CS 370 Spring 2015: Assignment 1

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Lec 001 :	MWF 10:30-11:20am MC2035	Office Hours: Tue 3:00pm-4:00pm
Lec 002 :	MWF 11:30-12:20pm MC2034	Office Hours: Tue 3:00pm-4:00pm
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	Tue May 26 2:00-3:00pm	Thu May 28 2:00-3:00pm

Web Site: www.student.cs.uwaterloo.ca/~cs370/

Friday May 29, 2015, 5:00 PM, in the Assignment Boxes, 4th Floor MC (across from the Tutorial Centre MC4065). Please attach a cover page, which you can download from the course website, to your submitted assignment.

1. **(8 marks)**

   (a) Carry out (by hand, with the aid of a calculator, if necessary) the following computation for $x = 0.001$ to obtain an approximation to y by simulating the 3-digit rounding arithmetic of the floating point number system $F(10, 3, -10, 10)$ based on (1) below:

   $$y = (1 + x)^{-\frac{1}{2}} - 1 \tag{1}$$

   Here we assume that, for $z \geq 0$, the computed value $\sqrt{z}$ equals $\text{fl}(\sqrt{z})$.

   (b) Show that (1) is equivalent to

   $$y = -\frac{x}{\sqrt{1 + x} + 1 + x} \tag{2}$$

   (Hint: multiplying both the numerator and denominator by the same factor). Using (2), repeat (a) to obtain another approximation to y.

   (c) Which result is a more accurate approximation to the value $\frac{1}{\sqrt{1+0.001}} - 1$? Explain.

2. **(6 marks)** Given a recursion

   $$q_n + aq_{n-1} + bq_{n-2} = 0$$

   with $a,\ b$ constant, it is known that the solution to this recursion is given by

   $$q_n = C_1(\mu_1)^n + C_2(\mu_2)^n$$

   where C_1, C_2 are constants, and μ_1, μ_2 are roots of the equation $\mu^2 + a\mu + b = 0$.

   Now consider the recurrence equation

   $$p_n = \frac{11}{6}p_{n-1} - \frac{1}{2}p_{n-2}, \quad n \geq 2 \tag{3}$$

   (a) Show that $\frac{1}{3}$ is a root to $\mu^2 - \frac{11}{6}\mu + \frac{1}{2} = 0$. This implies that $p_n = \left(\frac{1}{3}\right)^n$ is the solution to (3) with $p_0 = 1$ and $p_1 = \frac{1}{3}$.

   (b) Write a Matlab function to print out a computed value $\hat{p}_{50}$ based on (3) and the relative error estimate $\frac{|\hat{p}_{50} - p_{50}|}{|p_{50}|}$ where p_{50} is computed by $p_{50} = (\frac{1}{3})^{50}$. How many significant digits of $\hat{p}_{50}$ are accurate in comparison to p_{50}?

   (c) Carry out a stability analysis for this computation by considering the propagation of errors introduced in the initial values p_0 and p_1. Is computation (3) stable or unstable? Explain your observation in (b) using your stability analysis.

3. **(6 marks)**

   (a) Write down explicitly the linear system of equations to determine the derivative values s_1, s_2, s_3, s_4 for the cubic spline $S(x)$ going through the points $(0, -1)$, $(1, 0)$, $(2, 5)$ and $(3, 10)$ and having natural boundary conditions.

   (b) Determine values of s_1, s_2, s_3, s_4 (You can use matlab backslash command \).

   (c) Write down the resulting cubic spline $S(x)$.

   Explain your answers.

4. **(8 marks)** Consider the piecewise linear function $f(x) = \max(0, x)$ and the piecewise quadratic $S(x)$ defined below:

$$S(x) \equiv \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2}x^2 & \text{if } 0 \leq x \leq 1 \\ x - \frac{1}{2} & \text{if } x > 1 \end{cases} \tag{4}$$

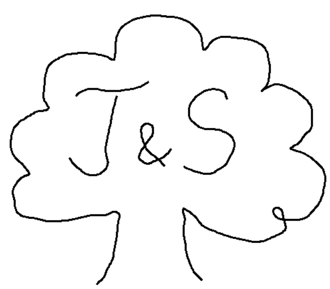
   (a) Using Matlab, in a single plot, graph the function $f(x)$ and $S(x)$ in $[-1, 2]$.

   (b) Show that $f(x)$ is continuous but not differentiable everywhere.

   (c) Show that both $S(x)$ and $S'(x)$ is continuous in $(-\infty, +\infty)$, i.e., $S(x)$ is a quadratic spline in $(-\infty, +\infty)$.

   (d) Comment on the difference between $S(x)$ with $f(x)$. Explain the pros and cons of using $S(x)$ to approximate $f(x)$.

5. **(14 marks)** Create parametric curves representing the outline of an image of any interesting object of your choice, with your first and last initials separated by an ampersand inside the image. For example,



   Follow the steps below:

   (a) Start with download the script file named **init.m** from the course website, which can be used/modified to initialize data arrays for your drawing. Use **help** command to learn any matlab command that is unfamiliar to you. You can trace the image of the object directly and draw initials on a piece of paper and then put the paper on the computer screen to trace with mouse clicks. Create your data using the mouse to select a few dozen points outlining the object and initials. Terminate the input sequence with a carriage return.

   Note: you will need to call `ginput` for each segment (i.e. each stroke of the pen). Modify the script as necessary. Hint: Consider using cell arrays. Also, you may find the `save` and `load` commands useful.

   (b) Using the data arrays from **init.m**, generate parametric curve representations based on smooth parametric curve interpolations as described in your course notes (3.1 and 3.2). At least one smooth segment of the image should be curved enough to be interesting (change your initials temporarily if need be). Show the output using piecewise linear splines and cubic splines. More precisely you are to do the following.

Prepare three Matlab.m scripts, one for each of the following tasks (as well as any functions you define). *Use the same axis scaling v from* **init.m** *for subsequent plots* (i.e. `axis(v)`).

(**part1.m**) A plot with grid lines and the interpolation data corresponding to the crude initial shape plotted with the circle symbol 'o'. Plot axis and grid lines for these plots. The plot should have a title and should show the axes and gridlines.

(**part2.m**) A plot of your letters and the object created by joining the original data points with straight lines. (The curves will not look very smooth). Plot axis and grid lines for this plot. The plot should have a title.

(**part3.m**) A smooth plot of your object and initial created by refining the parameter by a factor of 6. Use different spline end conditions (see matlab functions *csape* and *fnval*) for the letters and the object. For example, if the first uses not-a-knot conditions, you might try natural conditions for the second.

Plot this without grid or axes. The plots should have titles.

Submit all your Matlab code to the dropbox on Learn.