Chapter 2: Linear Time-Invariant (LTI) Systems

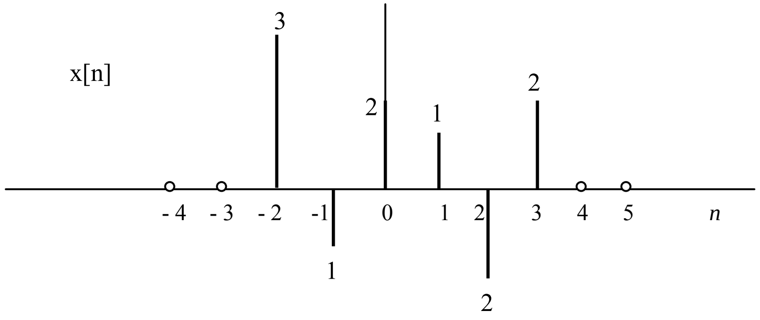
- LTI systems support *superposition*. Hence,

- If $x(t)$ or $x[n]$ is expressed as a linear combination of a set of basic functions, the output will be the *summation* of individual responses.

- A general signal may be expressed as a linear combination of *delayed* unit impulses. The output response will be summation (i.e. *Convolution sum* or *convolution integral* of the impulse responses.

Discrete Time Signal Representation:

Consider an arbitrary signal $x[n]$.



$x[3] = x[3].\ \delta[n-3]$,
$x[2] = x[2].\ \delta[n-2]$,
$x[1] = x[1].\ \delta[n-1]$,
$x[0] = x[0].\ \delta[n]$,
$x[-1] = x[-1].\ \delta[n+1]$,
$x[-2] = x[-2].\ \delta[n+2]$,
In general:
$x[k] = x[k].\ \delta[n-k]$.

Thus,

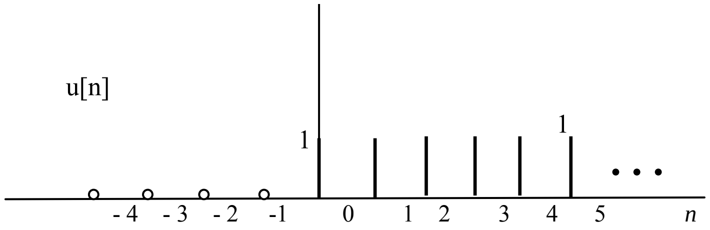
$$x[n] = \sum_{k=-\infty}^{\infty} x[k] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \qquad \ldots\ (1)$$

- x[n] may be expressed as a summation of *shifted* impulses with a respective weight of x[k].

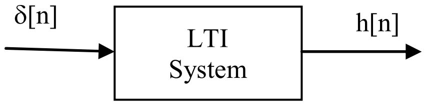
Example:

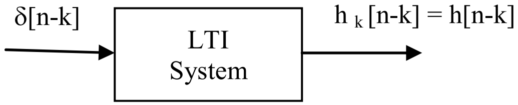
u[n]:

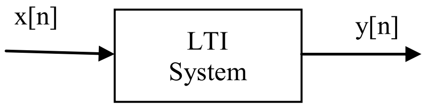
$$u[n] = \sum_{k=0}^{\infty} 1.\delta[n-k] = \sum_{k=0}^{\infty} \delta[n-k]$$

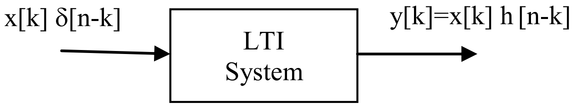


Discrete Time Unit Impulse Response:









$$x[n] = \sum_{-\infty}^{\infty} x[k]\,\delta[n-k] \qquad y[n] = \sum_{-\infty}^{\infty} x[k]\,h[n-k]$$

An input $x[n] = \sum_{-\infty}^{\infty} x[k]\,\delta[n-k]$ produces an output y[n] given by:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \qquad \text{... (3)}$$

- R.H.S. Is called the *convolution sum* and is represented by $x[n] * h[n]$.

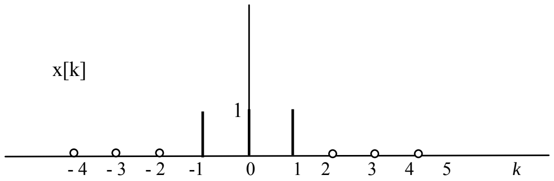
$$y[n] = x]n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \qquad \text{... (4)}$$

- Convolution may be found analytically using eqn. (4) or graphically.

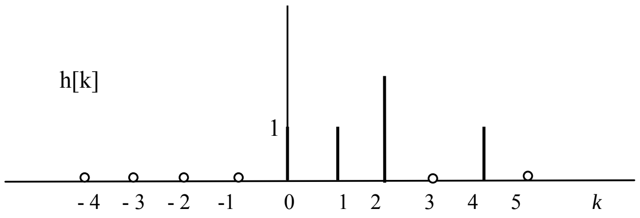
Graphical Method:

1.Sketch x[k] with k as x-axis;

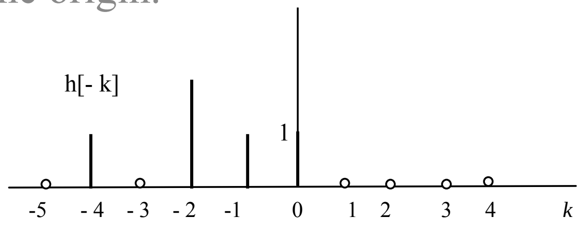
2.\\\



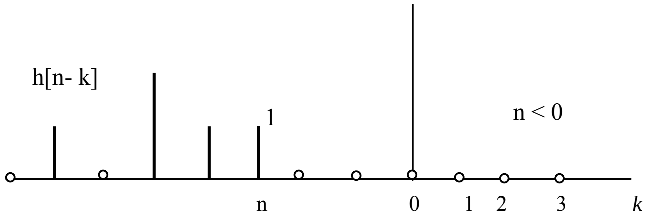
2. Sketch h[k] using k as x-axis.

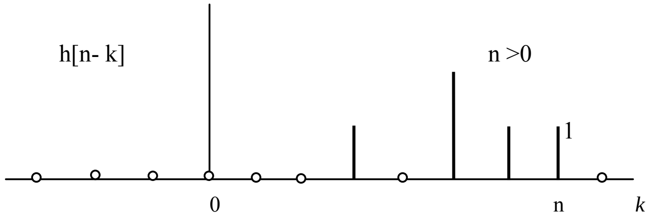


3.Obtain h[- k] by reflecting it about the origin.



4.Select a value of n. Shift h[- k] by n.





5. Multiply x[k] by h[n-k] for each k.

6. Calculate the sum

$$\sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \quad .$$

7. Take another value of n and repeat steps 4 to 6. Do this for

$$-\infty \le n \le \infty \quad .$$

Go through examples 2.3, 2.4 and 2.5 pp.81-90 of text.

Example:

Given: x[n]= u[n], and

$h[n] = a^{n}\,u[n], \quad 0 < a < 1.$. Find y[n].

Analytical Method:

$$h[n-k] = a^{n-k}\,u[n-k]$$

$$y[n] = \sum_{k=0}^{\infty} x[n]\,h[n\text{-}k] = \sum_{k=0}^{\infty} a^{n-k}\,u[n-k]$$

$$= \sum_{k=0}^{\infty} a^{n\text{-}k} = \sum_{m=n}^{0} a^{m} = \sum_{m=0}^{n} a^{m} = \frac{1-a^{n+1}}{1-a}$$

Graphical Method:

For $n < 0$,

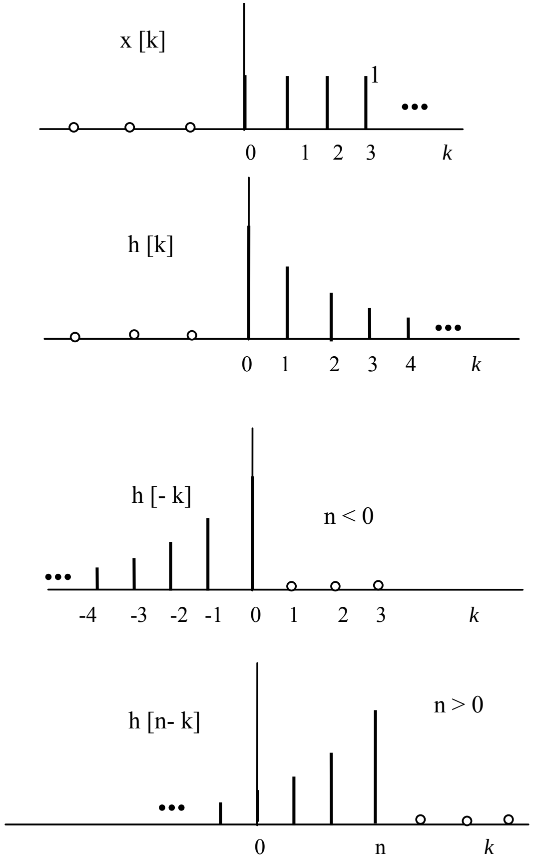
no overlap. $y[n] = 0$.

For $n \geq 0$,

overlap for 0 to n.

$$y[n] = \sum_{k=0}^{n} a^{n-k} = \sum_{m=n}^{0} a^{m}$$

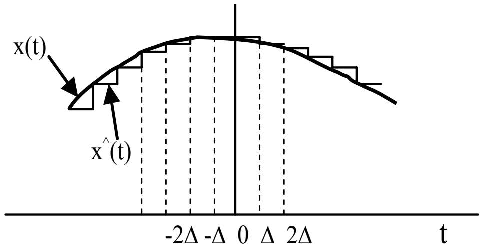
$$= \sum_{m=0}^{n} a^{m} = \frac{1-a^{n+1}}{1-a}.$$



Continuous-Time Signal Representation:

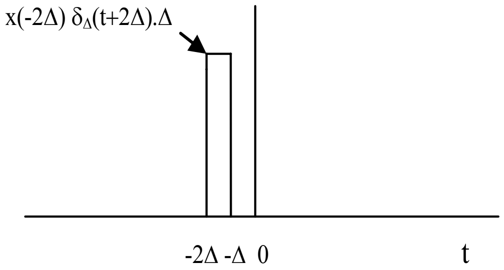
Signal x(t) and its staircase approximation $x^{\wedge}(t)$ are shown to the right.
As

$$\delta_{\Delta}(t) = \frac{1}{\Delta}, \qquad 0 \le t \le \Delta$$
$$= 0 \qquad \text{else.}$$



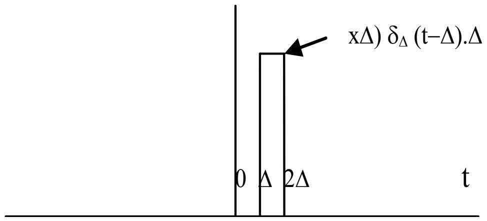
$x^{\wedge}(t)$ may be expressed as

$$x^{\wedge}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta(t-k\Delta).\Delta \quad ...(5)$$



Since $\delta(t) = \text{Lim}_{\Delta \to 0}\ \delta_{\Delta}(t)$

$$x(t) = \text{Lim}_{\Delta \to 0}\ x^{\wedge}(t) \quad .. (6)$$



From (5) and (6):

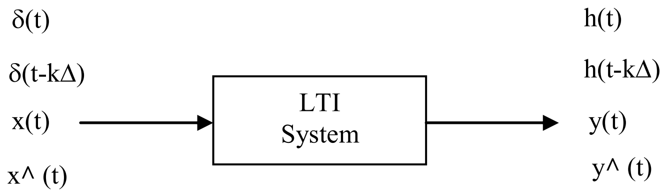
$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\, d\tau \quad ... (7)$$

Eqn (7) represents x(t) in terms of impulses δ(t).

If x(t) = u(t), then

$$u(t) = \int_{-\infty}^{\infty} u(\tau)\delta(t-\tau)d\tau = \int_{0}^{\infty} \delta(t-\tau)\,d\tau \qquad ..\ (8)$$

CT Unit Impulse Response and Convolution Integral:



$\delta(t)$ → LTI System → $h(t)$

$\delta(t-k\Delta)$ → LTI System → $h(t-k\Delta)$

$x(t)$ → LTI System → $y(t)$

$x\hat{}\,(t)$ → LTI System → $y\hat{}\,(t)$

From, eqn. (5),

$$x\hat{}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta).\delta_{\Delta}(t-k\Delta).\Delta \qquad \ldots\ (5)$$

d(t - kΔ) → h(t - kΔ) ;

x^(t) → y^(t) [since it is an LTI system].

Hence, from eqn (5) and above:

$$y\hat{}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta).h(t-k\Delta).\Delta \qquad (9)$$

As $\Delta \rightarrow 0$, $\hat{y}(t) \rightarrow y(t)$. Hence

$$\mathrm{Lim}_{\Delta\to 0}\ y\hat{\ }(t) = y(t), \quad \text{or}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \qquad \text{... (10)}$$

Eqn. (6) is called *convolution integral* and is represented by:

$$y(t) = x(t) * h(t) \qquad \text{... (11)}$$

Go through Examples 2.6 – 2.9 pp. 98-103 of text.

Important Results:

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau = x(t)$$

$$x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-t_0-\tau)\, d\tau = x(t-t_0)$$

Example 1:

Prob. 2.10, p.139.

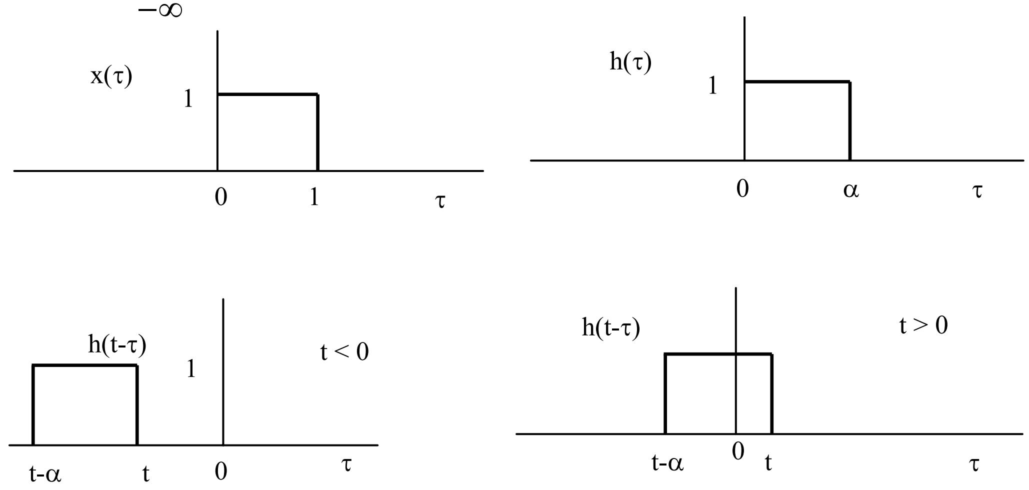
Given: $x(t) = 1, \quad 0 \le t \le 1$

$\quad = 0, \quad \text{else.}$

$h(t) = x(t/\alpha), \quad 0 < \alpha \le 1.$

Solution:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$



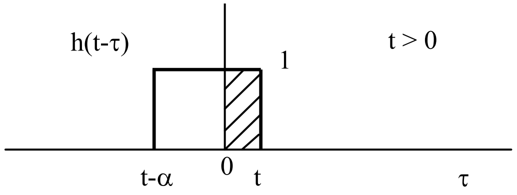
Case 1: $t < 0$.

No overlap. $y(t) = 0$.

Case 2: $0 \leq t < \alpha$

Partial overlap.

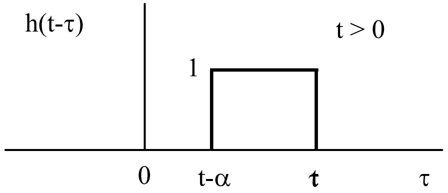
$$y(t) = \int_{0}^{t} 1.\,d\tau = \tau\,|_{0}^{t} = t.$$



Case 3: $\alpha \leq t \leq 1$

Complete overlap.

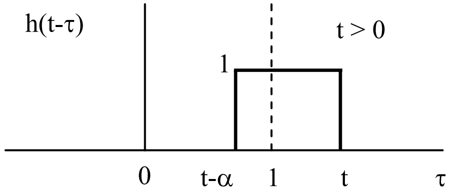
$$y(t) = \int_{t-\alpha}^{t} 1.\,d\tau = \tau\,|_{t-\alpha}^{t} = t-(t-\alpha) = \alpha$$



Case 4: $1 < t < 1+\alpha$.

Partial overlap.

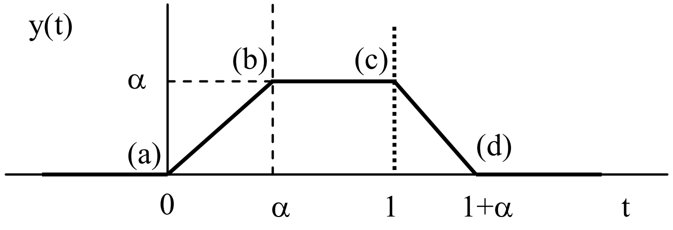
$$y(t) = \int_{t-\alpha}^{1} 1.\,d\tau = \tau\,|_{t-\alpha}^{1} = 1 - t + \alpha.$$



Case 5: t-α > 1:

No overlap. y(t) = 0.

Plot:



(b) $\frac{dy}{dt}$ has 4 discontinuities @ (a) – (d). Discontinuities at (a) and (d) are not removable. For 3 discontinuities, (b) and (c) are to be same, which happens when α = 1.

Example 2:

Find y(t), given: $x(t) = e^{\alpha t}\, u(-t), \qquad \alpha > 0$

$h(t) = e^{-\alpha t}\, u(t).$

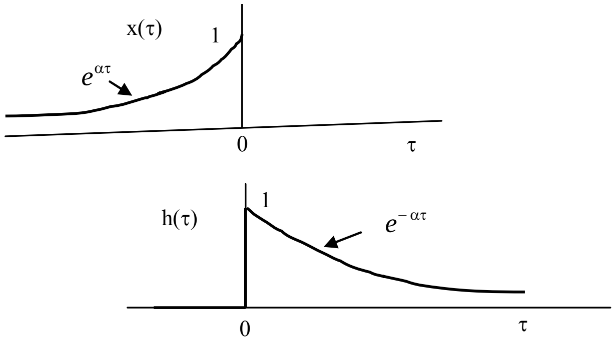
Solution:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Case 1: t < 0.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} e^{\alpha t} . \, e^{-\alpha(t-\tau)}\, d\tau$$

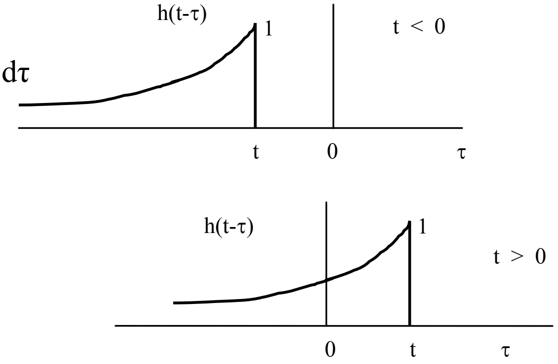
$$= e^{-\alpha t} \int_{-\infty}^{t} e^{2\alpha\tau}\, d\tau = \frac{1}{2\alpha} e^{\alpha t}$$



Case 2: t > 0.

$$y(t) = \int_{-\infty}^{0} x(\tau)\, h(t-\tau) d\tau = \int_{-\infty}^{0} e^{\alpha\tau} e^{-\alpha(t-\tau)} d\tau$$

$$= e^{-\alpha t} \int_{-\infty}^{0} e^{2\alpha\tau}\, d\tau = \frac{e^{-\alpha t}}{2\alpha}.$$



The two cases may be combined as:

$$y(t) = \frac{1}{2\alpha} e^{-\alpha|t|}.$$

LTI Properties:

LTI systems posses a number of properties - the important ones are discussed here.

Convolution Properties:

Commutative: (Summation and integration is commutative.)

$$x(t) * h(t) = h(t) * x(t),$$

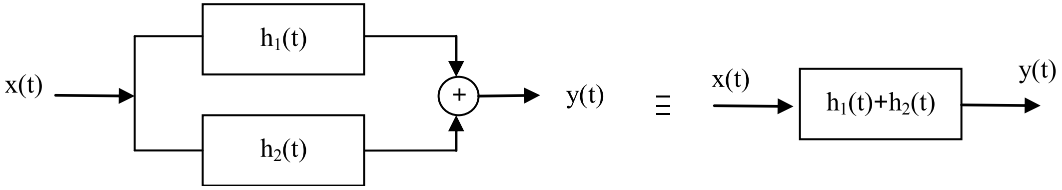
$$x[n] * h[n] = h[n] * x[n]$$

Distributive:

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t),$$

$$x[n] * \{h_1[n] + h_2[n]\} = x[n] * h_1[n] + x[n] * h_2[n].$$

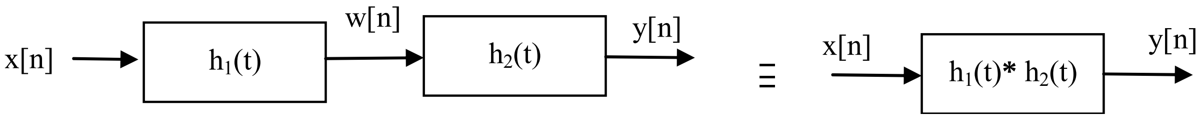
This implies:



Associative:

$$x(t)*[h_1(t)*h_2(t)]=[x(t)*h_1(t)]*h_2(t),$$
$$x[n]*\{h_1[n]*h_2[n]\}=\{x[n]*h_1[n]\}*h_2[n]$$

This property yields:



Important Relations:

Continuous-Time:

a) $x(t)*\delta(t)=x(t).$

b) $x(t)*\delta(t-t_0)=x(t-t_0).$

c) $x(t)*u(t)=\int_{-\infty}^{t} x(\tau)\,d\tau.$

d) $x(t)*u(t)=\int_{-\infty}^{t} x(\tau)\,d\tau.$

Proof:

b) $x(t)*\delta(t-t_0)=\int_{-\infty}^{\infty} x(\tau)\delta(t-t_0-\tau)d\tau=x(\tau)\Big|_{\tau=t-t_0}=x(t-t_0)$

d) $$x(t) * u(t - t_0) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t - t_0 - \tau)\, d\tau = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$$

Relations (a) and (c) are obtained with $t_0 = 0$.

Discrete-Time:

a) $$x[n] * \delta[n] = x[n].$$

b) $$x[n] * \delta[n - n_0] = x[n - n_0].$$

c) $$x[n] * u[n] = \sum_{k=-\infty}^{n} x[k].$$

d) $$x[n] * u[n - n_0] = \sum_{k=-\infty}^{n-n_0} x[k].$$

Proof:

b) $$x[n] * \delta[n - n_0] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n - n_0 - k] = x[n - n_0].$$

d) $$x[n] * u[n - n_0] = \sum_{k=-\infty}^{\infty} x[k]\, u[n - n_0 - k] = \sum_{k=-\infty}^{n-n_0} x[k].$$

For relations (a) and (c), put $n_0 = 0$.

Problem:

2.11 p. 139 of text.

Given:

$$x(t) = u(t-3) - u(t-5),$$

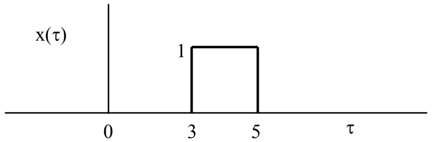
$$h(t) = e^{-3t}u(t).$$

Compute

a) $y(t) = x(t) * h(t)$,

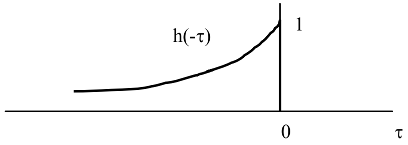
b) $g(t) = [\frac{dx(t)}{dt}] * h(t)$.

Solution:



a) Case 1: $t < 3$:

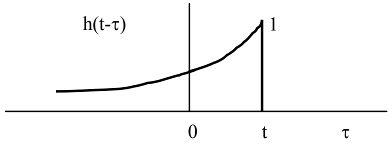
No overlap. $y(t) = 0$.



Case 2: $3 \le t < 5$:

Partial overlap.

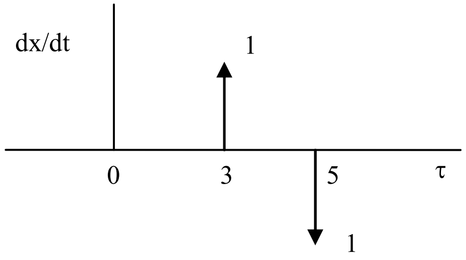
$$y(t) = \int_3^t e^{-3(t-\tau)}\,d\tau = e^{-3t}\int_3^t e^{3\tau}d\tau = \frac{1}{3}(1 - e^{9-3t}).$$



Case 3: $t \ge 5$:

$$y(t) = \int_3^5 e^{-3(t-\tau)}\,d\tau = e^{-3t}\int_3^5 e^{3\tau}d\tau = \frac{1}{3}(e^{-3t+15} - e^{9-3t}).$$

b) $$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$$



For t <3, no overlap. y(t) = 0.

For $3 \le t < 5$,

$$y(t) = e^{-3t} * \delta(t-3) = e^{-3(t-3)}$$

For $t \ge 5$,

$$y(t) = e^{-3t} * \delta(t-3) - e^{-3t} * \delta(t-5)$$
$$= e^{-3t+9} - e^{-3t+15}$$

LTI Systems with or without Memory

Without Memory:

y[n] depends on x[n] only. Now,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$
$$= \ ....+h[-1]\,x[n+1]+h[0]\,x[n]+h[1]\,x[n-1]+...$$
$$= k\,x[n], \quad \text{where} \quad k = h[0].$$

This requires:

$$h[n] = 0, \qquad n \neq 0,$$
$$= k, \qquad n = 0.$$

In general:

$$h[n] = k\,\delta[n] \qquad \ldots (12)$$

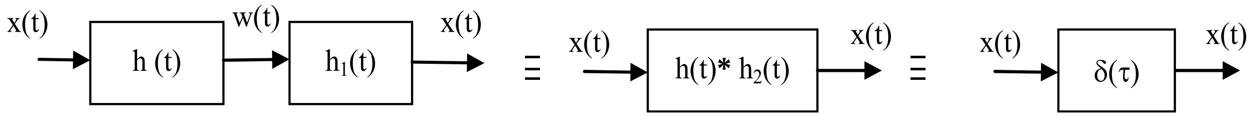
For a continuous–time system, the condition may be derived as:

$$h(t) = k\,\delta(t) \qquad \ldots (13)$$

If condition (12) or (13) is not satisfied, the system has memory.

Invertibility:

With $h(t)$ and $h_1(t)$ as the impulse responses of the system and its inverse respectively, we can represent them as follows:



This yields (for continuous-time):

$$h(t) * h_1(t) = \delta(t)\,.$$

For discrete-time,

$$h[n] * h_1[n] = \delta[n]\,.$$

Example: (Final 2009).

Two DT LTI systems have impulse responses:

h1[n]= δ[n] – δ[n-1],

h2[n]= u[n].

Show that they are inverse of each other.

Solution:

$$\begin{aligned} h_1[n] * h_2[n] &= u[n] * \delta[n] - u[n] \cdot \delta[n-1] \\ &= u[n] - u[n-1] = \delta[n]. \end{aligned}$$

Go through Examples 2.11 and 2,12 , pp. 110-111 of text.

Causality:

- Output depends on the present and past inputs only. Now,

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] \\ &= \;.... + h[-1]\, x[n+1] + h[0]\, x[n] + h[1]\, x[n-1] + ... \end{aligned}$$

For a causal systems, the future inputs x[n+1], x[n+2], etc. must be 0. Thus

h[-1], h[-2], ... = 0.

or $h[n] = 0, \quad \text{for} \quad n < 0.$ ... (14a)

For CT systems,

$h(t) = 0, \quad \text{for} \quad t < 0.$ ... (14b)

Stability:

To be stable, a system has to be BIBO (bounded-input bounded-output).

Bounded input: $|x[n]| < B$, B is finite.

For bounded output $|y[n]|$:

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k].x[n-k] \right| \leq \sum_{k=-\infty}^{\infty} |h[k]| . |x[n-k]|$$

$$< \sum_{k=-\infty}^{\infty} |h[n]| . B = B. \sum_{k=-\infty}^{\infty} |h[k]|$$

y[n] will be bounded iff:

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \qquad \text{... (15a)}$$

For CT systems:

$$\int_{-\infty}^{\infty} |h(\tau)| \, d\tau < \infty \qquad \text{... (15b)}$$

Examples:

Problem 2.28 p.144.

Is the system causal and/or stable?

b) $h[n] = (0.8)^n \ u[n+2]$

Not causal, since $h[-1] = (0.8)^{-1} u[1] \neq 0.$

Stability:

$$\sum_{-\infty}^{\infty} |h[k]| = \sum_{-\infty}^{\infty} 0.8^n \ \ u[n+2] = \sum_{-2}^{\infty} 0.8^n$$

$$= \frac{1}{0.8^2} + \frac{1}{0.8} + \sum_{0}^{\infty} 0.8^n = \frac{1}{0.64} + \frac{1}{0.8} + \frac{1}{1-0.8} = 7.8125$$

Stable.

e) $h[n] = (-\frac{1}{2})^n \ u[n] + (1.01)^n \ u[n-1]$

h[n] = 0 for n < 0. Hence causal.

Stability:

$$\sum_{k=-\infty}^{\infty} \left|(-\frac{1}{2})^k\right| |u[k]| + \sum_{k=-\infty}^{\infty} |(1,01)^k \ \ u[k-1]|$$

$$= \sum_{k=0}^{\infty} (\frac{1}{2})^k + \sum_{k=1}^{\infty} (1.01)^k = \infty, \text{ as } \ k \to \infty. \text{ Unstable.}$$

g) $h[n] = n.(\frac{1}{3})^n \; u[n-1]$

h[n] =0 for n < 0. Causal.

$$\sum_{k=-\infty}^{\infty} \left| k(\frac{1}{3})^k \; u[k-1] \right| = \sum_{k=1}^{\infty} \left| k(\frac{1}{3})^k \right| \le \sum_{0}^{\infty} \left| k(\frac{1}{3})^k \right|$$

$$= \frac{\frac{1}{3}}{(1-\frac{1}{3})^2} = \frac{3}{4}. \quad \text{Stable.}$$

Causal System Described by Differential and Difference Equations:

CT Systems:

- N-th order consant-coefficient DE of an LTI:

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \qquad \text{... (16)}$$

As the system is causal, we get x(t) =0 for $t \le 0.$

This gives the initial conditions:

$$y(t_0) = \frac{dy(t_0)}{dt} = \dots = \frac{d^{N-1}y(t_0)}{dt} = 0.$$

The solution of eqn. (16) has two parts :

- Particular solution : $y_p(t)$ → solution of eqn. (16),
- Homogeneous solution: $y_h(t)$ → solution of eqn. (16) with RHS = 0.

The complete solution:

$$y(t) = y_p(t) + y_h(t).$$

DT Systems:

•A linear DT system is given by a constant-coefficient difference equation:

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k] \qquad .. (17)$$

N is the order of the system, a_k and b_k are constant.

As the system is causal, we can use the conditions of initial rest, i.e.,

If x[n] = 0, for $n < n_0$, then
y[n]= 0, for $n < n_0$.

Instead of using particular and homogeneous solutions), equation (17) can be solved very conveniently using recursive method as follows:

Rewriting eqn. (17) as:

$$y[n] = \frac{1}{a_0}\{\sum_{k=0}^{M} b_k\, x[n-k] - \sum_{k=1}^{N} a_k\, y[n-k]\} \qquad \text{... (18)}$$

If the auxiliary conditions

y[-N], y[-n+1], ..., y[-1] are given,

y[n] can be computed directly using eqn. (18).

Equations (17) and (18) are called *recursive* equations - since a recursive procedure can be used to solve it.

Special cases:

1. *N =0*, eqn. (18) yields:

$$y[n] = \frac{1}{a_0}\sum_{k=0}^{M} b_k\, x[n-k] = \sum_{k=0}^{M} \left(\frac{b_k}{a_0}\right) x[n-k] \qquad ..\ (19)$$

The solution is direct and non-recursive. No initial conditions are necessary. The impulse response h[n] is given by:

$$h[n] = y[n] = \sum_{k=0}^{M} \frac{b_k}{a_0}\delta[n-k] = \frac{1}{a_0}(b_0 + b_1 + \ldots + b_M)$$

•H[n] has only (M+1) terms – thus it is of finite duration. Such a system is called *Finite Impulse Response* ('FIR') system.

2. *N> 0.* we have to use the recursive equation. The impulse response will be of infinite duration. Such a system is called *Infinite Impulse Response* ('IIR') system.

Examples:

Prob. 2.30 p.145 of text.

Given a first order system

$$y[n] + 2y[n-1] = x[n] \qquad \text{... (20)}$$

Find the system response with initial condition of rest.

Solution:

Initial rest: $y[n] = 0$, for $n < 0$.

Impulse response: $h[n] = y[n]$ with $x[n] = \delta[n]$.

Hence,

$$h[n]=y[n]= \delta[n] - 2y[n-1], \quad \text{giving}$$

$$h[0]=y[0]= \delta[0] - 2y[-1]= 1$$

$$h[1]=y[1]= \delta[1] - 2y[0] = -2$$

$$h[2]=y[2]= \delta[2] - 2y[1] = 4$$

$$h[3]=y[3]= \delta[3] - 2y[2] = -8$$

.

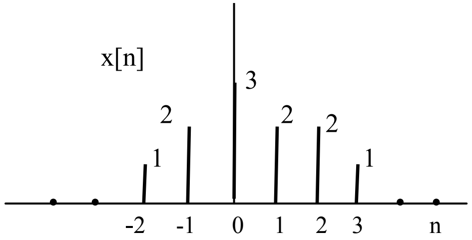
.

$$h[n]=y[n]= (-2)^n u[n].$$

Problem 2.31 p. 145 of text.

Given $y[n]+2y[n-1] = x[n]+2\,x[n-2]$,

find the system response with input x[n]:



Solution:

$x[n]= 0$, for $n< -2$, hence

$y[n] = 0$, for $n < -2$.

Now,

$y[n] = x[n] + 2x[n-2] – 2y[n-1]$. Thus,

$y[-2] = x[-2] + 2x[-4] – 2y[-3]= 1$

$y[-1] = x[-1] + 2x[-3] – 2y[-2] = 0$

$y[0] = x[0] + 2x[-2] – 2y[-1]= 5$

$y[1] = x[1] + 2x[-1] – 2y[0]= - 4$

$y[2] = x[2] + 2x[0] – 2y[1]= 16$

$y[4] = 58$, $y[5] = -114$

For $n > 5$,

$x[n]+ 2x[n-3] = 0$, hence

$y[n] = - 2\,y[n-1]$.

$y[6] = -2\,y[5]$

$y[7] = -2\,y[6] = 4\,y[5]$

$y[8] = -8\,y[5]$, ...

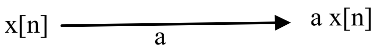
In general, $y[n] = (-2)^{n-5}\,y[5] = -114 \,.\, (-2)^{n-5}$, for $n > 5$.

Block Diagram Representation of a system

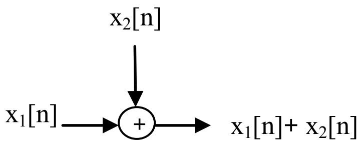
DT Systems:

Unit delay D:

Multiplier:

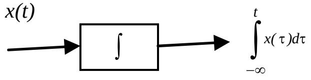


Adder:



CT Systems:

Integrator:



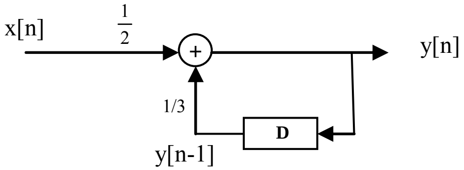
Problems:

Prob. 2.38(a), p.148 of text.

Represent the following in block diagram:

$$y[n] = \frac{1}{3} y[n-1] + \frac{1}{2} x[n]$$

The system diagram:



Prob. 2.39(a):

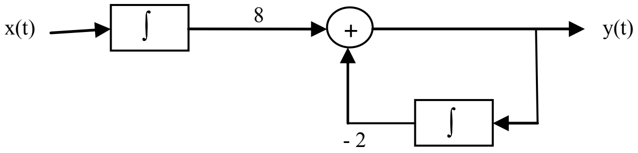
The system equation: $y(t) = -\frac{1}{2} \frac{dy(t)}{dt} + 4x(t)$

The system equation is rewritten as:

$$\frac{dy(t)}{dt} = -2\, y(t) + 8\, x(t), \quad \text{or}$$

$$y(t) = -2 \int y(t)\, dt + 8 \int x(t)\, dt$$

The system diagram:



Few Important Relations:

1.δ(t) and δ[n]:

$$\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)\ ;$$

$$\delta(\text{-}t) = \ \delta(t);$$

$$\delta[\alpha n] = \frac{1}{|\alpha|}\delta[n]\,;$$

$$\delta[\text{-}n] = \delta[n].$$

2.Convolution:

$$x(t) * \delta(t) = x(t);$$

$$x(t) * \delta(t - t_0) = x(t - t_0);$$

$$x[n] * \delta[n] = x[n];$$

$$x[n] * \delta[n - n_0] = x[n - n_0];$$

With x[n] = δ[n],

$$\delta[n] * \delta[n] = \delta[n];$$

$$\delta[n] * \delta[n - n_0] = \delta[n - n_0];$$

$$\delta[n - n_1] * \delta[n - n_0] = \delta[n - n_1 - n_0];$$

Example:

$$\{\delta[n-3] - \delta[n+10]\} * \{\delta[n+15] - \delta[n-8]\}$$

$$= \delta[n+12] -\ \delta[n+25] - \delta[n-11] +\ \delta[n+2]$$

3. Step response $s(t)$ and $s[n]$:

$$h(t) = \frac{ds(t)}{dt}$$

$$h[n] = s[n] - s[n-1]$$

$$s(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)\, d\tau = \int_{-\infty}^{t} h(\tau(\, d\tau$$

$$s[n] = h[n] * u[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k] = \sum_{k=-\infty}^{n} h[k]$$