ASSIGNMENT 4 (DUE Tuesday, March 8h 2011)

PROBLEM 1 (15 points)

During a wind tunnel test on a sphere of radius, $\boldsymbol{r}$ = 150mm it is found that the velocity of flow $\boldsymbol{u}$ along the longitudinal axis of the tunnel passing through the centre of the sphere at a point upstream which is a distance $\boldsymbol{x}$ from the centre of the sphere is given by

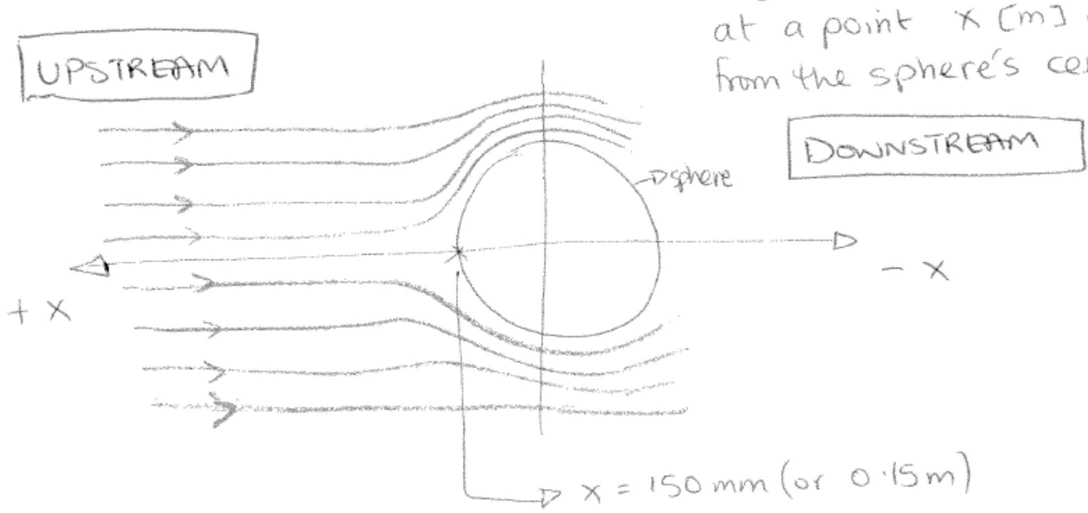
$$u = U_0\left(1 - \frac{r^3}{x^3}\right)$$

Where $\boldsymbol{U_0}$ is the mean velocity of the undisturbed airstream. If $\boldsymbol{U_0}$ = 60m/s, what is the convective acceleration when the distance $\boldsymbol{x}$ is (a) 300mm, (b) 150mm.

Given
- * Wind tunnel test
- * Object in the wind tunnel is a sphere ($r = 0{\cdot}15$m)
- * $u = U_0\left(1 - \frac{r^3}{x^3}\right)$
- * $U_0 = 60$ m/s

x: distance upstream from sphere's centre

u: velocity of flow along longitudinal axis (i.e. @ r=0), at a point x [m] away from the sphere's centre.



$$a_x = a_x)_{convective} + \underbrace{a_x)_{local}}_{=0}$$

$a_x)_{local} = 0$ since flow is steady $\left(\frac{\partial u}{\partial t} = 0\right)$
$[u \neq f(t)]$

Also, only the convective component of acceleration is required, so the local acceleration is irrelevant

$$\therefore\ a_x = u \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left[U_0 - U_0 r^3/x^3\right]$$

$$= \frac{3U_0 r^3}{x^4}$$

$$a_x = U_0\left(1 - \frac{r^3}{x^3}\right) * \frac{3U_0 r^3}{x^4}$$

$$\boxed{a_x = \frac{3U_0^2 r^3}{x^4}\left(1 - r^3/x^3\right)}$$

(a) a_x when $x = 300mm$ (i.e. $0.3\ m$) Very Important!!
has to be same units as $[r]$ → see eq$^{\underline{n}}$ above

$$a_x = \frac{3(60)^2(0.15^3)}{(0.3^4)}\left(1 - 0.15^3/0.3^3\right)$$

$$= 3,937.5\ m/s$$

(b) a_x when $x = 0.15m$ (i.e. $x = r$)

$$a_x = \frac{3(60)^2(0.15^3)}{0.15^4}\left(1 - \frac{0.15^3}{0.15^3}\right)$$

$= 0$ m/s $\Rightarrow$ makes sense since there is no flow @ $x = r$ (stagnation)

Even $u = 0$ @ $x = r$ due to stagnation

PROBLEM 2 (20 points)

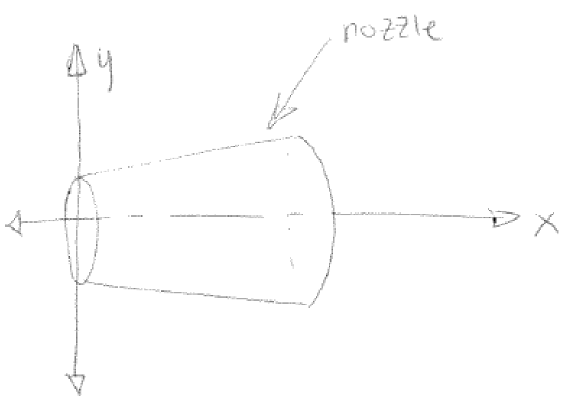
The velocity along the centreline of a nozzle of length ***L*** is given by

$$u = 2t\left(1 - 0.5\frac{x}{L}\right)^2$$

Where ***u*** is the velocity in m/s, ***t*** is the time in seconds from the commencement of flow; ***x*** is the distance from the inlet to the nozzle. Find the convective acceleration and the local acceleration when t = 3s, x = 0.5L and L = 0.8m

* Nozzle length $\Rightarrow L = 0.8m$
* $u = 2t\left(1 - 0.5\frac{x}{L}\right)^2$

u: velocity along centreline of nozzle in m/s (ie. y=0)
t: time
L: Nozzle length
x: distance away from nozzle inlet.



$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$ [1-dimensional flow, at centre line]

$$\frac{\partial u}{\partial x} = \frac{-2t}{L}\left(1 - 0.5\frac{x}{L}\right)$$

$$\frac{\partial u}{\partial t} = 2\left(1 - 0.5\frac{x}{L}\right)^2$$

Convective Acceleration

$$a_x)_{conv.} = u\frac{\partial u}{\partial x} = 2t\left(1 - 0.5\frac{x}{L}\right)^2 * -\frac{2t}{L}\left(1 - 0.5\frac{x}{L}\right)$$

$$= -\frac{4t^2}{L}\left(1 - 0.5\frac{x}{L}\right)^3$$

$$= -\frac{4(3^2)}{0.8}\left(1 - 0.5\frac{(0.5\cancel{L})}{\cancel{L}}\right)^3 = -18.98\,m/s^2$$

Local Acceleration

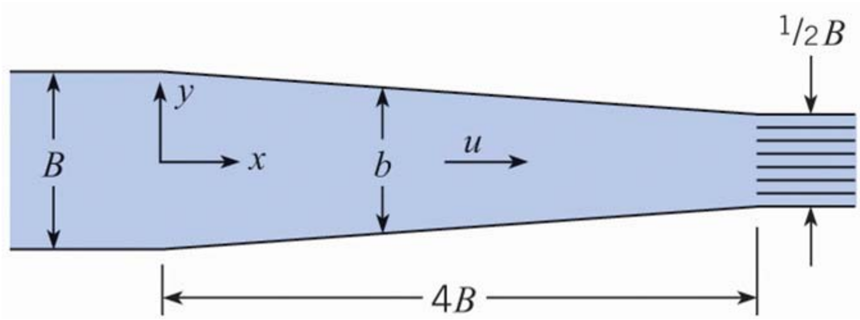
$$a_x)_{local} = \frac{\partial u}{\partial t} = 2\left(1 - 0.5\frac{x}{L}\right)^2$$

$$= 2\left(1 - 0.5\frac{(0.5\cancel{L})}{\cancel{L}}\right)^2$$

$$= 1.125\,m/s^2$$

PROBLEM 3 (15 points)

Liquid flows through this two-dimensional slot with a velocity of $V = 2\,(q_0/b)(t/t_0)$, where q_0 and t_0 are reference values. What will be the local acceleration at $x = 2B$ and $y = 0$ in terms of B, t, t_0 and q_0? **Hint:** Begin by identifying which are the variables and which are the constants before solving the problem.



SOLUTION

$$V = 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right)$$ But @ **x = 2B, b = 3B/4**

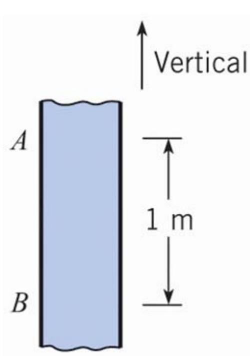
$$V = \left(\frac{8q_o}{3B}\right)\left(\frac{t}{t_o}\right)$$

$$a_l = \frac{\partial V}{\partial t}$$

$$a_l = \frac{8q_o}{3Bt_o}$$

PROBLEM 4 (15 points)

The hypothetical liquid in the tube shown in the figure has zero viscosity and a specific weight of 10 kN/m^3. If p_B-p_A is equal to 12 kPa, one can conclude that the liquid in the tube is being accelerated (a) upward, (b) downward, or (c) neither (*i.e.* acceleration = 0)?



$\boxed{\text{SOLUTION}}$ Euler's equation

$$\begin{aligned} \rho a_\ell &= -\frac{\partial}{\partial \ell}(p + \gamma z) \\ a_\ell &= \frac{1}{\rho}\left(-\frac{\partial p}{\partial \ell} - \gamma \frac{\partial z}{\partial \ell}\right) \end{aligned}$$

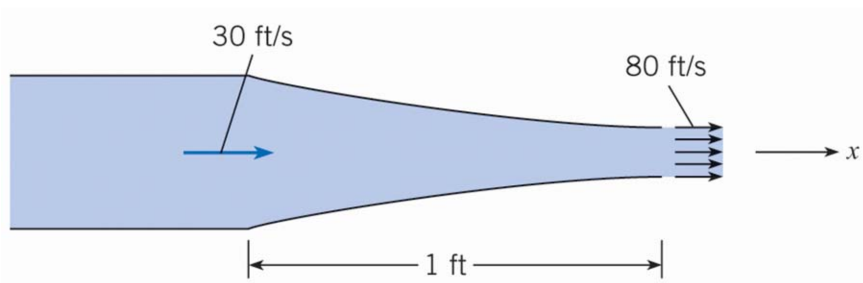
Let ℓ be positive upward. Then $\partial z/\partial \ell = +1$ and $\partial p/\partial \ell = (p_A - p_B)/1 = -12{,}000$ Pa/m. Thus

$$\begin{aligned} a_\ell &= \frac{g}{\gamma}(12{,}000 - \gamma) \\ a_\ell &= g\left(\frac{12{,}000}{\gamma} - 1\right) \\ a_\ell &= g(1.2 - 1.0) \text{ m/s}^2 \end{aligned}$$

a_ℓ has a positive value; therefore, acceleration is upward. Correct answer is $\boxed{\text{a).}}$

PROBLEM 5 (15 points)

If the velocity varies linearly with distance through this water nozzle, what is the pressure gradient ***dp/dx***, halfway through the nozzle? ($\rho = 62.4$ lb_m/ft^3)



SOLUTION

Euler's equation

$$\frac{\partial}{\partial x}(p+\gamma z) = -\rho a_x$$

but z =const.; therefore

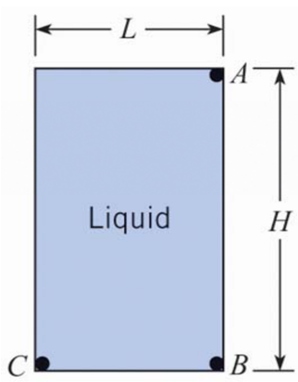
$$\frac{\partial p}{\partial x} = -\rho a_x$$
$$a_x = a_{\text{convective}} = V\frac{\partial V}{\partial x}$$
$$\frac{\partial V}{\partial x} = (80-30)/1 = 50\ \text{s}^{-1}$$
$$V_{\text{mid}} = (80\,\text{ft/s} + 30\,\text{ft/s})/2 = 55\ \text{ft/s}$$
$$= (55\ \text{ft/s})(50\ \text{ft/s/ft}) = 2,750\ \text{ft/s}^2$$

Finally

$$\frac{\partial p}{\partial x} = (-1.94\ \text{slug/ft}^3)(2,750\ \text{ft/s}^2)$$
$$\boxed{\frac{\partial p}{\partial x} = -5,330\ \text{psf/ft}}$$

PROBLEM 6 (20 points)

The closed tank shown, which is full of liquid, is accelerated downward at ***(2/3)g*** and to the right at ***1g***. Here ***L*** = *2.5m*, *H* = *3m*, and the liquid has a specific gravity of 1.3. Determine p_C-p_A and p_B-p_A.



SOLUTION

Euler's equation in z direction

$$
\begin{aligned}
\frac{dp}{dz} + \gamma &= -\rho a_z \\
\frac{dp}{dz} &= -\rho(g + a_z) \\
\frac{dp}{dz} &= -1.3\left(1,000\,\mathrm{kg/\,m^3}\right)\left(9.81\,\mathrm{m/\,s^2} - 6.54\,\mathrm{m/\,s^2}\right) \\
&= -4,251\ \mathrm{N/m^3} \\
p_B - p_A &= \left(4,251\,\mathrm{N/\,m^3}\right)(3\,\mathrm{m}) \\
&= 12,753\ \mathrm{Pa} \\
&\boxed{p_B - p_A = 12.7\ \mathrm{kPa}}
\end{aligned}
$$

Euler's equation in x-direction

$$
\begin{aligned}
-\frac{dp}{dx} &= \rho a_x \\
p_C - p_B &= \rho a_x L \\
&= 1.3 \times 1,000 \times 9.81 \times 2.5 \\
&= 31,882\ \mathrm{Pa} \\
p_C - p_A &= p_C - p_B + (p_B - p_A) \\
p_C - p_A &= 31,882 + 12,753 \\
&= 44,635\ \mathrm{Pa} \\
&\boxed{p_C - p_A = 44.6\ \mathrm{kPa}}
\end{aligned}
$$