

ASSIGNMENT 3 (DUE Tuesday, March 1st 2011)**PROBLEM 1 (25 points)**

A piece of wood of relative density 0.651 is 76 mm square and 1.5 m long. How many kilograms of lead (density 11.2 Mg/m^3) must be fastened at one end of the stick so that it will float upright with 0.3 m out of water?

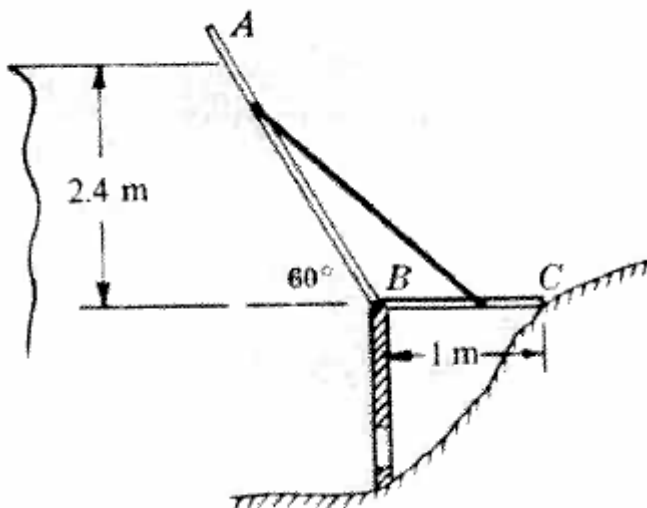
Solution:

$$\begin{aligned} \text{Total weight of wood and lead} &= \text{weight of displaced water} \\ [651 \times 1.5(0.076)^2 + 11\,200t] &= 1000[(0.076)^2 \times 1.2 + t] \end{aligned}$$

from which $t = 0.000\,126\,6 \text{ m}^3$ and mass of lead $= 11.2 \times 10^3 t = 1.418 \text{ kg}$

PROBLEM 2 (25 points)

In the figure bellow, gate **ABC** is hinged at **B** and is 1.2 m long. Neglecting the weight of the gate, determine the unbalanced moment due to the water acting on the gate.



Solution:

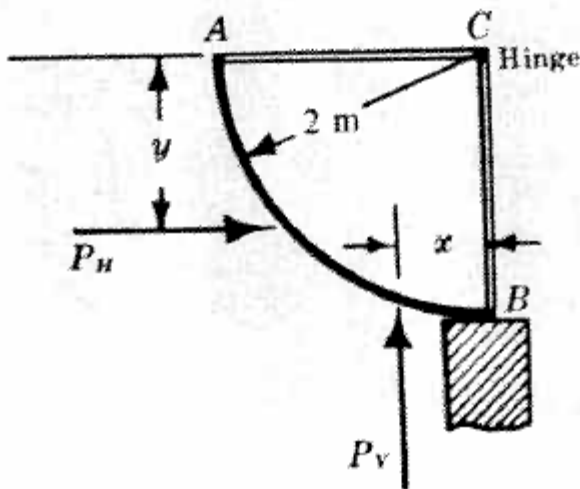
$P_{AB} = 9810(1.2)(2.77 \times 1.2) = 39\,100\text{ N}$, acting $\frac{2}{3}(2.77) = 1.85\text{ m}$ from A .

$P_{BC} = 9810(2.4)(1 \times 1.2) = 28\,200\text{ N}$, acting at the center of gravity of BC since the pressure on BC is uniform. Taking moments about B , (clockwise plus).

$$\begin{aligned}\text{Unbalanced moment} &= + 39\,100 \times 0.92 - 28\,200 \times 0.5 \\ &= + 21\,900\text{ Nm}\end{aligned}$$

PROBLEM 3 (25 points)

Determine and locate the components of the force due to the water acting on the curved area AB in the figure below, per meter of its length.

**Solution:**

$$\begin{aligned}P_H &= \text{force on vertical projection } CB = \rho g h_{cg} A_{CB} \\ &= 9810(1)(2 \times 1) = 19\,620\text{ N} \quad \text{acting } \frac{2}{3}(2) = \frac{4}{3}\text{ m from } C\end{aligned}$$

$$P_V = \text{weight of water above area } AB = 9810(\pi 2^2/4 \times 1) = 30\,820\text{ N}$$

acting through the center of gravity of the volume of liquid. The center of gravity of a quadrant of a circle is located at a distance $4/3 \times r/\pi$ from either mutually perpendicular radius. Thus

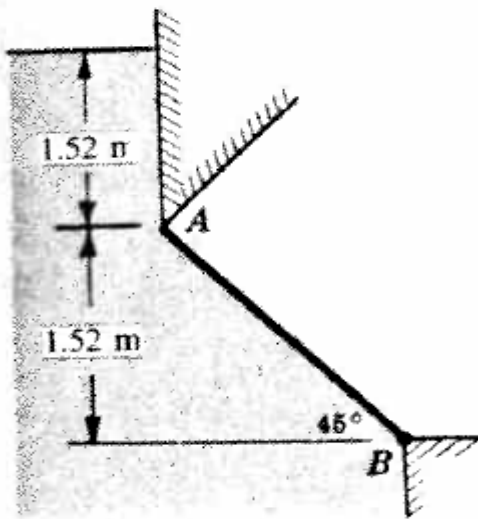
$$x_{cg} = 4/3 \times 2/\pi = 0.85\text{ m to the left of line } BC$$

Note: Each force dP acts normal to the curve AB and would therefore pass through hinge C upon extended. The total force should also pass through C . In order to confirm this statement, take moments components about C , as follows.

$$\Sigma M_C = -19\,620 \times \frac{4}{3} + 30\,820 \times 0.85 = 0 \quad (\text{satisfied})$$

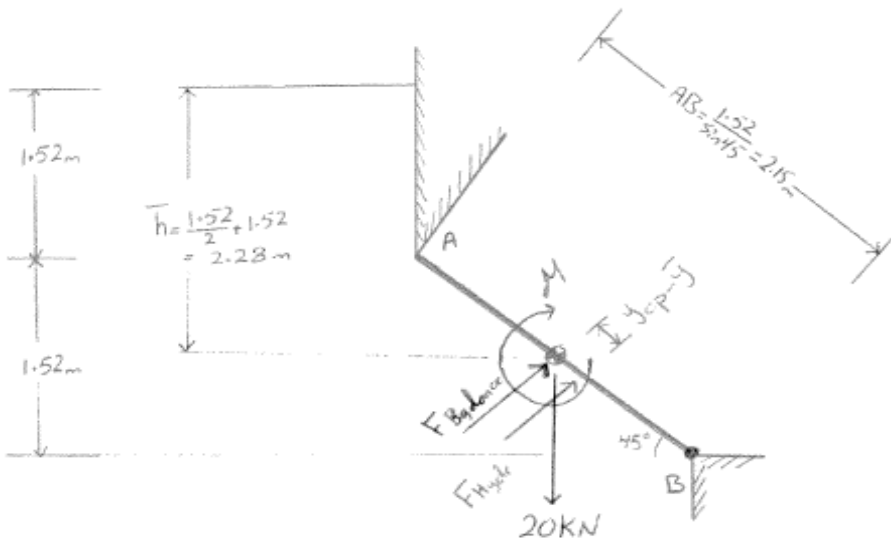
PROBLEM 4 (25 points)

In the figure below, the gate **AB** is hinged at **B** and is 1.2 m wide. What vertical force applied at the center of gravity of the 20 kN gate, will keep it in equilibrium?



Problem 4.

The gate AB is hinged
the gate is 1.2 m wide
the gate is 20 kN (wt.)



$$\begin{aligned}
 F_{Hydr} &= \bar{P}A \\
 &= \gamma \bar{h} A \\
 &= 9810 \cdot 2.28 \cdot (1.2 \cdot 2.15) \\
 &= \underline{\underline{57.706 \text{ kN}}}
 \end{aligned}$$

$$\bullet \quad y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$$

$$y_{cp} - \bar{y} = \frac{0.994}{3.225(1.2 \cdot 2.15)} = \boxed{0.1194 \text{ m}}$$

$$\bar{y} = \frac{2.15}{2} + \frac{1.52}{\sin 45^\circ} = 3.225 \text{ m}$$

$$\bar{I}_x = \frac{bh^3}{12} = \frac{1.2(2.15)^3}{12} = 0.994 \text{ m}^4$$

$$\bullet \quad \left(\sum M_B \right) = M + F_{\text{hydr}} \cdot \left(\frac{AB}{2} - (y_{cp} - \bar{y}) \right) - 20 \text{ kN} (\cos 45^\circ) \left(\frac{AB}{2} \right) = 0$$

$$M = -39.94 \text{ kN}\cdot\text{m}$$

$$F_{\text{Balance}} = \frac{M}{(AB/2)} = 37.15 \text{ kN}$$

$$F_{\text{vertical}} = \frac{F_{\text{Balance}}}{(\cos 45^\circ)} = \boxed{52.5 \text{ kN}}$$