{4}\right)(0.06^2) = 0.0333 \qquad \mathbf{V_1 = 5.45\ m/s} \quad \textit{Ans. (a)}$$

From mass conservation, $Q_4 = V_4A_4$

$$(0.0333\ m^3/s) = V_4(\pi)(0.06^2)/4 \qquad \mathbf{V_4 = 5.24\ m/s} \quad \textit{Ans. (c)}$$

PROBLEM 4 (15 points)
The pipe flow in the figure below fills a cylindrical tank as shown. At time t=0, the water depth in the tank is 30cm. Estimate the time required to fill the remainder of the tank.

Solution: For a control volume enclosing the tank and the portion of the pipe below the tank,

$$\frac{d}{dt}\left[\int \rho\, dv\right] + \dot{m}_{out} - \dot{m}_{in} = 0$$

$$\rho\pi R^2 \frac{dh}{dt} + (\rho AV)_{out} - (\rho AV)_{in} = 0$$

$$\frac{dh}{dt} = \frac{4}{998(\pi)(0.75^2)}\left[998\left(\frac{\pi}{4}\right)(0.12^2)(2.5-1.9)\right] = 0.0153\ m/s,$$

$$\Delta t = 0.7/0.0153 = \mathbf{46\ s} \quad \textit{Ans.}$$

PROBLEM 5 (15 points)
According to Torricelli's theorem, the velocity of a fluid draining from a hole in a tank is $V = (2gh)^{1/2}$, where *h* is the depth of water above the hole, as shown in the figure below. Let the hole have area A_o and the cylindrical tank have bottom area A_b. Derive a formula for the time to drain the tank from an initial depth h_o.

Solution: For a control volume around the tank,

$$\frac{d}{dt}\left[\int \rho\, dv\right] + \dot{m}_{out} = 0$$

$$\rho A_b \frac{dh}{dt} = -\dot{m}_{out} = -\rho A_o \sqrt{2gh}$$

$$\int_{h_o}^{0} \frac{dh}{\sqrt{h}} = -\int_{0}^{t} \frac{A_o\sqrt{2g}}{A_b}\, dt; \qquad \boldsymbol{t = \frac{A_b}{A_o}\sqrt{\frac{h_o}{2g}}} \quad \textit{Ans.}$$

PROBLEM 6 (20 points)
A solid steel cylinder, 4.5cm in diameter and 12cm long, with a mass of 1500 grams, falls concentrically through a 5-cm-diameter vertical container filled with oil (SG=0.89). Assuming the oil is incompressible, estimate the oil average velocity in the annular clearance between cylinder and container (a) relative to the container and (b) relative to the cylinder.

Solution: (a) The *fixed* CV shown is relative to the *container*, thus:

$$Q_{cyl} = Q_{oil}, \quad or: \quad \frac{\pi}{4}d^2 V_{cyl} = \frac{\pi}{4}(D^2 - d^2)V_{oil}, \quad thus \quad \mathbf{V_{oil}} = \frac{\mathbf{d^2}}{\mathbf{D^2 - d^2}}\mathbf{V_{cyl}} \quad \textit{Ans. (a)}$$

For the given dimensions (d = 4.5 cm and D = 5.0 cm), V_{oil} = **4.26** $V_{cylinder}$.

(b) If the CV moves *with* the cylinder we obtain, relative to the cylinder,

$$V_{oil\ relative\ to\ cylinder} = V_{part(a)} + V_{cyl} = \frac{\mathbf{D^2}}{\mathbf{D^2 - d^2}}\mathbf{V_{cyl}} \approx \mathbf{5.26}V_{cyl} \quad \textit{Ans. (b)}$$