MATH 1005 A **Name:**________________________

Sample Final **Student Number:**________________________

Recall that only **non-programmable** calculators are permitted. Books and/or notes are not permitted.

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1. [**6 marks**] Consider the first-order equation

$$\frac{x}{x^2+y^2+1} - 2x + \left(\frac{y}{x^2+y^2+1} + 2y\right)\frac{dy}{dx} = 0 \tag{1}$$

   (a) Show that equation (1) is exact.

   (b) Find the general solution to (1).

2. [**9 marks**] Consider the second-order equation

$$y'' - 6y' + 9y = 4e^{3x}. \tag{2}$$

(a) Find the general solution to the corresponding homogenous equation.

(b) Find a particular solution to (2) using the method of undetermined coefficients.

(c) What is the general solution to (2)?

3. [**12 marks**] Consider the second-order equation

$$x^2y'' + 5xy' + 3y = 4xe^{x^2}, \quad x > 0. \tag{3}$$

(a) Find the general solution to the corresponding homogenous equation.

(b) Find a particular solution to (3) using variation of parameters.

(c) What is the general solution to (3)?

4. [**6 marks**] Find a fundamental matrix for the linear system

$$\mathbf{x}' = A\mathbf{x}; \quad A = \begin{pmatrix} 3 & 1 \\ -4 & -2 \end{pmatrix}.$$

5. [**6 marks**] Consider the series

$$\sum_{n=1}^{\infty} \frac{\ln(n)^3}{n^2}. \tag{4}$$

(a) Show that the sequence $a_n = \frac{\ln(n)^3}{\sqrt{n}}$, $n \geq 1$, converges to 0 (note that $a_n \neq \frac{\ln(n)^3}{n^2}$).

(b) Use part (a) to show that the series (4) converges.

6. [**7 marks**] Consider the function $f(x) = \cos(\sqrt{x})$.

(a) Compute the Taylor series for $f(x)$ centered at $a = 0$. What is its radius of convergence?

(b) Approximate the integral

$$\int_0^1 f(x)dx$$

using a second degree Taylor approximation. Show that your answer is correct to 3 decimal places.

7. [**8 marks**] Consider the second-order equation

$$y'' + xy' - 2y = 0. \tag{5}$$

(a) Find the coefficient recursion relation for the general series solution about $x_0 = 0$.

(b) Solve the recursion relation to obtain the general solution to (5).