ASSIGNMENT 1 (SOLUTION)

(DUE Friday, January 21st 2011)

PROBLEM 1 (15 points)

At 32ºC and 2 bar absolute the specific volume $\boldsymbol{v_s}$ of a certain gas was 0.74 m^3 /kg. Determine the gas constant ***R*** and the density $\boldsymbol{\rho}$.

Solution:

$$\text{Since } \rho = \frac{p}{RT}, \text{ then } R = \frac{p}{\rho T} = \frac{pv_s}{T} = \frac{(2 \times 10^5)(0.74)}{(273 + 32)} = 485.2$$

$$\text{Density } \rho = \frac{1}{v_s} = \frac{1}{0.74} = 1.35 \text{ kg/m}^3$$

PROBLEM 2 (20 points)

(a) Find the change in volume of 1.00 m^3 of water at 26.7ºC when subjected to a pressure increase of 20 bar. (b) From the following test data determine the bulk modulus of elasticity of water: at 35 bar the volume was 1.000 m^3 and at 240 bar the volume was 0.990 m^3.

Solution:

(*a*) From Table 1C in the Appendix, E at 26.7°C is 2.24×10^9 Pa. Using formula (12),

$$dv = -\frac{v\,dp'}{E} = -\frac{1.00 \times 20 \times 10^5}{2.24 \times 10^9} = -0.000\,89 \text{ m}^3$$

(*b*) The definition associated with formula (12) indicates that *corresponding* changes in pressure and volume must be considered. Here an increase in pressure corresponds to a decrease in volume.

$$E = -\frac{dp'}{dv/v} = -\frac{(240 - 35)10^5}{(0.990 - 1.000)/1.000} = 2.05 \times 10^9 \text{ Pa} = 2.05 \text{ GPa}$$

PROBLEM 3 (15 points)

Convert 15.14 poises to kinematic viscosity in m^2 /s units if the liquid has relative density 0.964.

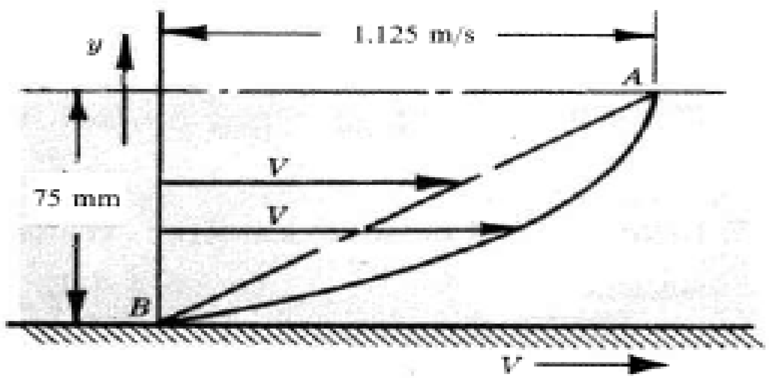
Solution:

The steps illustrated in Problem 6 may be taken or an additional factor may be established for water from

$$\frac{1}{10} \times \frac{1}{1000} = 10^{-4} = 0.001\,078. \text{ Hence } \nu \text{ in m}^2/\text{s} = \frac{15.14 \times 10^{-4}}{\text{rl dn} = 0.964} = 1.57 \times 10^{-3}$$

PROBLEM 4 (30 points)

A fluid has absolute viscosity 0.048 Pa s and relative density 0.913.Calculate the velocity gradient and the intensity of shear stress at the boundary and at points 25 mm, 50 mm, and 75 mm from the boundary, assuming (a) a straight line velocity distribution and (b) a parabolic velocity distribution. The parabola in the sketch has its vertex at ***A***. Origin is at ***B***.



Solution:

(*a*) For the straight line assumption, the relation between velocity and distance y is $V = 15y$. Then $dV = 15\,dy$, or the velocity gradient is $dV/dy = 15$.

$$\text{For } y = 0,\ V = 0,\ dV/dy = 15\ \text{sec}^{-1} \text{ and}$$
$$\tau = \mu(dV/dy) = 0.048 \times 15 = 0.72\ \text{Pa}$$

Similarly, for other values of y we also obtain $\tau = 0.72$ Pa.

(*b*) The equation of the parabola must satisfy the condition that the velocity is zero at the boundary B. The equation of the parabola is $V = 1.125 - 200(0.075 - y)^2$. Then $dV/dy = 400(0.075 - y)$ and tabulation of results yields the following:

y	V	dV/dy	$\tau = 0.048(dV/dy)$
0	0	30	1.44 Pa
0.025	0.625	20	0.96 Pa
0.050	0.880	10	0.48 Pa
0.075	1.125	0	0

It will be observed that where the velocity gradient is zero (which occurs at the center line of a pipe flowing under pressure, as will be seen later) the shear stress is also zero.

Note that the units of velocity gradient are s^{-1}, and so the product $\mu(dV/dy) = (\text{Pa s})(\text{s}^{-1}) = \text{Pa}$, the correct dimensions of shear stress τ.

PROBLEM 5 (20 points)

A cylinder of 0.12 m radius rotates concentrically inside of a fixed cylinder of 0.13 m radius. Both cylinders are 0.3 m long. Determine the viscosity of the liquid which fills the space between the cylinders if a torque of 0.880 Nm is required to maintain an angular velocity of 2π.

Solution:

(*a*) The torque is transmitted through the fluid layers to the outer cylinder. Since the gap between the cylinders is small, the calculation may be made without integration.

Tangential velocity of the inner cylinder $= r\omega = (0.12\ \text{m})(2\pi\ \text{rad/s}) = 0.754\ \text{m/s}$.

For the small space between cylinders, the velocity gradient may be assumed to be a straight line and the mean radius can be used. Then $dV/dy = 0.754/(0.13 - 0.12) = 75.4\ (\text{m/s})/\text{m}$ or s^{-1}.

The torque applied = the torque resisting

$$0.88 = \tau(\text{area})(\text{arm}) = \tau(2\pi \times 0.125 \times 0.3)(0.125) \quad \text{and} \quad \tau = 29.9\ \text{Pa}$$

Then $\mu = \tau/(dV/dy) = 29.9/75.4 = 0.397\ \text{Pa s}$.