Chapter 4

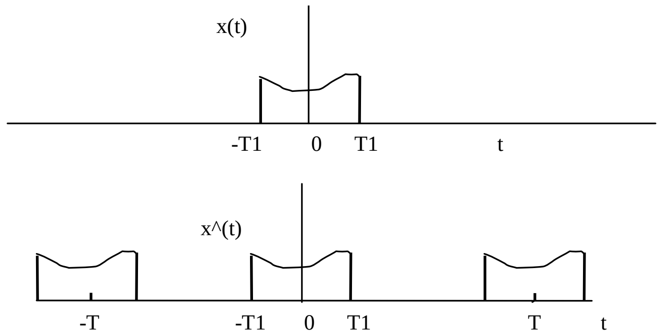
Continuous Time Fourier Transforms

• Periodic signals represented by Fourier Series of frequencies of fundamental and harmonics.

• Non-periodic signals are represented by Fourier integrals. Frequencies are continuous.

Representation of continuous aperiodic signals:

Consider an aperiodic signal x(t) below:



x^(t) is a periodic signal such that for :

$$\hat{x}(t) = x(t), \quad \text{for} \quad -\frac{T}{2} \le t \le \frac{T}{2}$$

Fourier series of the periodic x^(t):

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j\omega_0 kt}, \qquad \omega_0 = \frac{2\pi}{T}. \qquad \text{...(1)}$$

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x\hat{}(t)\, e^{-j\omega_0 kt}\, dt \;= \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\omega_0 kt}\, dt$$

$$= \frac{1}{T}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega_0 kt}\, dt = \frac{1}{T} X(jk\omega_0) \qquad \text{... (2)}$$

where

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad \text{... (3)}$$

or

$$a_k = \frac{1}{T} X(jk\omega_0) \qquad \text{... (4)}$$

From eqns. (1) and (4):

$$x\hat{}(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} X(jk\omega_0)\, e^{jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{\infty} (\frac{\omega_0}{2\pi})\, X(jk\omega_0)\, e^{jk\omega_0 t} = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} X(jk\omega_0)\, e^{jk\omega_0 t} \,.\, \omega_0 \qquad \text{... (5)}$$

As $T \rightarrow \infty, \omega \rightarrow 0,\ x\hat{}(t) = x(t),$ and summation becomes integral. So,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega \qquad \text{... (6)}$$

Thus for an aperiodic signal x(t):

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega \qquad \text{... (7)}$$

or,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad \text{... (8)}$$

- X(jω) is called the *Fourier transform* or *Fourier integral* of x(t).

Convergence of Fourier transforms:

X(jω) of eqn. (8) may or may not converge. To be convergent, x(t) must satisfy the *Dirichlet's* conditions:

- x(t) is absolutely integrable, i.e.

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

- x(t) has a finite number of maxima and minima in any finite interval,
- x(t) may have a finite number of *finite* discontinuities in any time interval.

Examples:

Problem 4.1(b) p. 334 of text.

Find the Fourier transform of $e^{-2|t-1|}$.

Solution:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

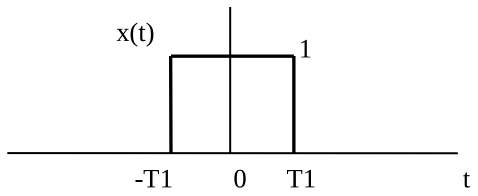
$$= \int_{-\infty}^{1} e^{2(t-1)}\, e^{-j\omega t}\, dt + \int_{1}^{\infty} e^{-2(t-1)}\, e^{-j\omega t}\, dt$$

$$= e^{-2} \int_{-\infty}^{1} e^{(2-j\omega)t}\, dt + e^{2} \int_{1}^{\infty} e^{-(2+j\omega)t}\, dt$$

$$= \frac{e^{-j\omega}}{2-j\omega} + \frac{e^{-j\omega}}{2+j\omega} = e^{-j\omega} \frac{4}{4+\omega^2}.$$

Example 4.4 p.293. [Important result.]

Find the Fourier transform X(jω) of x(t):



Solution:

$$X(j\omega) = \int_{-T_1}^{T_1} 1.e^{-j\omega t}\, dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-T_1}^{T_1}$$

$$= \frac{1}{j\omega}(e^{j\omega T_1} - e^{-j\omega T_1}) = 2\frac{\sin \omega T_1}{\omega}.$$

The function $\frac{\sin \omega T_1}{\omega}$, more generally, $\frac{\sin \theta}{\theta}$ is called '*sync*' function sinc(θ) and is very important in signal analysis.

Fourier Transform of Periodic Signals

Consider x(t) with the Fourier transform X(jω) given by:

$$X(j\omega) = 2\pi\, \delta(\omega - \omega_0) \qquad \text{... (9)}$$

Taking inverse transform:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\, \delta(\omega - \omega_0)\, e^{j\omega t}\, d\omega$$

$$= \int_{-\infty}^{\infty} \delta(\omega - \omega_0)\, e^{j\omega t}\, d\omega = e^{j\omega_0 t}$$

Now,

$$e^{j\omega t} \quad \rightarrow \quad 2\pi\, \delta(\omega - \omega_0)$$

$$e^{jk\omega_0 t} \quad \rightarrow \quad 2\pi\, \delta(\omega - k\omega_0)$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_0 t} \rightarrow \sum_{k=-\infty}^{\infty} 2\pi\, a_k\, \delta(\omega - k\omega_0)$$

Thus, with

$$x(t) = \sum_{-\infty}^{\infty} a_k \, e^{jk\omega_0 t},$$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi \, a_k \, \delta(\omega - k\omega_0) \qquad \text{... (10)}$$

Go through examples 4.6, 4.7 and 4.8 p. 297-300 of text.

Problem 4.3 (a) p. 334

Find the FT of $\sin(2\pi t + \frac{\pi}{4})$

Solution:

$$\sin(2\pi t + \frac{\pi}{4}) = \frac{1}{2j}[e^{j(2\pi t + \frac{\pi}{4})} - e^{-j(2\pi t + \frac{\pi}{4})}]$$

$$= \frac{1}{2j} e^{j\frac{\pi}{4}} \, e^{j\omega_0 t} - \frac{1}{2j} e^{-j\frac{\pi}{4}} \, e^{-j\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k \, e^{jk\omega_0 t}$$

giving:

$$a_1 = \frac{e^{j\frac{\pi}{4}}}{2j}, \quad a_{-1} = -\frac{e^{-j\frac{\pi}{4}}}{2j}.$$

Other coefficients = 0.

Thus,

$$X(j\omega) = 2\pi\, a_1\, \delta(\omega - \omega_0) + 2\pi\, a_{-1}\, \delta(\omega + \omega_0)$$

$$= \frac{2\pi\, e^{j\frac{\pi}{4}}}{2j}\, \delta(\omega - \omega_0) - \frac{2\pi\, e^{-j\frac{\pi}{4}}}{2j}\, \delta(\omega + \omega_0)$$

$$= \frac{\pi}{j}[e^{j\frac{\pi}{4}}\, \delta(\omega - 2\pi) - e^{-j\frac{\pi}{4}}\, \delta(\omega + 2\pi)].$$

Properties of Fourier Transforms

Linearity:

$$\text{If} \quad x(t) \rightarrow X(j\omega)$$

$$y(t) \rightarrow Y(j\omega), \quad \text{then}$$

$$ax(t) + by(t) \rightarrow aX(j\omega) + bY(j\omega)$$

Time shift:

$$x(t - t_0) \rightarrow e^{j\omega t_0}\, X(j\omega)$$

Conjugate and conjugate symmetry:

$$x(t) \rightarrow X(j\omega)$$

$$x^*(t) \rightarrow X^*(-j\omega)$$

$$Ev\{x(t)\} \rightarrow Re\{X(j\omega)\}$$

$$Od\{x(t)\} \rightarrow j\, Im\{X(j\omega)\}$$

Differentiation and Integration:

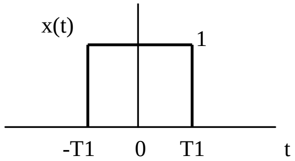
$$\frac{dx(t)}{dt} \rightarrow j\omega X(j\omega)$$

$$\int_{-\infty}^{t} x(\tau)\, d\tau \rightarrow \frac{1}{j\omega} X(j\omega) + \pi X(0)\, \delta(\omega)$$

Time and Frequency Scaling:

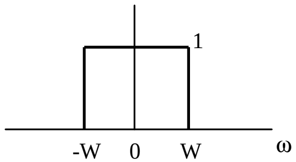
$$x(at) \rightarrow \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

Duality:



$$\Rightarrow \quad X(j\omega) = \frac{2\sin \omega T_1}{\omega}$$

$$x(t) = \frac{\sin(Wt)}{\pi t} \quad \Rightarrow$$

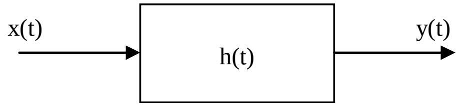


Parseval's Relation:

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega.$$

Consult Table 4.1 p. 328 of text for more properties.

Convolution Properties:



Here,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t - \tau)\, d\tau.$$

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau)\, h(t - \tau)\, d\tau \right] e^{-j\omega t}\, dt$$

Interchanging the order of integration,

$$Y(j\omega) = \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(t - \tau)\, e^{-j\omega t}\, dt \right] d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) \left[e^{-j\omega\tau} H(j\omega) \right] d\tau, \qquad \text{[using time shift property]}$$

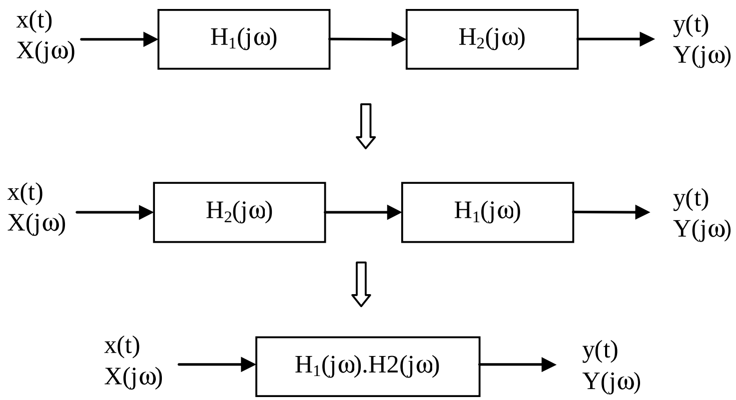
$$= H(j\omega) \int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, d\tau = H(j\omega).\, X(j\omega).$$

$$Y(j\omega) = H(j\omega).\, X(j\omega). \qquad \text{... (10)}$$

i.e.,

$$y(t) = h(t) * x(t) \qquad \rightarrow \qquad Y(j\omega) = H(j\omega).\, X(j\omega)$$

Eqn. (10) indicates that if two systems are cascaded , they may be represented as follows:



Problems:

Given $h(t) = 2\,e^{-5t}\,u(t),\qquad x(t) = 9\,e^{-2t}\,u(t);$ find $y(t)$.

Solution:

From Table 4.2:

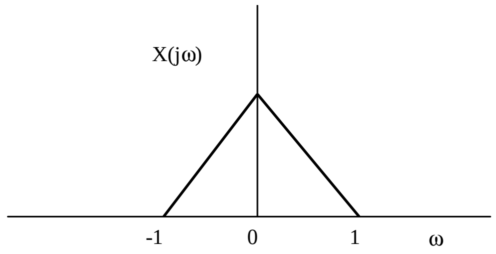
$$H(j\omega) = \frac{2}{5 + j\omega},\qquad X(j\omega) = \frac{9}{2 + j\omega}.$$

$$Y(j\omega) = \frac{18}{(5 + j\omega)(2 + j\omega)} = -\frac{6}{5 + j\omega} + \frac{6}{5 + j\omega}.$$

$$y(t) = -6\,e^{-5t}\,u(t) + 6\,e^{-2t}\,u(t).$$

Problem 4.28 (b) p. 342

Given $X(j\omega)$ as:



Sketch $Y(j\omega)$, where,

$y(t) = x(t) \,.\, p(t),$

$p(t) = \cos t.$

Solution:

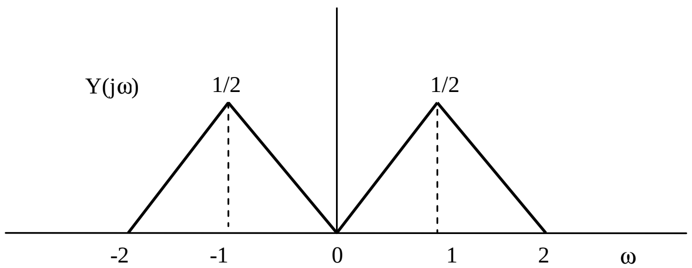
$$p(t) = \cos t = \frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt}$$

$$P(j\omega) = \pi\,\delta(\omega-1) + \pi\,\delta(\omega+1)$$

$$Y(j\omega) = \frac{1}{2\pi}X(j\omega) * P(j\omega) = \frac{1}{2\pi}\times\pi\{X(j\omega)*[\delta(\omega-1)+\delta(\omega+1)]\}$$

$$= \frac{1}{2}[X(j(\omega-1)) + X(j(\omega+1))]$$

The plot of $Y(j\omega)$ versus ω :



Problem 4.36 p.346.

Given an LTI system with

$$x(t) = [e^{-t} + e^{-3t}]\,u(t), \qquad y(t) = [2e^{-t} - 2e^{-4t}]\,u(t).$$

Find the system's

(a) frequency response,

(b) impulse response.

Solution:

$$Y(j\omega) = H(j\omega).X(j\omega), \qquad \text{or,} \quad H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

From Table 4.2:

$$X(j\omega) = \frac{1}{1+j\omega} + \frac{1}{3+j\omega} = \frac{4+j2\omega}{(1+j\omega)(3+j\omega)}$$

$$Y(j\omega) = \frac{2}{1+j\omega} - \frac{2}{4+j\omega} = \frac{6}{(1+j\omega)(4+j\omega)}$$

(a)

$$H(j\omega) = \frac{6}{(1+j\omega)(4+j\omega)} \times \frac{(1+j\omega)(3+j\omega)}{4+j2\omega} = \frac{3(3+j\omega)}{(4+j\omega)(2+j\omega)}$$

(b) $H(j\omega) = \frac{3}{2}\left(\frac{1}{4+j\omega} + \frac{1}{2+j\omega}\right).$, hence

$$h(t) = \frac{3}{2}(e^{-4t} + e^{-2t})\,u(t).$$

Go through Examples 4.10, 4,11, 4.12, 4.15, 4.18 and 4.19 pp. 305-321 of text.

Problem 4.5 p.335.

Find:

(a) x(t) given $|X(j\omega)| = 2[u(\omega+3) - u(\omega-3)], \quad \angle X(j\omega) = -\frac{3}{2}\omega + \pi.$

(b) t for which x(t) = 0.

Solution:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega.$$

$$X(j\omega) = |X(j\omega)|\, e^{\angle X(j\omega)}$$

$$= 2\, e^{j(-\frac{3}{2}\omega+\pi)}, \quad -3 \le \omega \le 3$$

$$= 0. \quad \text{else.}$$

(a)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\, e^{j(-\frac{3}{2}\omega+\pi)}\, e^{j\omega t}\, d\omega = \frac{e^{j\pi}}{\pi}\int_{-3}^{3} e^{j(t-\frac{3}{2})\omega}\, d\omega$$

$$= -\frac{2}{\pi}\,\frac{\sin[3(t-\frac{3}{2})]}{(t-\frac{3}{2})}$$

(b) x(t)= 0, if

$$t - \frac{3}{2} \ne 0, \text{ and } 3(t-\frac{3}{2}) = k\pi, \; k \ne 0.$$

or

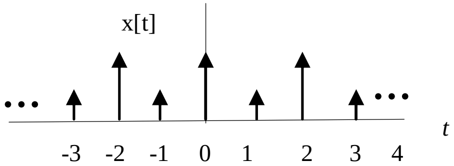
$$t = \frac{3}{2} + \frac{k}{3}\pi.$$

Problem 4.21 p. 238 of text.

Find the Fourier transform of:

(b) $x(t) = e^{-3|t|}\sin 2t$.

(h) x(t):



Solution:

(b)

$$x(t) = e^{-3|t|}\sin 2t = e^{3t}\sin 2t\, u(-t) + e^{-3t}\sin 2t\, u(t).$$

With

$$x_1(t) = \sin 2t\, u(t) = \frac{1}{j2}[e^{j2t} - e^{-j2t}]\,u(t)$$

$$X_1(j\omega) = \int_0^\infty \frac{1}{j2}[e^{j2t} - e^{-j2t}]\,e^{-j\omega t}\,dt$$

$X(j\omega) =$

$$= \frac{1}{j2}\int_0^\infty [e^{j(2-\omega)t} - e^{-j(2+\omega)t}]\,dt = \frac{1}{j2}\left[\frac{1}{j(\omega-2)} - \frac{1}{j(\omega+2)}\right]$$

With

$$x_2(t) = e^{-3t}x_1(t),$$

$$X_2(j\omega) = \frac{1}{j2}\left[\frac{1}{3+j(\omega-2)} - \frac{1}{3+j(\omega+2)}\right] \qquad \text{(shift property)}$$

Similarly, for

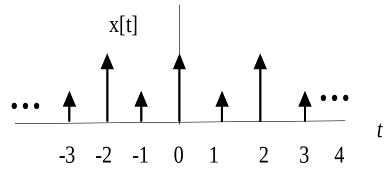
$$x_3(t) = e^{3t}\sin 2t\, u(-t),$$

$$X_3(j\omega) = \frac{1}{j2}\left[\frac{1}{3-j(\omega-2)} - \frac{1}{3-j(\omega+2)}\right]$$

Now,

$$X(j\omega) = X_2(j\omega) + X_3(j\omega)$$
$$= \frac{1}{j2}\left[\frac{1}{3+j(\omega-2)} + \frac{1}{3-j(\omega-2)} - \frac{1}{3+j(\omega+2)} - \frac{1}{3-j(\omega+2)}\right]$$
$$= 3j\left[\frac{1}{9+(\omega+2)^2} - \frac{1}{9+(\omega-2)^2}\right].$$

(h) x(t) is periodic with T =2. It may be expressed as:



$$x(t) = 2x_1(t) + x_2(t), \qquad \text{where,}$$

$$x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k), \qquad \text{for even values of } t;$$

$$x_2(t) = \sum_{k=-\infty}^{\infty} \delta(t-1-2k), \qquad \text{for odd values of } t.$$

$X_1(j\omega)$:

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-1}^{1} \delta(0)e^{-jk\omega_0 t}\,dt = \frac{1}{2}\delta(0) = \frac{1}{2}.$$

From eqn. (11):

$$X_1(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi.\frac{1}{2}.\delta(\omega-k\pi) = \pi\sum_{k=-\infty}^{\infty}\delta(\omega-k\pi).$$

Similarly,

$$X_2(j\omega) = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)\, e^{-j\omega}.$$

Now,

$$X(j\omega) = 2X_1(j\omega) + X_2(j\omega)$$

$$= 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi) + \pi\, e^{-j\omega} \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)$$

$$= (2 + e^{-j\omega}) \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)$$

Problem 4.26(a) p. 341 of text.

Find the convolution $x(t) * h(t)$ of:

i) $x(t) = t\, e^{-2t}\, u(t), \qquad h(t) = e^{-4t}\, u(t)$

ii) $x(t) = e^{-t}\, u(t), \qquad h(t) = e^{t}\, u(-t).$

Solution:

$$t\, e^{-2t}\, u(t) \rightarrow \frac{1}{(2 + j\omega)^2} \qquad \text{(from Table 4.2)}$$

$$t\, e^{-4t}\, u(t) \rightarrow \frac{1}{(4 + j\omega)}$$

$X(j\omega).H(j\omega) \quad = \frac{1}{(2+j\omega)^2} \cdot \frac{1}{4+j\omega} = \frac{\frac{1}{2}}{(2+j\omega)^2} - \frac{\frac{1}{4}}{2+j\omega} + \frac{\frac{1}{4}}{4+j\omega}$

whence,

$$x(t) * h(t) = \frac{1}{4} e^{-4t}\, u(t) - \frac{1}{4} e^{-2t}\, u(t) + \frac{1}{2} t\, e^{-2t}\, u(t).$$

iii)

$$X(j\omega) = \frac{1}{1+j\omega}; \qquad h(t) = e^{t}\, u(-t) = x(-t), \text{ hence, } \quad H(j\omega) = \frac{1}{1-j\omega}.$$

$$X(j\omega).H(j\omega) = \frac{1}{(1+j\omega)(1-j\omega)} = \frac{\frac{1}{2}}{1+j\omega} + \frac{\frac{1}{2}}{1-j\omega}.$$

$$x(t) * h(t) = \frac{1}{2} e^{-t}\, u(t) - \frac{1}{2} e^{t}\, u(-t) = \frac{1}{2} e^{-|t|}.$$

Problem 4.21 (d), p. 238.

Find the Fourier transform of :

$$x(t) = \sum_{k=0}^{\infty} \alpha^k\, \delta(t - kT), \qquad |\alpha < 1.|$$

Solution:

$$X(j\omega) = \int_{-\infty}^{\infty} (\sum_{k=0}^{\infty} \alpha^k\, \delta(t-kT)\, e^{-j\omega t}\, dt = \sum_{k=0}^{\infty} \alpha^k \int_{-\infty}^{\infty} \delta(t-kT)\, e^{-j\omega t}\, dt$$

$$= \sum_{k=0}^{\infty} \alpha^k\, .\, e^{-j\omega KT} = \sum_{k=0}^{\infty} (\alpha e^{-j\omega T})^k = \frac{1}{1 - \alpha e^{-j\omega T}}.$$

Problem 4.22 (b), p. 339.

Find x(t), given : $X(j\omega) = \cos(4\omega + \frac{\pi}{3}).$

Solution:

$$X(j\omega) = \frac{1}{2}[e^{j\frac{\pi}{3}} . e^{j4\omega} + e^{-j\frac{\pi}{3}} . e^{-j4\omega}].$$

From Table 4.2:

$$x(t) = \frac{1}{2} e^{j\frac{\pi}{3}} \delta(t+4) + \frac{1}{2} e^{-j\frac{\pi}{3}} \delta(t-4).$$

Problem 4.33 (a) and (b), p. 345.

Given a causal LTI system:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8\,y(t) = 2\,x(t).$$

Find: a) system impulse response,

b) system response with $x(t) = t\,e^{-2t}\,u(t)$.

Solution:

(a) $$-\omega^2\,Y(j\omega) + 6\,j\omega\,Y(j\omega) + 8\,Y(j\omega) = 2\,X(j\omega), \quad \text{or}$$

$$[-\omega^2 + 6j\omega + 8]\,Y(j\omega) = 2X(j\omega)$$

Giving

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{-\omega^2 + 6j\omega + 8} = \frac{2}{(j\omega+2)(j\omega+4)}$$

$$= \frac{1}{j\omega+2} - \frac{1}{j\omega+4}.$$

$$h(t) = e^{-2t}\, u(t) - e^{-4t}\, u(t)$$

(b)

$$X(j\omega) = \frac{1}{(j\omega+2)^2}$$

$$Y(j\omega) = H(j\omega)\,.\,X(j\omega) = \frac{2}{(j\omega+2)(j\omega+4)} \times \frac{1}{(j\omega+2)^2}$$

$$= \frac{2}{(j\omega+2)^3(j\omega+4)} = \frac{1}{(j\omega+2)^3} - \frac{1/2}{(j\omega+2)^2} + \frac{1/4}{j\omega+2} - \frac{1/4}{(j\omega+4)}$$

$$y(t) = t^2\, e^{-2t}\, u(t) - \frac{1}{2} t\, e^{-2t}\, u(t) + \frac{1}{4} e^{-2t}\, u(t) - \frac{1}{4} e^{-4t}\, u(t).$$

Problem on Parseval's Relation:

Find the energy of $x(t)= e^{-at}$, $a > 0$.

Solution:

Parseval's:

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega$$

$$X(j\omega) = \frac{1}{a + j\omega}.$$

$$\text{Energy } = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{1}{a + j\omega} \right|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{a^2 + \omega^2} \, d\omega$$

$$= \frac{1}{\pi} \int_{0}^{\infty} \frac{1}{a^2 + \omega^2} \, d\omega = \frac{1}{\pi a} \tan^{-1} \frac{\omega}{a} \Big|_{0}^{\infty} = \frac{1}{2a}.$$

Duality Properties:

If $\quad x(t) \rightarrow X(j\omega)$, then

$X(jt) \rightarrow 2\pi x(-\omega).$

Problem: Find the Fourier transform of $x(t) = 1$.

Solution:

$$\delta(t) \rightarrow X(j\omega) = 1$$

$$X(jt) = 1 \rightarrow 2\pi\delta(-\omega) = 2\pi\delta(\omega).$$