Final Exam Instructions 1

- 3 hour exam. 6 problems, all of equal weight. Each problem consists of 3-5 parts.
- It is a closed book exam.
- One crib sheet single page (both sides) allowed, and a formula sheet will be provided. It will be attached on the exam paper. The formula sheet is available in examination.
- All answers on the **exam booklet**
- Always redraw the diagrams in your exam booklet.
- The marking will be based on what is written on the exam booklet and **not** on the exam paper.
- Always justify your answers. A correct answer without justification will receive no credit.

Final Exam Instructions 2

- Part marks will be given for the correct method even when the numerical answer is incorrect.
- Write answers with symbols as much as possible and then put numbers into the symbolic expression when required.
- Any equation not on the formula sheet that you want to use must be derived
- Any type of calculator will be allowed in the exam room.

List of sections excluded from the final exam

- Chapter 21 section 21.12

Chapter 16

Coulomb's Law

The magnitude of the force between two point charges is given by Coulomb's Law,

$$F = \frac{k\,|q_1|\,|q_2|}{r^2}$$

The magnitude is always a positive quantity.
Do not use the signs of the charges in this equation when calculating the magnitude of the force.

The direction of the force is determined from

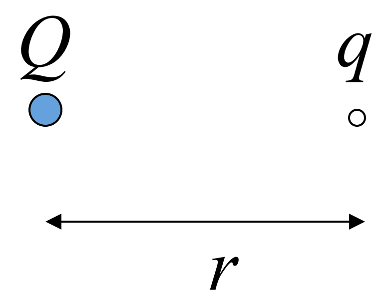
Charges of the same sign repel one another

Charges with opposite signs attract one another.

Electric Field

magnitude of the electric field, *E*, by a point charge *Q* at a distance *r* is defined as the electric force on the test charge at that point divided by the value of the test charge.

$$E = \frac{F}{q} = \left(\frac{k\,q\,Q}{r^2}\right)\frac{1}{q}$$



$$E = \frac{kQ}{r^2}$$

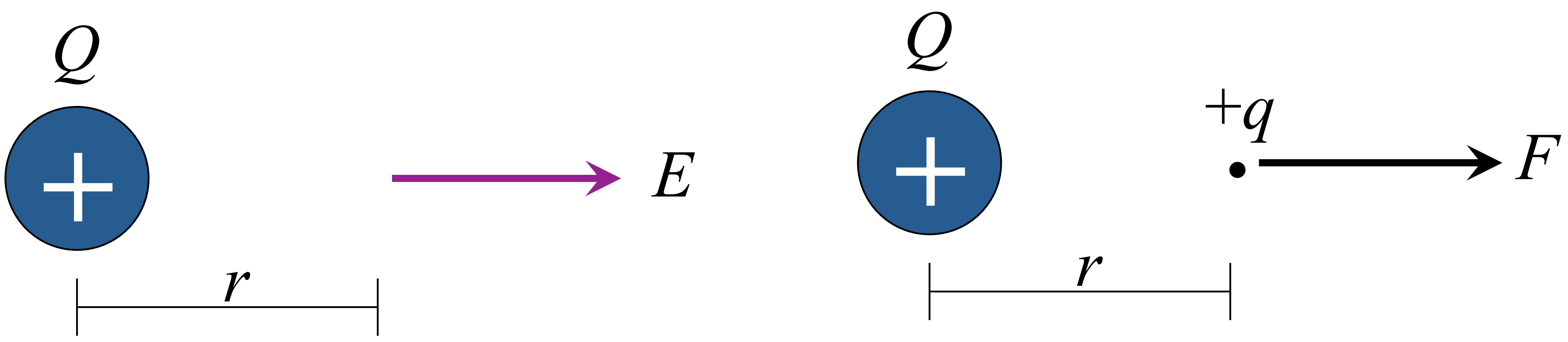
Units for *E*: N/C

Direction: Electric field is vector whose direction is defined by force on a positive test charge

In vector form: $\vec{E} = k\frac{Q}{r^2}\hat{r}$

Definition of the Electric Field

Knowing magnitude and direction of *E* can be used to calculate force on any charge

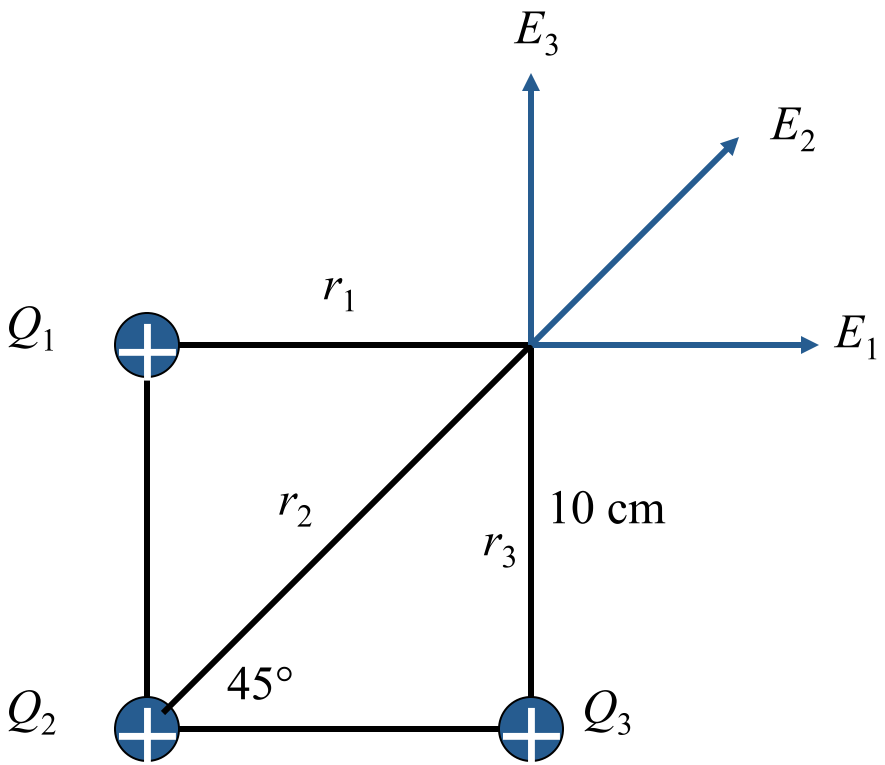


If we know the E-field at a point, then if a charge *Q´* is placed at that point it will experience an electric force given by

$$\vec{F} = Q'\vec{E}$$

Example 4:

Electric field by multiple point charges) Three charges are positioned at 3 of the 4 corners of a square of side 10 cm. Each have a charge of 5×10^{-10} C. Calculate the E-field at the empty corner.



For more than one charge use the superposition principle:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

Chapter 17

Potential in a *Uniform* Electric Field

$$\Delta U = -q\vec{E} \cdot \Delta\vec{s} = -q|\vec{E}|\cos\theta|\Delta\vec{s}|$$

Increment of potential energy

External force

Magnitude of parallel displacement

$$\Delta V = -\vec{E} \cdot \Delta\vec{s} = -|\vec{E}|\cos\theta|\Delta\vec{s}|$$

Increment of potential

Electric Potential Due to A *Point* Charge

Potential of a point charge, *Q´*

$$\implies V = \frac{kQ'}{r}$$

This is the potential a distance r from the charge *Q´*.

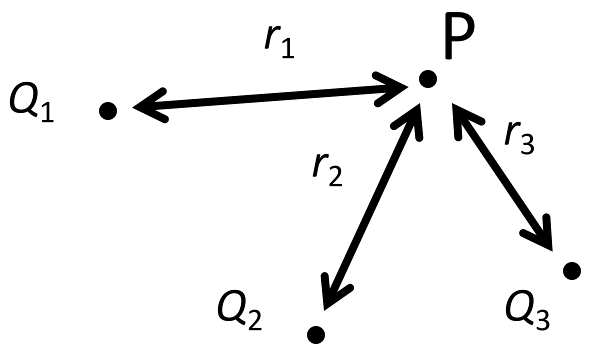
Potential energy (PE) = work to bring a charge Q from infinity to a distance r away from a charge Q' given by:

$$\text{PE} = QV$$

Electric Potential Due to *Multiple* Point Charges

For many charges, Q_1, Q_2, Q_3, ... , the potential at a point P is given by the superposition principle:

$$V_p = V_1 + V_2 + V_3 + \ldots$$
$$= \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \frac{kQ_3}{r_3} + \ldots$$



* Remember to include the sign of the charges Q_1, Q_2, Q_3, ... , when calculating V_p.

* ***If a charge Q is placed at point P its potential energy will be:***

$PE = Q\,V_p$

Potential Energy of a System of Charges

The potential energy (*U*) of a collection of charges is the amount of work needed in assembling the charges.

1. Start with all the charges at infinitely away.

2. Bring one charge at a time and calculate the work done in doing so.

3. When all the charges are assembled, the potential energy is equal to the algebraic sum of all the work done.

Conservation of Energy Calculations With Charges

In many situations, it is very useful to use the conservation of energy principle.

$$[\text{Total Energy at point A}] = [\text{Total Energy at point B}]$$

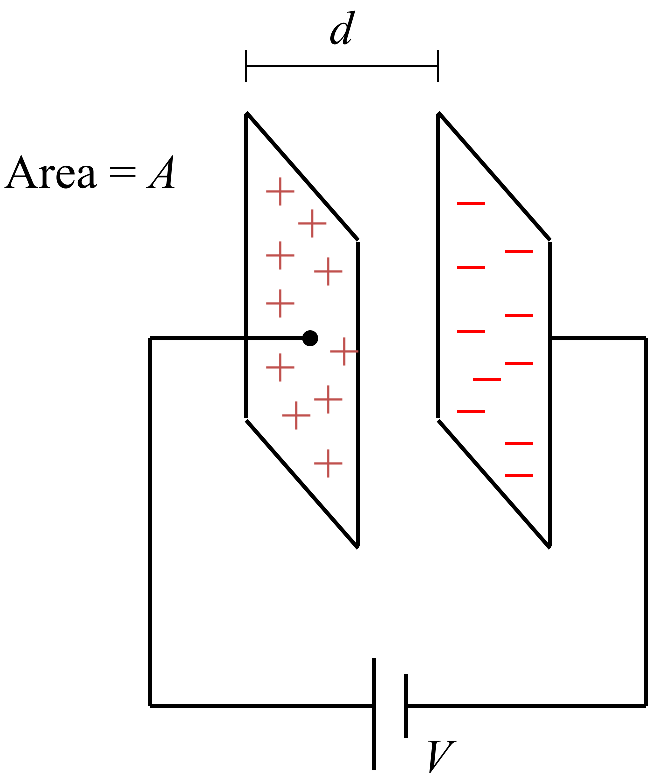
$$(\text{KE} + \text{PE})_{\text{A}} = (\text{KE} + \text{PE})_{\text{B}}$$

$$\frac{1}{2}mv_{\text{A}}^2 + QV_{\text{A}} = \frac{1}{2}mv_{\text{B}}^2 + QV_{\text{B}}$$

Parallel late Capacitor:

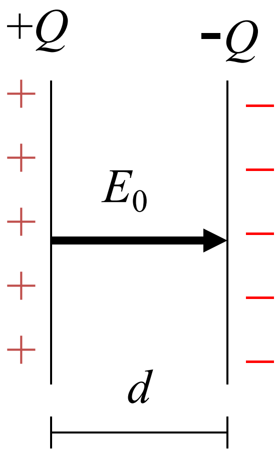
$$Q = CV$$

$$C = \varepsilon_0 \frac{A}{d}$$



ε_o: permittivity of vacuum. $\varepsilon_o = 8.85 \times 10^{-12}\ C^2/Nm^2$

Summary of Capacitor Concepts



$$Q = CV$$

$$C = \epsilon_o \kappa \frac{A}{d} = \epsilon \frac{A}{d} \qquad (\kappa > 1)$$

$$V = Ed$$

$$Q = \sigma A$$

$$E = \frac{\sigma}{\epsilon_o}$$

Stored electrical potential energy

$$U = \frac{Q^2}{2C} \text{ or } U = \frac{1}{2}CV^2 \text{ or } U = \frac{1}{2}QV$$

$$\text{Energy density} = \frac{PE}{Ad} = \frac{1}{2}\kappa\varepsilon_0 E^2$$

- Charge on a disconnected capacitor cannot escape.
- A capacitor connected to a battery remains at a fixed voltage.

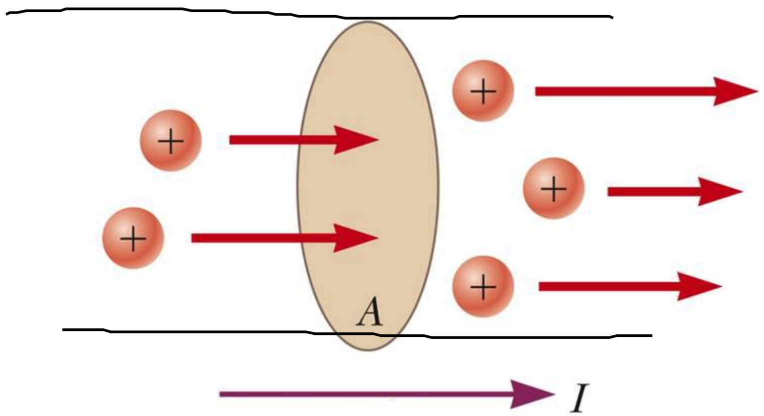
Chapter 18

Definition of Electric Current:

Electric current is the rate of flow of charge across a surface.

$$I = \frac{\Delta Q}{\Delta t} \quad \text{or} \quad I = \frac{dQ}{dt}$$

Units for current: C/s or Ampère [A]

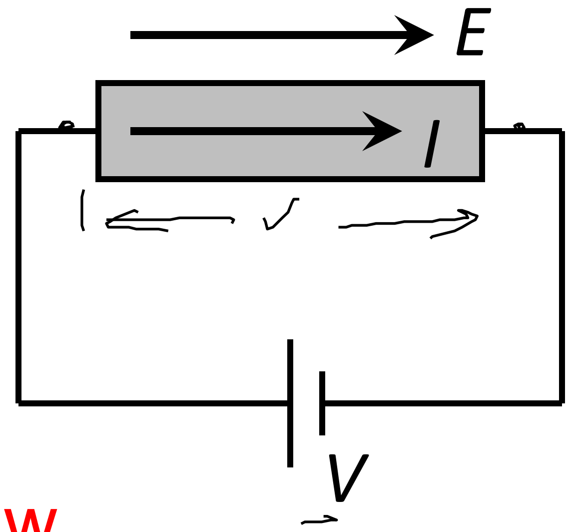


The direction of electric current is defined as the direction of motion of a positive charge.

Electrons contribute a current in the direction opposite to their flow.

Electrical Resistance and Ohm's Law (18-3)

The larger the potential difference, the larger the current inside the conductor.



$$I \propto V$$

$$V = RI$$

Ohm's Law

R : Resistance

V : Voltage across the resistor

I : Current passing through the resistor.

Units for Resistance: $\mathrm{V/A} \equiv \mathrm{Ohm}\ [\Omega]$

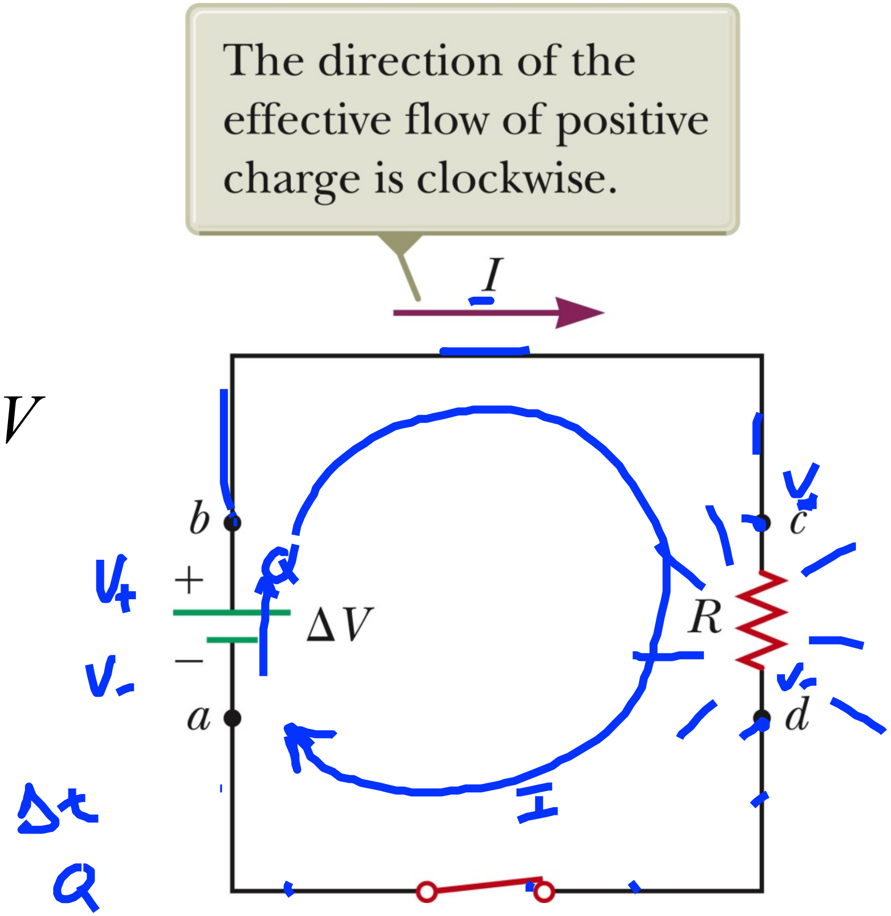
Electric power (18.5)

$$\text{Work done} = QV = (I \times \Delta t)V$$

$$\text{Power (rate of work)} = \frac{I\Delta t\, V}{\Delta t} = IV$$

$$P = IV$$

[unit : watts W]

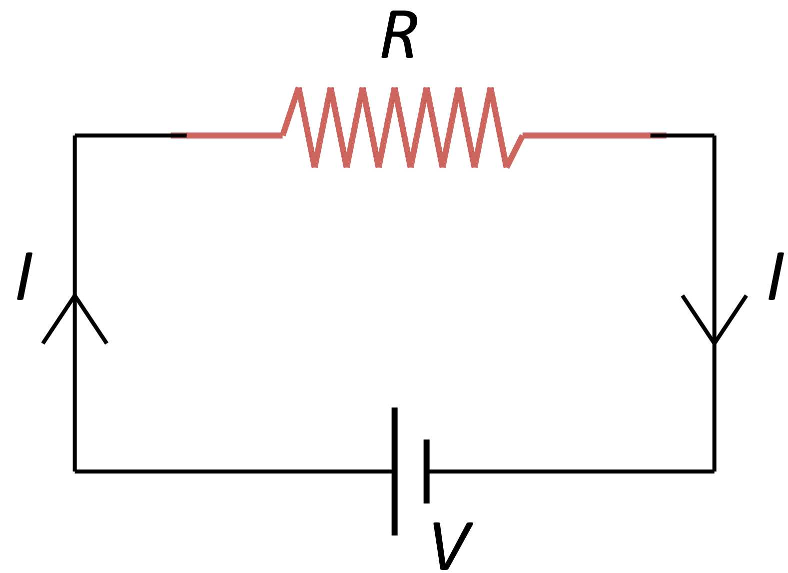


Power **transferred or received** by any device is given by the product of the current passing through it and the potential difference across it.

Joule Heating in Resistor

The electric power generated by the battery must equal the electric power delivered and dissipated (as heat) in the resistor.

(conservation of energy)



$P = IV$ and for the resistor $V = IR$

$I = V/R$

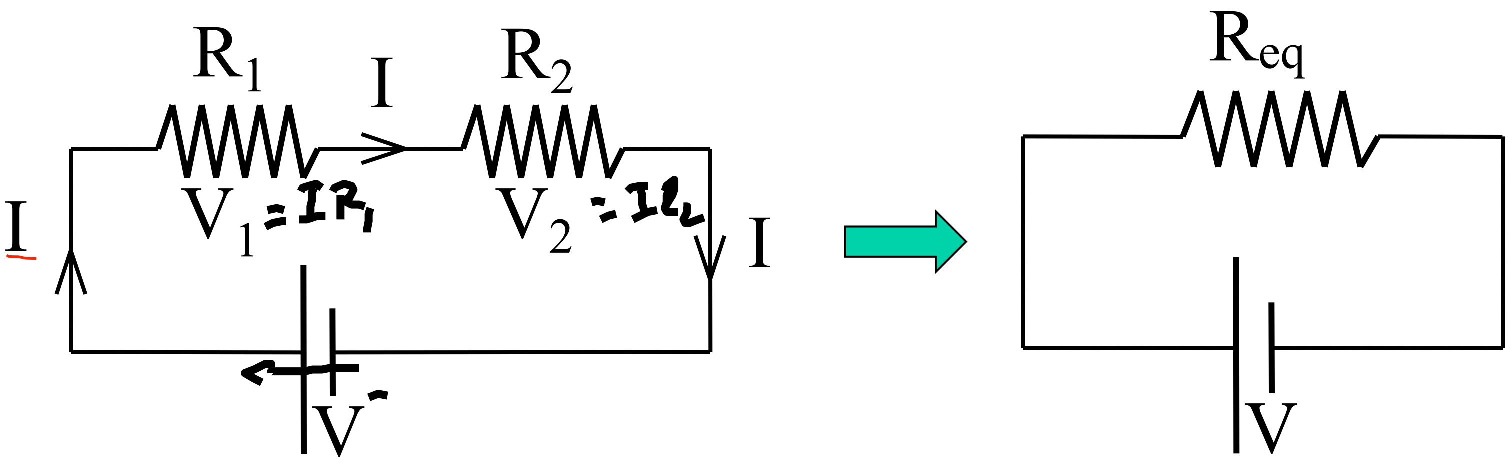
$$P = I^2R \quad \text{or} \quad P = V^2/R$$

This expression is sometimes referred to as Joule heating or Joule's Law.
Also called as Ohmic heating, resistive heating

Chapter 19

Resistors in Series and Parallel (19-2)

1. Resistors in series



- Neglect the resistance of the wires that make up the connections in the circuit.

•The same current flows through both resistors.

•Total potential difference

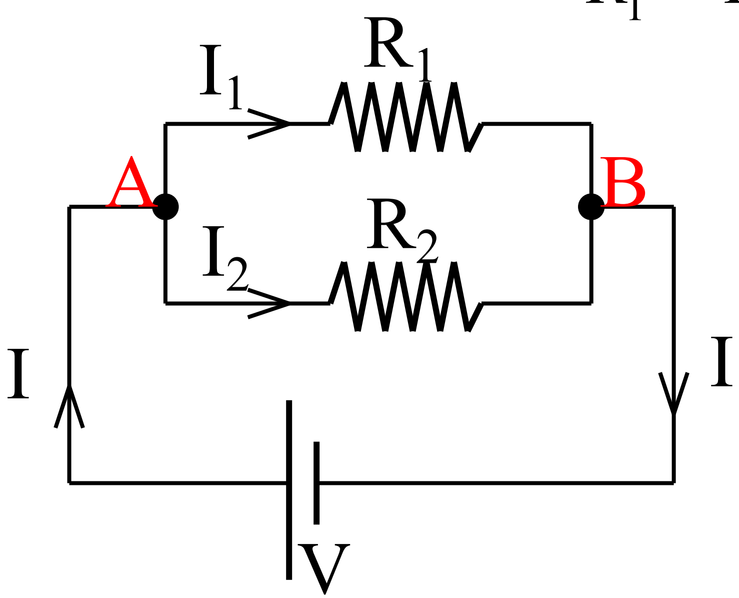
$$V = V_1 + V_2 = I_1 R_1 + I_2 R_2$$
$$= I\,(R_1 + R_2) = IR_{eq}$$

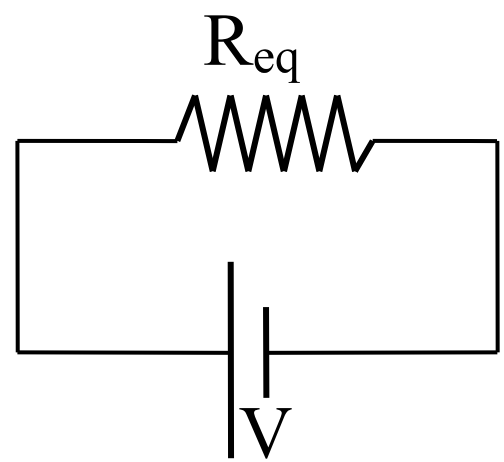
$$R_{eq} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Resistors in parallel

$$I = I_1 + I_2 \implies I = \frac{V_1}{R_1} + \frac{V_2}{R_2} = V\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \implies \frac{I}{V} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$





Where the equivalent resistance is given by:

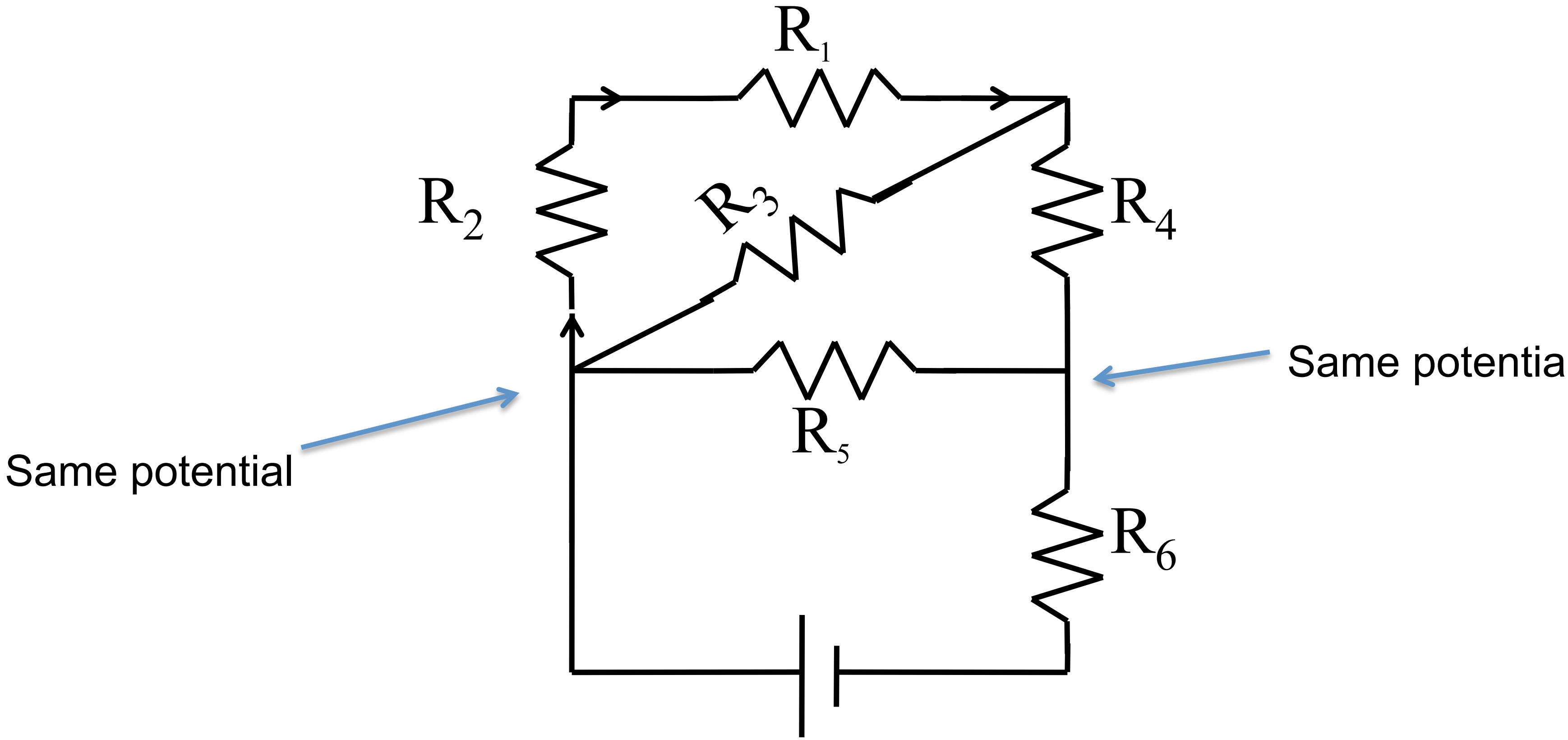
$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\implies \quad R_{\mathrm{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

Multiple resistors in parallel

$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Application to Complex Resistor Networks

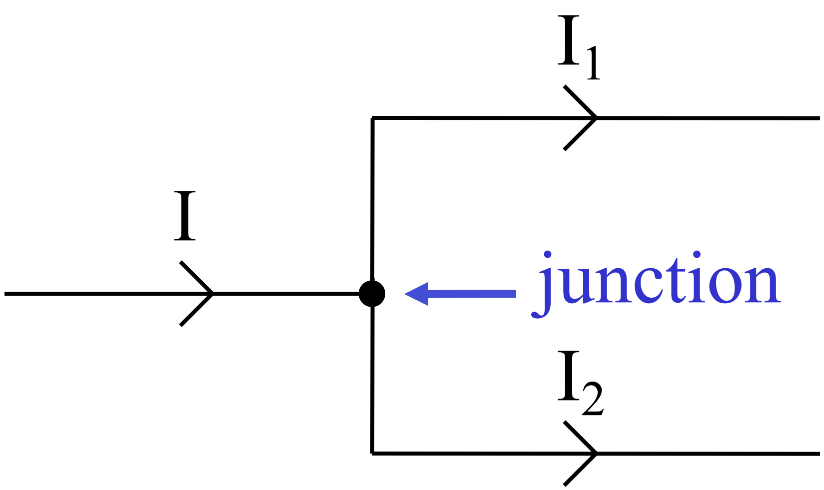


Statement of Kirchhoff's Rules

1. Junction/Node Rule

"The sum of the currents entering a junction equals the sum of the currents leaving that junction"

Conservation of charge



$$I = I_1 + I_2$$

Statement of Kirchhoff's Rules

2. Loop Rule

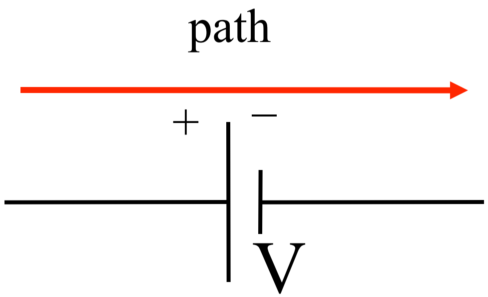
"The algebraic sum of the **changes** in potential around any loop must be zero."

→ Conservation of energy

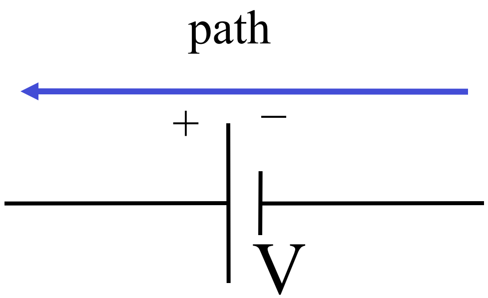
* The potential increases for a path traversing the - terminal to the + terminal of a battery.

* The current flows from a higher potential to a lower potential. So, a path through a resistor in the direction of the current gives negative change in potential. A path opposite to the direction of current in the resistor gives a positive change in potential.

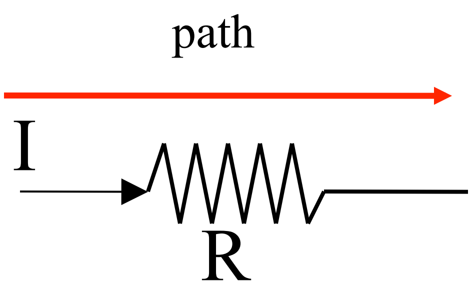
Summary of Voltage Paths (Important!)



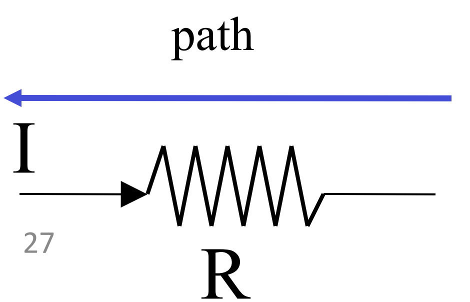
Decrease in potential, $-V$



Increase in potential, $+V$



Decrease in potential, $-IR$



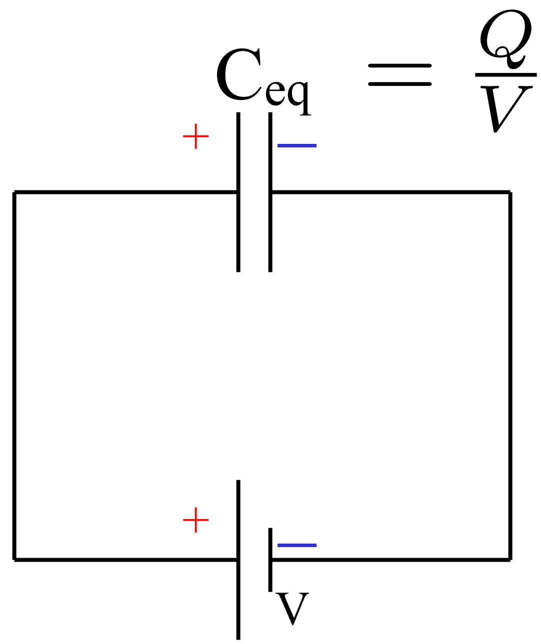
Increase in potential, $+IR$

Capacitors in Parallel

The rule for adding multiple capacitances connected in parallel is:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

The equivalent circuit is:



Capacitors in Series

The equivalent, total or effective capacitance can be determined from:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

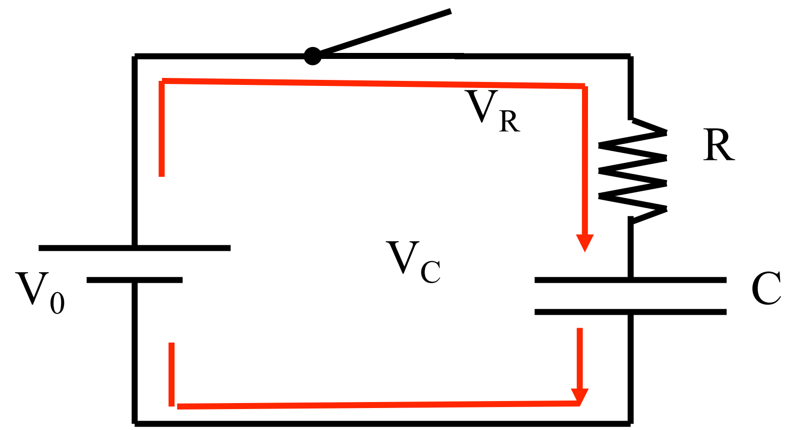
$$C_{eq} = \frac{1}{\frac{C_1 + C_2}{C_1 \times C_2}}$$

The rule for adding multiple capacitances in series is:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Capacitor at Early and Late Times

Right after closing switch at t=0



$$Q = 0 \quad \Rightarrow V_C = Q/C = 0$$

By Kirchhoff's law $V_0 = V_R + V_C$

$$\therefore V_R = V_0$$

After a long time ($t \rightarrow \infty$)

The capacitor is fully charged and no more charges can flow from the battery to the plates of the capacitor.

$$I = 0 \text{ and } V_R = IR = 0$$

By Kirchhoff's rules $V_0 = V_R + V_C \implies V_c = V_0$

Charge is maximized $\quad Q_0 = V_0 C$

Charging of a Capacitor

$$1-\frac{Q}{CV_0}=1-\frac{Q}{Q_0}=e^{-\frac{t}{RC}}$$

$$\boxed{\frac{Q}{Q_0}=1-e^{-\frac{t}{RC}}}$$

Discharging of a Capacitor

$$Q = Q_0 e^{-t/\tau}$$

τ= RC
time constant

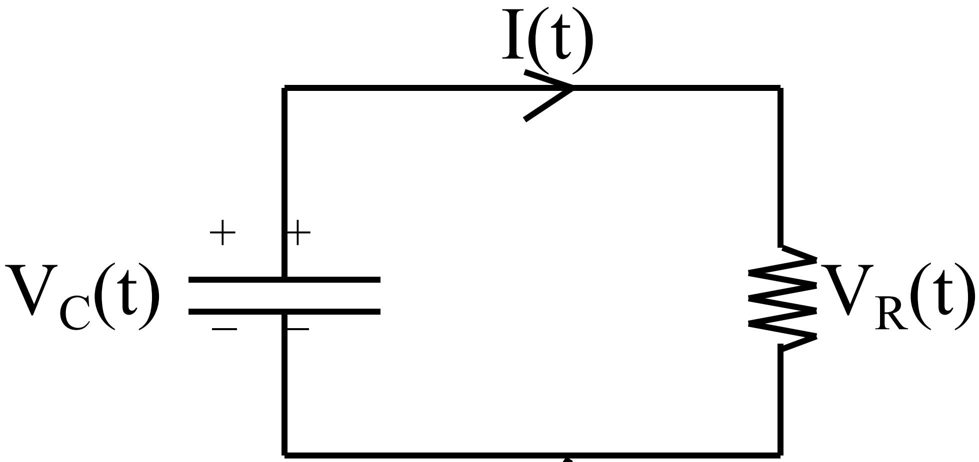
When $t = 1\tau$

$Q = Q_0\, e^{-1} = Q_0/e$

$Q = 0.368\, Q_0$

[reminder: e = 2.7183]

Discharging of a Capacitor



$Q = CV$

$Q(t) = Q_o e^{-t/\tau}$

$V_c = \frac{Q_o}{C} e^{-t/\tau}$

As in the charging case, can find $V_C(t)$, $V_R(t)$ and $I(t)$ from $Q(t)$.

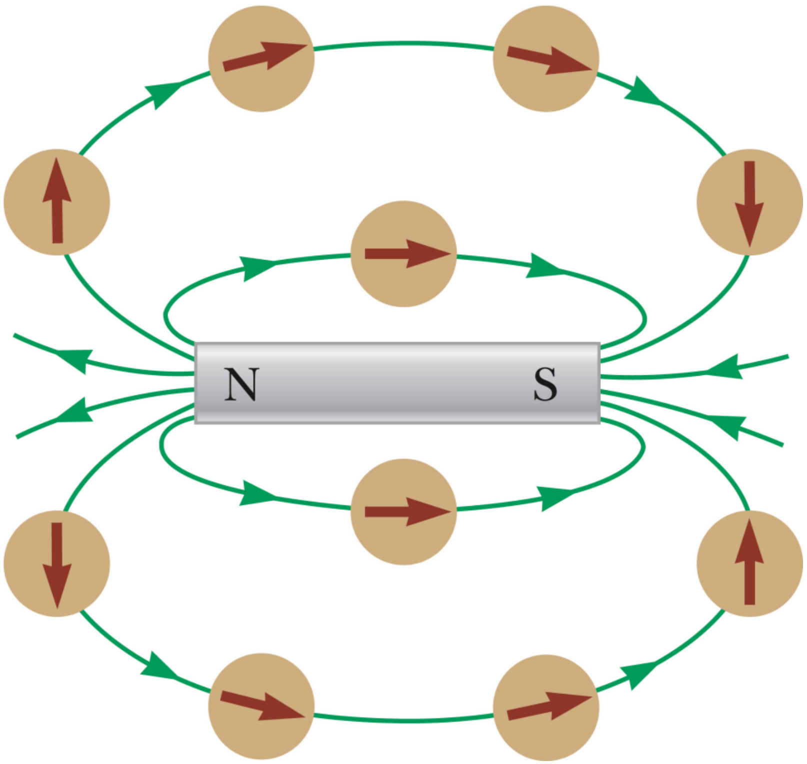
$$V_C(t) = \frac{Q(t)}{C} \Rightarrow V_C(t) = V_o e^{-t/\tau}$$

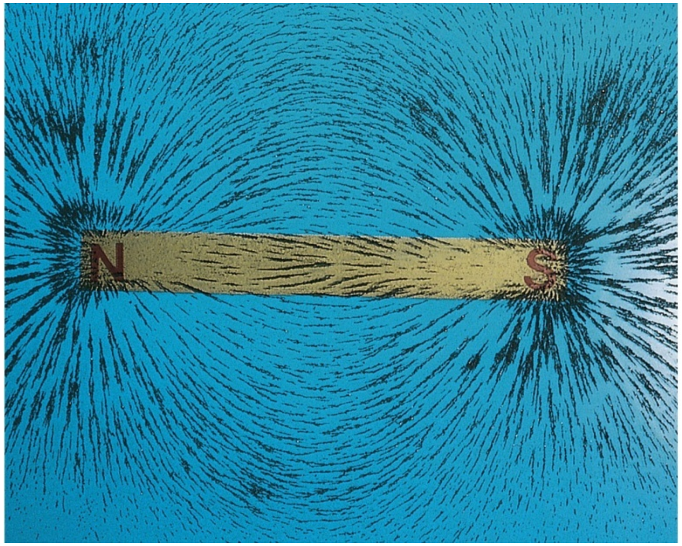
$$V_R(t) = V_C(t)$$

$$I(t) = \frac{V_R(t)}{R} \Rightarrow I(t) = \frac{V_o}{R} e^{-t/\tau}$$

Chapter 20

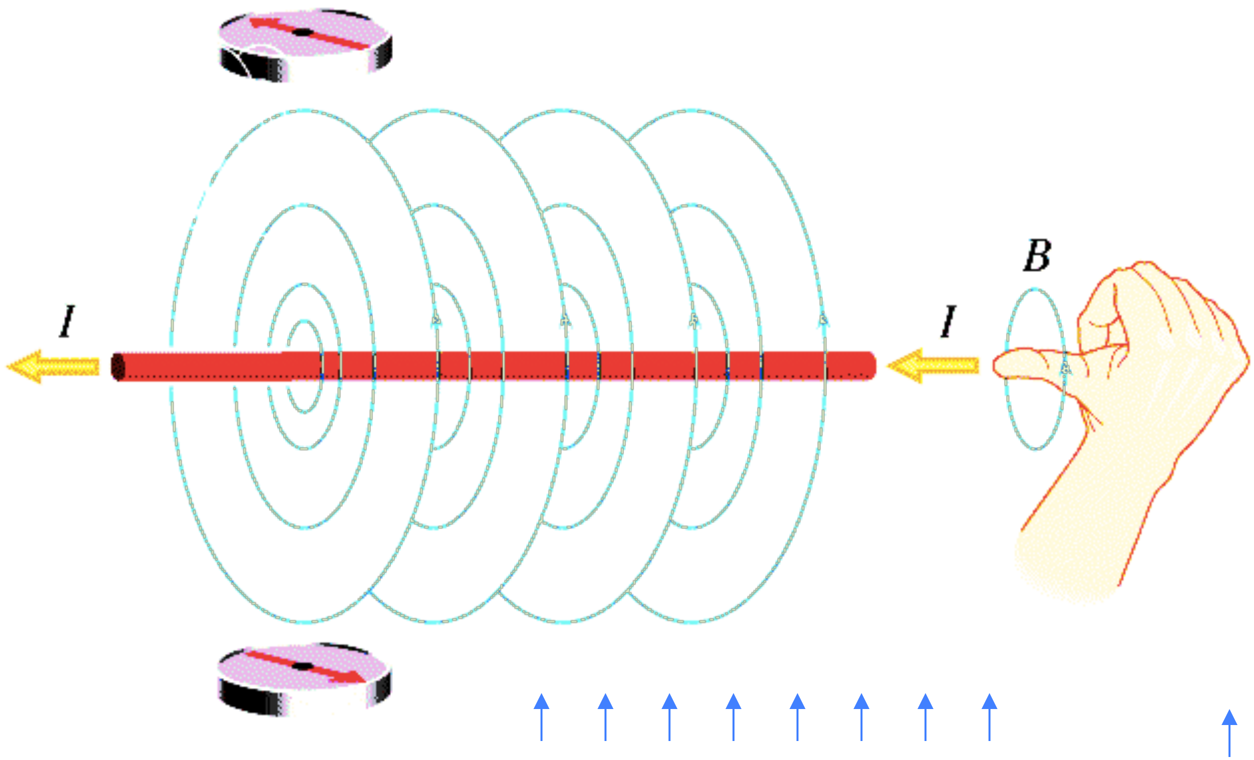
Magnetic Field Lines of a Bar Magnet

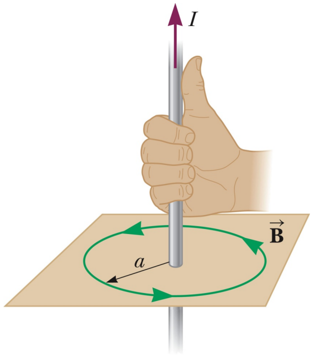




Magnetic Field Due to a Current-Carrying Wire

$$B = \frac{\mu_0 I}{2\pi a}$$



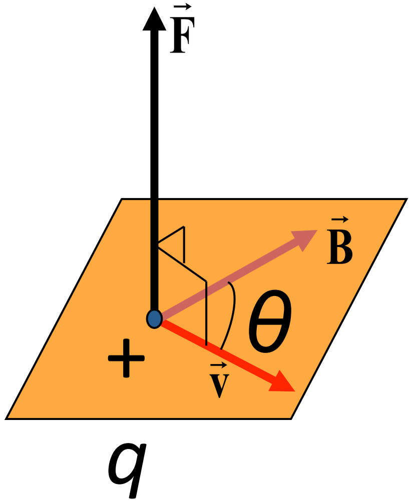


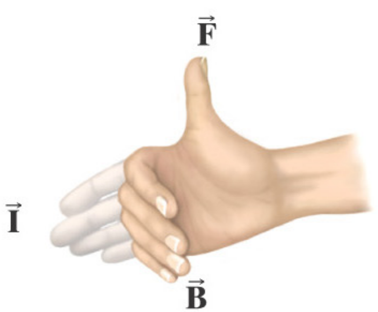
Remember, B is a vector. Fields from multiple wires ADD UP as vectors

Magnetic Forces and Direction of Forces

$$F = |q| \mathrm{v} B \sin\theta$$ ← Force on a point charge

$$F = IlB\sin(\theta)$$ ← Force on current

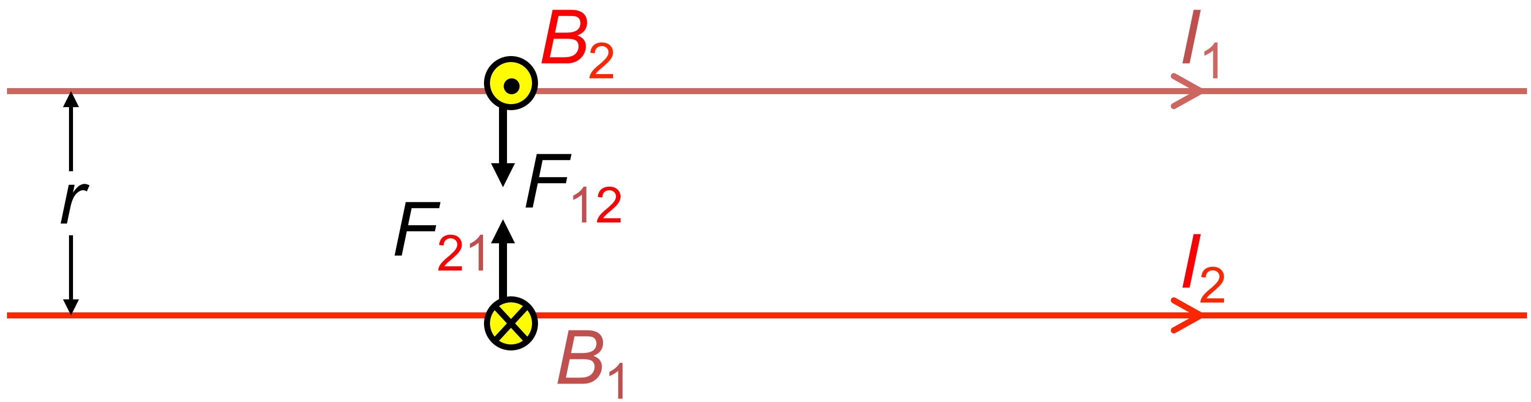




Right-hand rule

(c)

Magnetic Force Between two Parallel Conductors



The magnitude of the force on I_2 is:

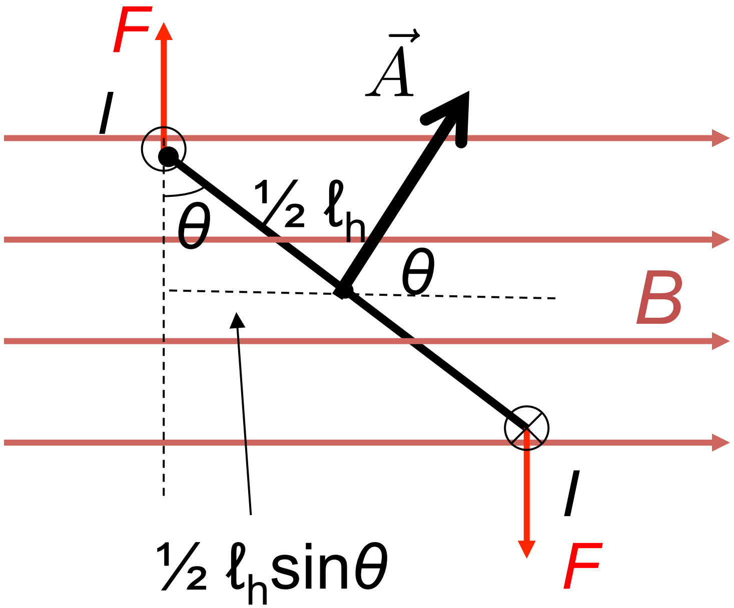
$$F_{21} = I_2 l B_1 \sin\theta = I_2 l \left(\frac{\mu_o I_1}{2\pi r}\right) = \frac{\mu_o l I_1 I_2}{2\pi r} = F_{12}$$

There is an attractive force between the two wires.

Force on a Rotating Current Loop –Torque

Torque on a loop:

$$\tau = NIAB\sin\theta$$
$$= N\mu B\sin\theta$$
$$= \mu_{\text{coil}}B\sin\theta$$



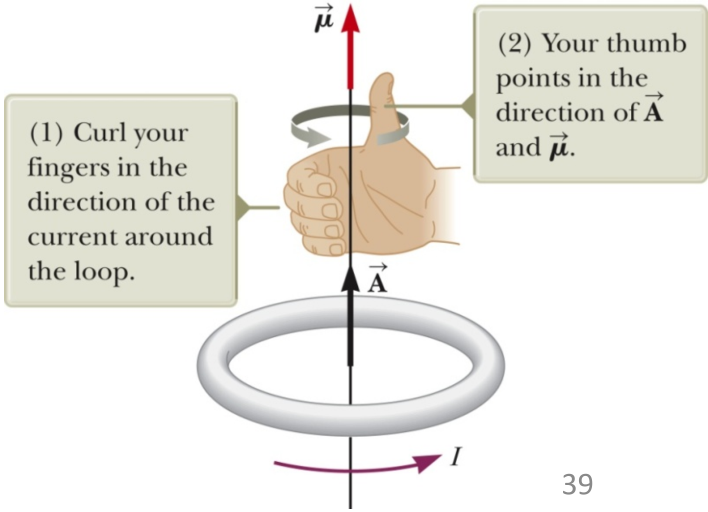
Magnetic dipole moment

$$\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}}$$

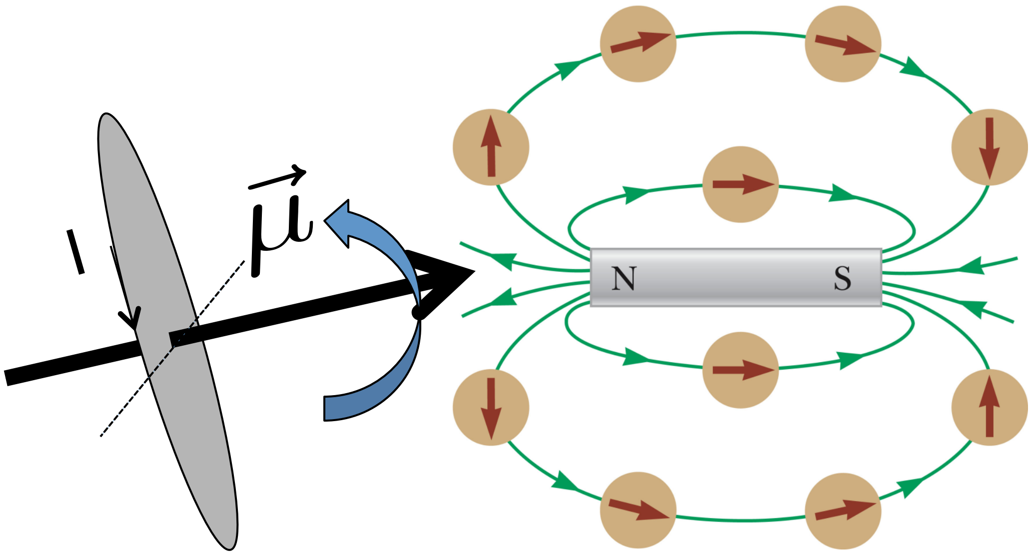
Unit [A ·m^2]

$$\vec{\boldsymbol{\mu}}_{\text{coil}} \equiv NI\vec{\mathbf{A}}$$

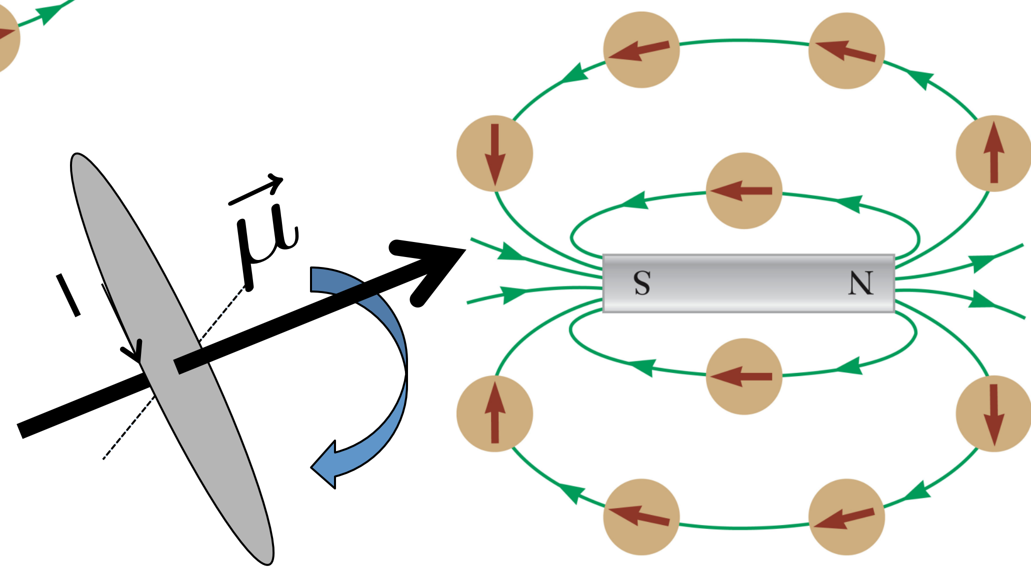
Magnetic moment of coil



Intuitively, Magnetic Moment is Like a Magnet



North faces north: current loop deflects counterclockwise



North faces south: current loop deflects clockwise to align

Vector symbol for torque

$$\vec{\tau} = \vec{\mu}_{\text{coil}} \times \vec{B}$$

Circular Motion of Charged Particles in *Uniform* B-Fields

The magnetic force acting on an electric charge is a centripetal force.

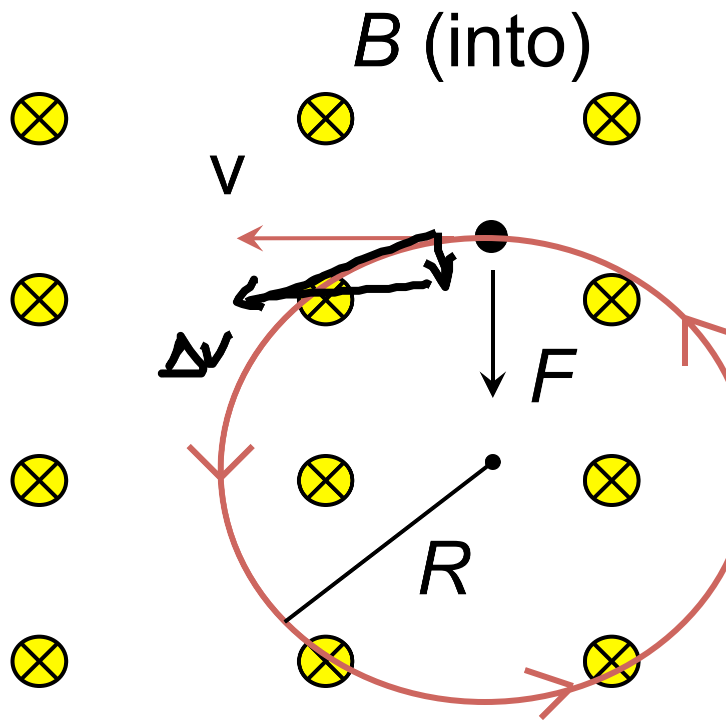
Its direction is always toward the centre of the circle.

$$F = ma_c = \frac{mv^2}{R}$$

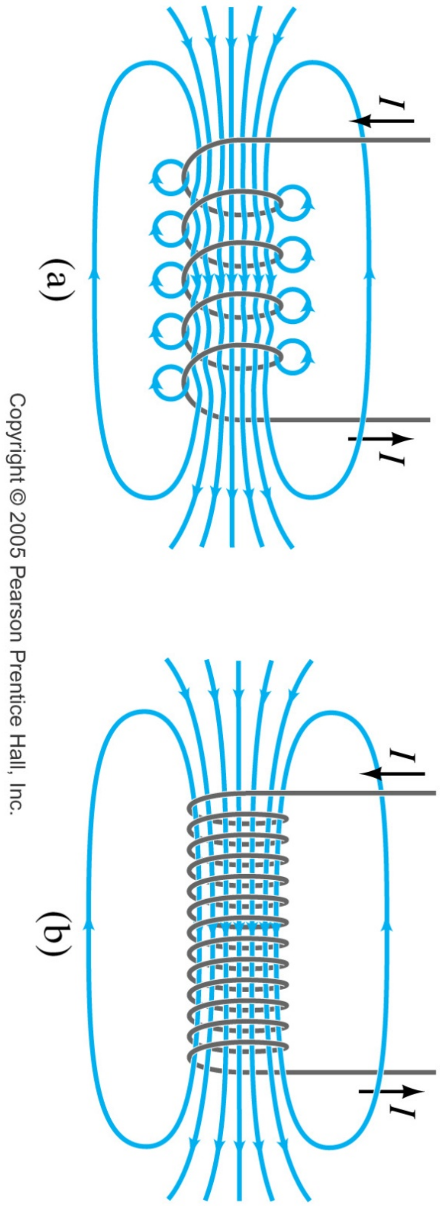
a_c : centripetal acceleration
R : the radius of the circular path

$$F = qvB = \frac{mv^2}{R}$$

Solving for R → $R = \frac{mv}{qB}$



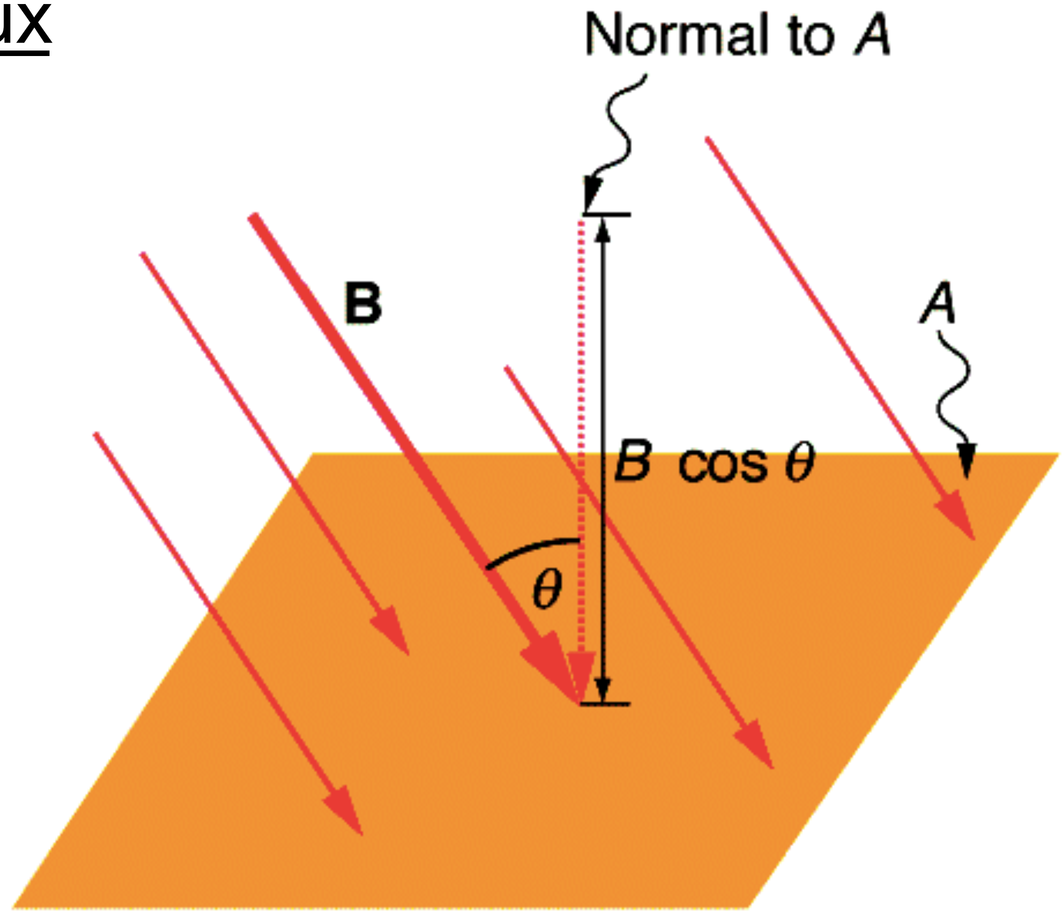
Magnetic Field of a solenoid



$$B = \mu_0 n I$$

Chapter 21

Magnetic Flux



$$\Phi = \sum \vec{B}_{||} dA = BA\cos\theta$$

$\sum \rightarrow$ rectangular areas through which B passes

$\vec{B}_{||} \rightarrow$ magnetic field parallel to normal pointing out of plane

$dA \rightarrow$ A

Faraday's Law of Induction

$$\epsilon = -\frac{\Delta \Phi_B}{\Delta t} = -\frac{\Delta(B\,A\cos\theta)}{\Delta t}$$ ✓

To produce/induce an EMF in a circuit, one must change one or more of the following:

(i) The B-field $\left(\frac{\Delta B}{\Delta t}\right)$

(ii) The area which is penetrated by the B-field $\left(\frac{\Delta A}{\Delta t}\right)$

(iii) Rotating the area in the presence of the B-field $\left(\frac{\Delta(\cos\theta)}{\Delta t}\right)$

(iv) If the circuit contains N loops, closely wrapped, the EMF of the circuit is the sum of that induced in each loop.

Faraday's Law of Induction and Ohm's Law

It is convenient to assume that during electromagnetic induction an EMF is induced in the conductor, as if an imaginary battery is introdued in the circuit.

If induced EMF is in a conductor making a closed circuit through a resistor, then the induced current will flow through the circuit according to Ohm's law:

$$\epsilon = I\,R$$

ϵ is the induced EMF

I is the induced current

R is the resistance of the circuit

Lenses Law -Direction of Induced EMF

$$\epsilon = -\frac{\Delta\Phi_B}{\Delta t}$$

$$I_{\text{ind}} = \frac{\epsilon}{R}$$

Direction: Lens's law:

- Increasing magnetic flux induces an EMF that opposes the increased flux
- Decreasing magnetic flux induces an EMF that compensates for the decreasing flux
- **In both cases RHR is used to "back out" what induced current direction is necessary to cause the required induced mag field (i.e. same or opposite directions of the existing field).**
- **Careful about multi-loop circuits**

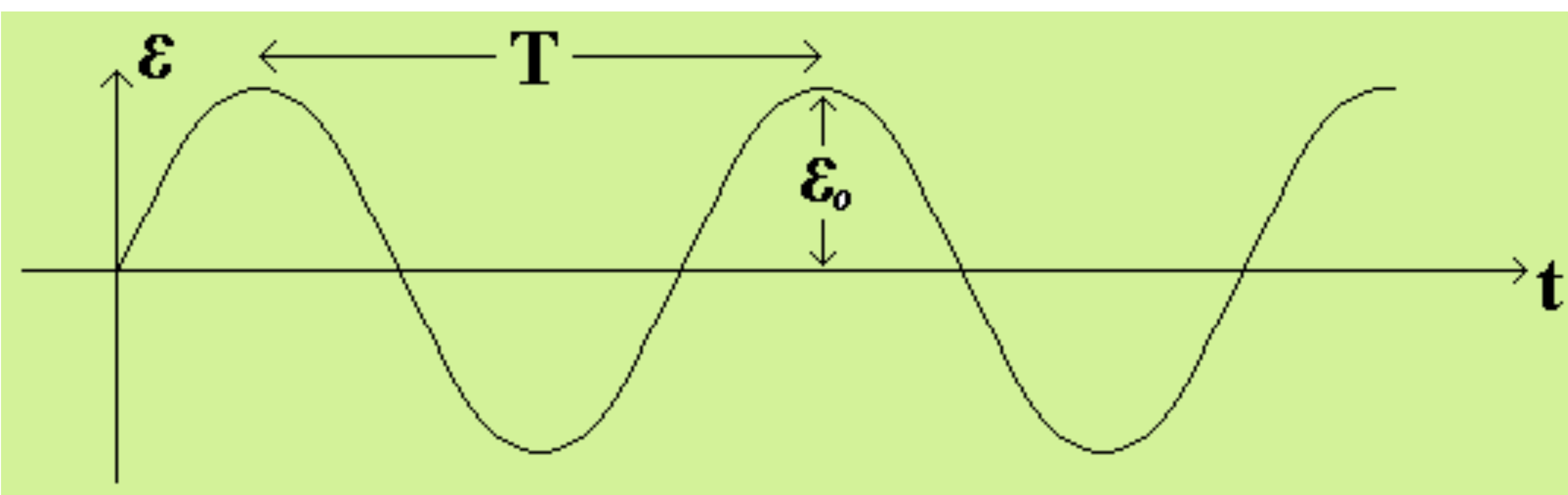
Induced EMF in a Loop -Generator

$$\varepsilon = -NBA\frac{d(\cos\theta)}{dt} = -NBA\frac{d(\cos\omega t)}{dt}$$

$$\varepsilon = \omega NBA\sin(\omega t)$$

$$\varepsilon = \epsilon_o \sin(\omega t) \Longrightarrow \epsilon_o = \omega NBA$$

This is called *alternating* EMF or simply an "*AC* voltage"



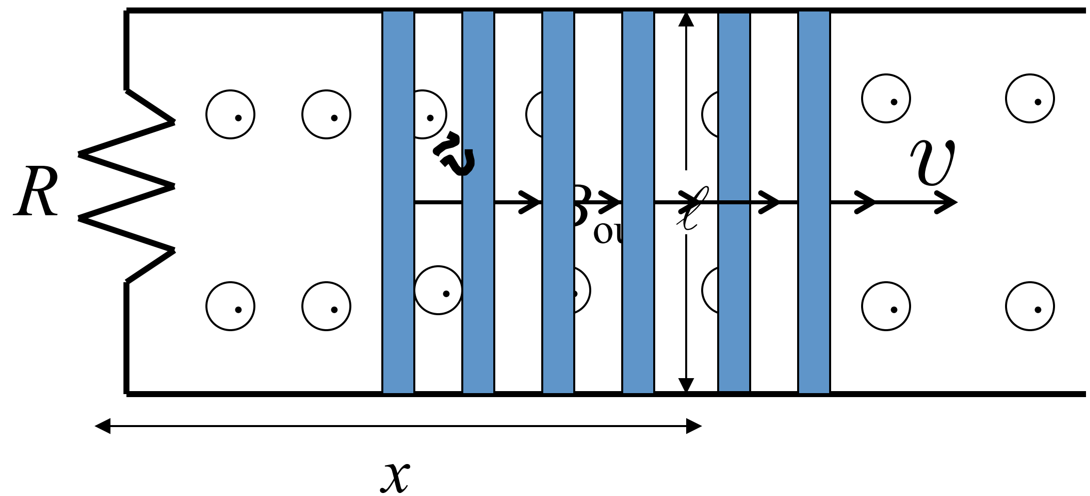
Moving Conductor Attached to a Circuit

Once again we have a moving bar in a magnetic field!

Now attach it to a circuit, and let the bar move on sliding rails….

Use Farady's law:

$$\Phi = BA$$



$$\epsilon = \frac{\Delta \Phi_B}{\Delta t}$$

$$= \frac{\Delta (BA)}{\Delta t}$$

$$= B\frac{\Delta A}{\Delta t}$$

$$= B\frac{\Delta (lx)}{\Delta t} = Bl\underbrace{\frac{\Delta x}{\Delta t}}_{v}$$

$$\Longrightarrow \; \epsilon = Blv$$

$$I_{\text{ind}} = \frac{\epsilon}{R} = \frac{Blv}{R}$$

Counter Torque or Counter Force

Induced currents in rotating loop and sliding bar feel a force by the very magnetic field that gave rise to them (same a the forces we studied in Ch 20). This force attempts to counter the process of induction that gave rise to them

- Sliding conductor branch in expanding circuit (previous example) will feel an induced force slowing it down. This can be called a "counter force"

- Similarly a rotating generator induces a current (two slides back) . This current also feels a torque the same way a "usual" current loop would. This torque will always be opposing the initial rotation of the generator. As such it is called a "counter torque.

Back EMF

- When we apply an external current in a loop and the loop feels a torque Ch 20). This torque causes a rotation of the loop, which in turn leads to an induced EMF as in page according to:

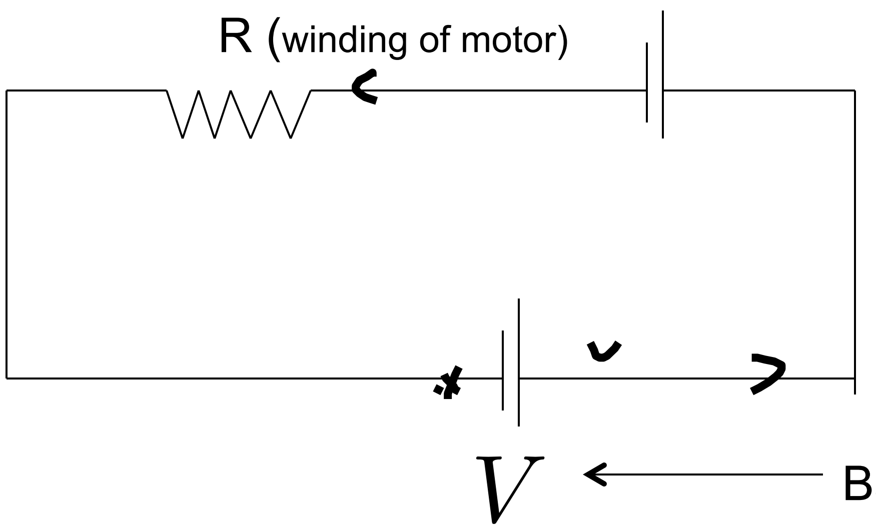
$$\epsilon = -\frac{d\Phi_B}{dt}$$

- This induced EMF attempts to resist the initial EMF that gave a current in the generator in the first place. This induced EMF is called a “BACK EMF”.

Modeling Back EMF in a Motor

While a motor rotates, an induced EMF arises in the coil. That induced EMF (back EMF) reduces the current provided by battery.

$$\epsilon_{\text{back EMF}} = \omega NBA \sin(\omega t)$$



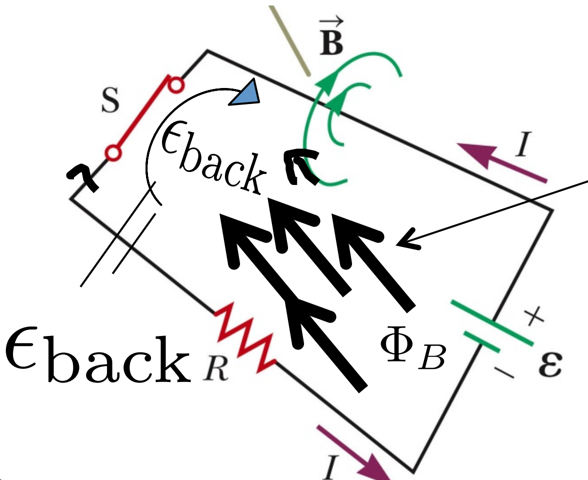
Battery or eternal source of EMF

$$I = \frac{V - \epsilon_{back}}{R}$$

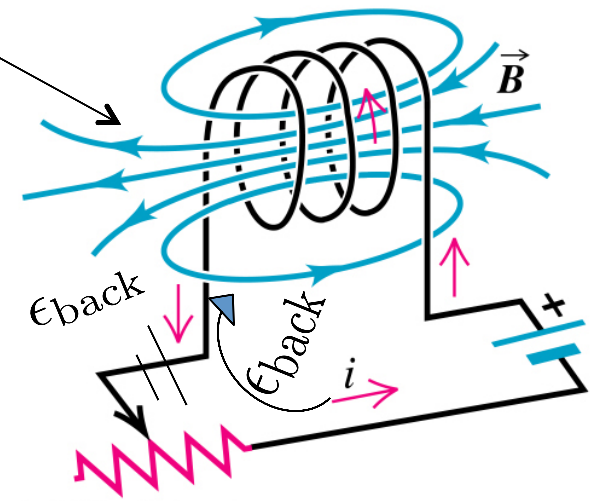
$$V - \epsilon_{\text{bemf}} - IR = 0$$

Self Inductance and Back EMF

As **current increases with time**, magnetic flux through the circuit loop due to this current also increases with time.



changing flux up through the circuit loop



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$\frac{d\Phi_B}{dt} \rightarrow 0$ as current reaches steady state

A time-varying current in a circuit produces an induced emf opposing the emf that initially set up the time-varying current. **This is a back emf**

Even if there is a single turn, there will be an induced emf in the coil as long as the current is changing.

Self Inductance and Induced (back) EMF

If the current, *I*, is changing then a back EMF will be induced in the coil.

$$\epsilon_L \propto -\frac{dI}{dt} \implies \epsilon_L = -L\frac{dI}{dt}$$

The proportionality constant *L* is called the inductance.
Its value depends on the geometry of the coil and the medium inside the coil.

SI units for *L*: V s / A → henry [H]

Any device that has an inductance is called an inductor.

The "−" sign in the expression of ϵ is due to Lenz's rule.
This induced EMF is also called back-EMF or counter EMF.

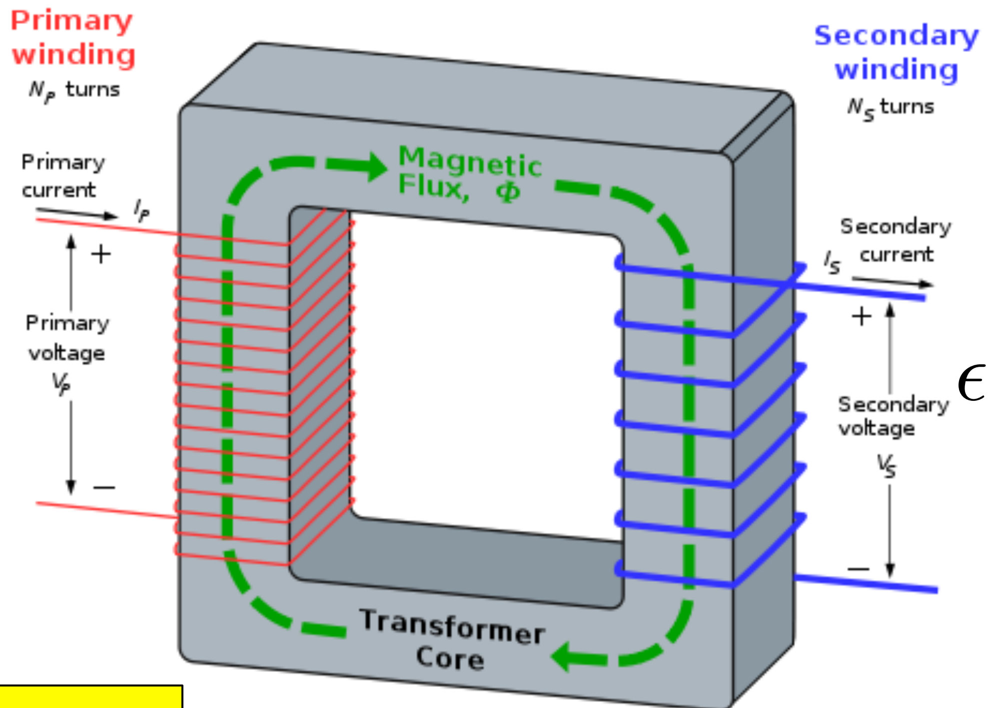
Transformers (21-7)

I_p induces flux in left coil windings, transferred through the core to the coils on the right side transformer core, inducing an EMF

Back EMF

$$\epsilon_1 = -N_p \frac{d\Phi}{dt}$$
$$= V_p$$

Supply voltage



$$\epsilon_2 = -N_s \frac{d\Phi}{dt}$$
$$= V_s$$

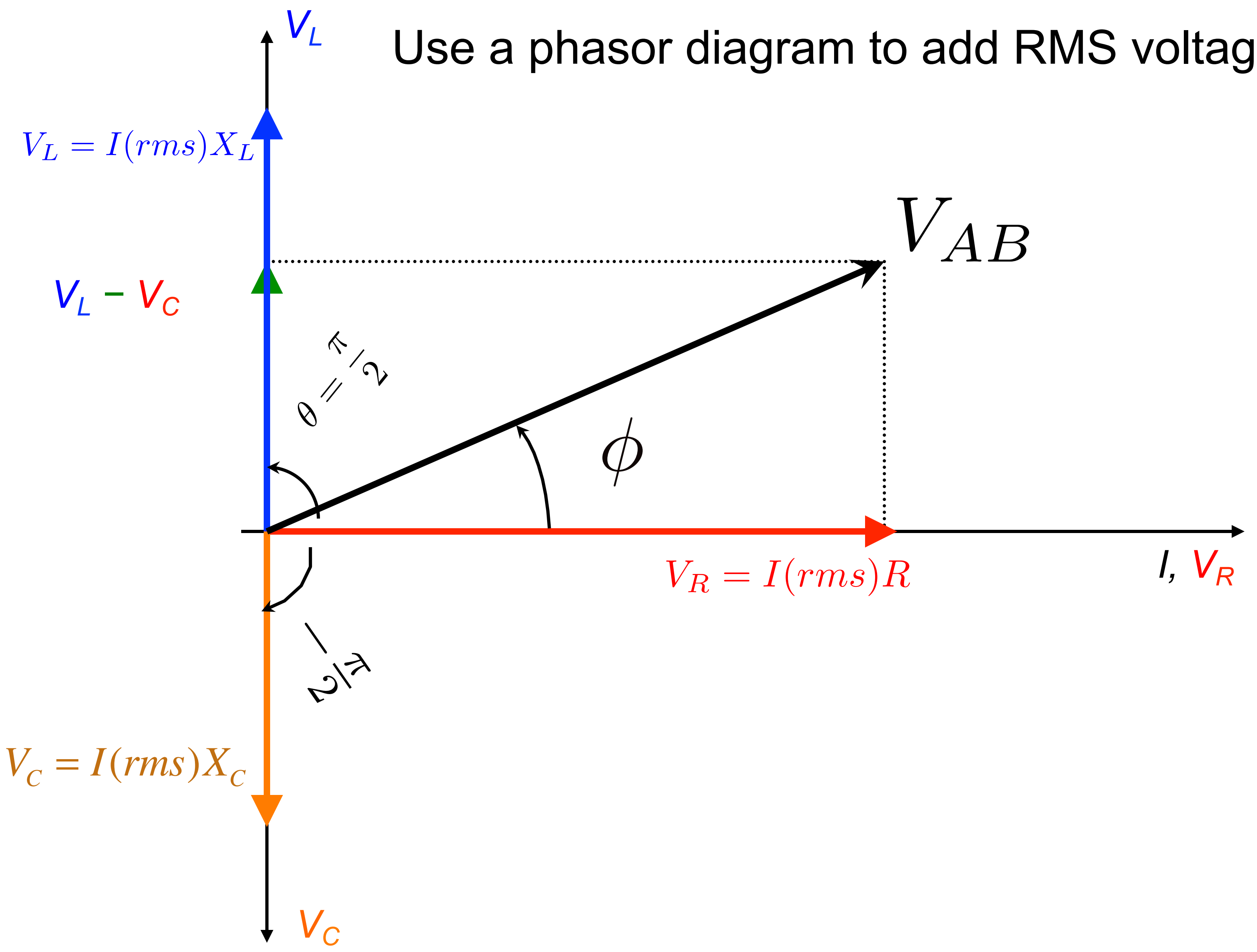
$$\left|\frac{\epsilon_1}{\epsilon_2}\right| = \left|\frac{V_p}{V_s}\right| = \frac{N_p}{N_s}$$

Since power on left side=power on right:

$$I_p V_p = I_s V_S \Rightarrow \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

Phasor Diagram for RLC Circuit

Use a phasor diagram to add RMS voltages

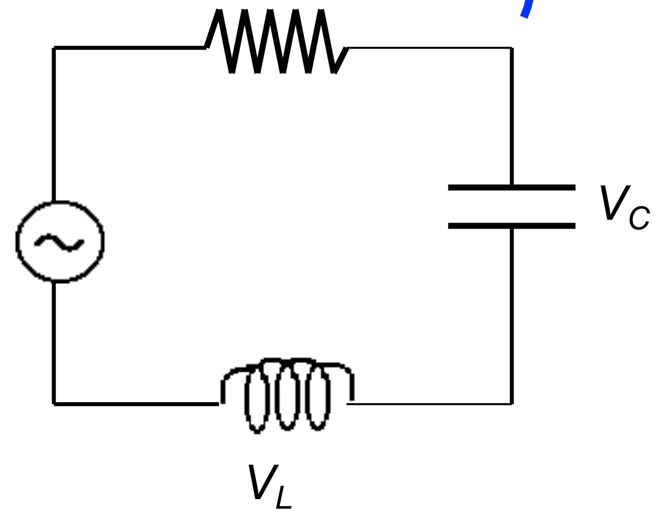


Resonance in RLC circuits

$$I_{rms} = \frac{V_{rms}}{Z(\omega)}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



For some specific angular frequency

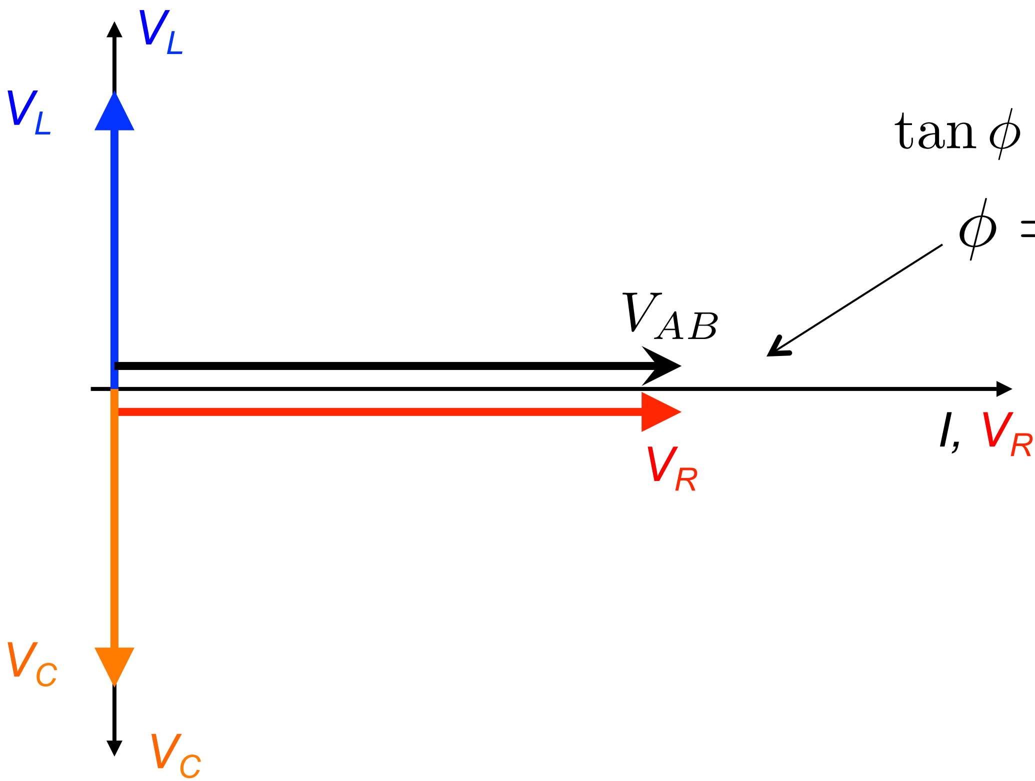
$$\omega L - \frac{1}{\omega C} = 0$$

→When this occurs, $Z = R$.

This gives the minimum value for the impedance, and the maximum current.

Condition for Resonance in RLC circuits: Phasor Diagram

The phase angle between current and voltage will be zero



$$\tan\phi = \frac{V_L - V_C}{V_R} = 0$$

$$\phi = 0$$

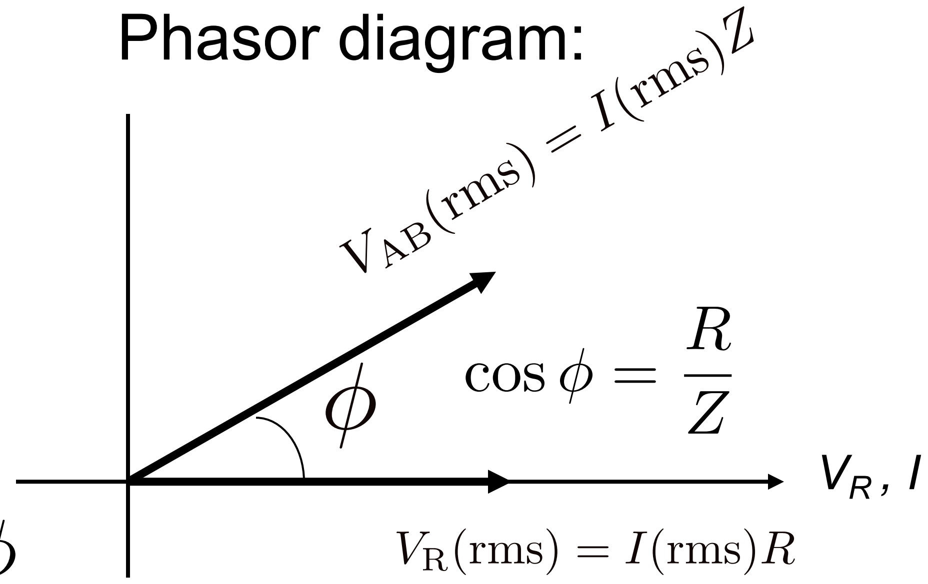
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$(X_L - X_C) = 0$$

$$Z = R$$

Power Factor of Circuit

Phasor diagram:



$$\implies \bar{P} = I_{\mathrm{rms}}^2 R$$

$$= I_{\mathrm{rms}}^2 (Z\cos\phi)$$

$$= I_{\mathrm{rms}}(I_{\mathrm{rms}} Z)\cos\phi$$

$$\bar{P} = I(\mathrm{rms})V_{\mathrm{AB}}(\mathrm{rms})\cos\phi$$

Do Example 16 & 17 in Ch 21 notes!

$\cos\phi$ is called the power factor.

At resonance, the power factor, $\cos\phi = 1$

(i.e. maximum power delivered to the resistor/device)