# Assignment 2 with solution

## Question (1)

### (A)

A contractor must purchase one of three trucks for his business. The trucks vary in both costs and benefits. What criterion should be used to determine which truck to purchase if PW analysis is to be used? Choose an answer from below
A. Choose truck with lowest PW (costs)
B. Choose truck with highest PW (benefits)
C. Choose truck with coolest color
D. Choose truck with highest [PW (benefits) - PW (costs)]

Solution: D

### (B)

PW analysis with equal lives
A company with a minimum attractive rate of return (MARR) of 10% plans to install one of two elevators in one of its office buildings. Doing nothing is not an option. The elevators provide equivalent service and each has an estimated 20-year service life. Choose the appropriate elevator.

- The installed cost of elevator X is $200,000 with an annual operating and maintenance (O&M) cost of $30,000.
- The installed cost of elevator Y is $250,000 with an annual operating and maintenance (O&M) cost of $20,000.

This is the easiest case of PW analysis. Set the analysis period equal to the lives of the alternatives, choose an appropriate decision criterion, then make the PW calculations and choose an alternative.

Solution
Because the elevators provide equivalent service (same benefits), the appropriate PW analysis criterion is to minimize PW (costs).
X: PW (costs) = $200k + $30k (P/A,10%,20)
PW (costs) = $200k + $30k (8.514) = $200k + $255.42k = $455,420
Y: PW (costs) = $250k + $20k (P/A,10%,20)
PW (costs) = $250k + $20k (8.514) = $250k + $170.28k = $420,280
Install elevator Y because it has the lower PW (costs).

## Question (2)

### (A)

Consider the following cash flows and compute the equivalent annual worth at $i = 12\%$

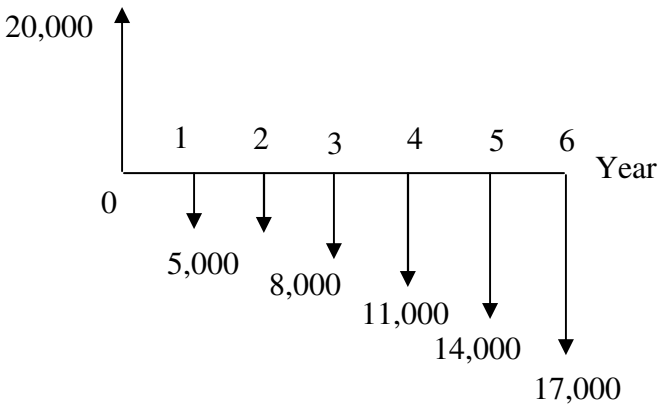
| $A_n$ | | |
|---|---|---|
| **n** | **Investment** | **Revenue** |
| 0 | -10,000 | |
| 1 | | 2000 |
| 2 | | 2000 |
| 3 | | 3000 |
| 4 | | 3000 |
| 5 | | 1000 |
| 6 | +2000 | 500 |

Solution

AE(12%)=-10,000 (A/P,12%,6)+[2,000(P/F,12%,1)+…..+2,500(P/F,12%,6)
= 180.96

### (B)

Consider the cash flow diagram. Compute the equivalent annual worth at $i = 12\%$




Solution

AE(12%)=-20,000 (A/P,12%,6)-5,000-3,000(P/G, 12%,5)(P/F,12%,1)((A/P, 12%,6)
= 4,303.13

## Question (3)

### (A)

PW analysis with unequal lives

When alternatives under consideration have unequal lives, one approach is to use an analysis period that is the least common multiple of the alternative lives. For example, if X has a 3-year life, Y has a 4-year life, and Z has a 6-year life, then a 12-year analysis period is used. In such cases, we assume that each alternative can be identically replaced at the end of its service life.

A company with an MARR of 10% plans to install one of three production machines (X, Y or Z) that provide equivalent service (same benefits). Doing nothing is not an option. The machines have zero salvage values at the end of their lives. The machines are expected to have the same annual operating and maintenance (O&M) costs, although their initial costs and service lives differ, as follows:

| Machine: | X | Y | Z |
|---|---|---|---|
| Initial cost ($) | 25,000 | 30,000 | 50,000 |
| Service life (years) | 3 | 4 | 6 |

Which machine the company should choose?

Solution

Select a 12-year analysis period, the least common multiple of 3, 4 and 6. The appropriate criterion is to select the machine with the lowest PW (cost) over the 12-year analysis period, assuming that X is identically replaced at EOY 3, EOY 6 and EOY 9; that Y is identically replaced at EOY 4 and EOY 8; and Z is identically replaced EOY 6.

X: PW (cost) = $25k + $25k (P/F,10%,3) + $25k (P/F,10%,6) + $25k (P/F,10%,9)
PW (cost) = $25k [ 1 + (P/F,10%,3) + (P/F,10%,6) + (P/F,10%,9) ]
PW (cost) = $25k [ 1 + 0.7513 + 0.5645 + 0.4241 ]
PW (cost) = $25k ( 2.7399 ) = $68,498

Y: PW (cost) = $30k [ 1 + (P/F,10%,4) + (P/F,10%,8) ]
PW (cost) = $30k [ 1 + 0.6830 + 0.4665 ]
PW (cost) = $30k ( 2.1495 ) = $64,485

Z: PW (cost) = $50k [ 1 + (P/F,10%,6) ]
PW (cost) = $50k [ 1 + 0.5645 ] = $50k ( 1.5645 ) = $78,225
Select machine Y because it has the lowest PW (cost).

**(B)**

A large food-processing corporation is considering using laser technology to speed up and eliminate waste in the potato-peeling process. To implement the system, the company anticipates needing $3 million to purchase the industrial-strength lasers. The systems will save $1,200,000 per year in labor and materials. However, it will require an additional operating and maintenance cost of $250,000. Annual income taxes will also increase by $150,000. The system is expected to have a 10-year service life and will have a salvage value of about $200,000. If the company's MARR is 18%, justify the economics of the project based on:

(a) PE method
(b) FE method
(c) AE method

Solution

**(A)**
PE(18%) = -3,000,000+[1,2000,000-250,000-150,000](P/A,18%,10)+200,000(P?f,18%,10)
=663,482
**(B)**
FE (18%) =633,482(F/P, 18%, 10)

**(C)**
AE (18%) =633,482(A/P, 18%, 10)

## Question (4)

### (A)

Consider the following project balances for a typical investment project with a service life of 4 years:

| n | $A_n$ | Project Balance |
|---|---|---|
| 0 | -1000 | -1000 |
| 1 | ( 100 ) | -1100 |
| 2 | (520 ) | -800 |
| 3 | 460 | -500 |
| 4 | ( 600 ) | 0 |

(a) Construct the original cash flows of the project.
(b) Determine the interest rate in computing the project balance.
(c) At $i$ =15%, would this project be acceptable?

Solution

***(b)***
PB(i)3 -500 = -800(1+i) +460
-800 i=-160
i= 20%

***(c )***
FE(15%) = -1000(F/P, 15%,4) +-1000(F/P, 15%,3)+ -520(F/P, 15%,2) +460(F/P, 15%,1)+600
219.78>0
Yeas the project is acceptable

(B)

Infinite analysis period (capitalized cost)
Capitalized cost is defined as the present worth of costs over an infinite analysis period.
It can be shown that the factor (*P/A,i%, N* = infinity) is equal to (1 / *i* ), with the interest rate *i* in decimal form. For example, (*P/A,5%, N* = infinity) = 1 / 0.05 = 20

Determine the capitalized cost at 15% interest of a structure with an initial cost of $200,000 and annual operating and maintenance costs of $40,000.

Solution:
P = $200,000 + $40,000 (P/A,15%,n = infinity)
P = $200,000 + $40,000 (1 / 0.15 )
P = $200,000 + $266,667 = $466,667

## Question (5)

### (A)

Consider the following two mutually exclusive investment alternatives:

| Net Cash Flow | | |
|---|---|---|
| **End of year** | **Machine A** | **Machine B** |
| 0 | -1000 | -2000 |
| 1 | 900 | 2500 |
| 2 | 800 | 800+200 |
| 3 | 700 | |

Suppose that your firm needs either machine for only 2 years. The net proceeds from the sale of machine B is estimated to be $200. What should be the required net proceeds from the sale of machine A so that both machines could be considered economically indifferent at an interest rate of 10%?

Solution

$$-1000+900(P/F,10\%,1)+((800+S_A)(P/F,10\%,2)=-2000+2500(P/F,10\%,1)+1000(P/F,10\%,2)$$

$$S_A=750 \quad (\textit{salvage value of machine A after } 2 \textit{ years})$$

**(B)**

Consider the following two mutually exclusive investment projects. Assume that the MARR=12%.

| Project's Cash Flow | | |
|---|---|---|
| **n** | **A** | **B** |
| 0 | -4200 | -2500 |
| 1 | 2610 | 1210 |
| 2 | 2930 | 1720 |
| 3 | 2300 | 1500 |

(a) Which alternative would be selected using the PE criterion?
(b) Which alternative would be selected using the FE criterion?

Solution

$(A)$

$PE(12\%)_A = -4200 + 2610(P/F,12\%,1) + 2930(P/F,12\%,2) + 2300(P/F,12\%,3)$

$= 2103.23$

$PE(12\%)_B = -2500 + 1210(P/F,12\%,1) + 1720(P/F,12\%,2) + 1500(P/F,12\%,3)$

$= 1019.20$

$select\ \text{Pr}oject\ A$

$(B)$

$FE(12\%)_A = 2103.23(F/p,12\%,3) = 2954.48$

$FE(12\%)_B = 1019.20(F/P,12\%,3) = 1431.90$

$select\ \ project\ A$

## Question (6)

An electric motor is rated at 10 horsepower (HP) and costs $800. Its full load efficiency is specified to be 85%. A newly designed, high-efficiency motor of the same size has an efficiency of 90%, but costs $1200. it is estimated that the motors will operate at a rated 10 HP output for 1500 hours a year, and the cost of energy will be $0.07 per kilowatt-hour. Each motor is expected to have a 15-year life. At the end of 15 years, the first motor will have a salvage value of $100. Consider the MARR to be 8%. (Note: 1HP = 0.7457 kW.)

(a) Determine which motor should be installed based on the PE criterion.
(b) In (a), what if the motors operated 2500 hours a year instead or 1500 hours a year? Would the same motor in (a) be the choice?

Solution

**(A)**

Required HP to produce 10 HP:

Motor A: $X_1 = 10/0.85 = 11.765$ HP

Motor B:
$X_2 = 10/0.90 = 11.11$ HP

Annual energy cost:

Motor A:
11.765(0.7457) (1,500) (0.07) = 921.18

Motor B:
11.111(0.7457) (1500)(0.07) = 869.97

Present equivalent cost
$PE\ (8\%)_A = -800-921.18(P/A, 8\%, 15) +100(P/F, 15)$

=
$PE\ (8\%)_B = -1200-869.97(P/A, 8\%, 15) =$

Motor B is preferred

**(B)**

With 2,500 operating hours:
$PE\ (8\%)_A = -800-1535.26(P/A, 8\%, 15) +50(P/F, 8\%, 15) +100(P/F, 8\%, 15)$
= -
$PE\ (8\%)_B = -1200-1449.97(P/A, 8\%, 15)$
=-
Motor B is still preferred.

## Question (7)

Consider the following two mutually exclusive investment projects, which have unequal service lives.

| Project Cash Flow | | |
|---|---|---|
| **n** | **A1** | **A2** |
| 0 | -900 | -1800 |
| 1 | -400 | -300 |
| 2 | -400 | -300 |
| 3 | -400+200 | -300 |
| 4 | | -300 |
| 5 | | -300 |
| 6 | | -300 |
| 7 | | -300 |
| 8 | | -300+500 |

(a) What assumption(s) do you need to compare a set of mutually exclusive investments with unequal service lives?
(b) With the assumption(s) defined in (a) and using $i$ = 10%, determine which project should be selected based on PE analysis.
(c) If your analysis period (study period) is just 3 years, what should be the salvage value of project A2 at the end of year 3 to make the two alternatives economically indifferent?

Solution

**(A)**
Without knowing the future replacement opportunities, we may assume that both alternatives will be available in the future with the same investment and expenses. We further assume that the required service period will be indefinite.

**(B)**
With common service period of 24 years,

Project A1:
PE (10%) $_{cycle}$ = -900-400(P/A, 10%, 3) +200(P/F, 10%, 3)
= - 1744.48
Effective interest rate for a 3 year cycle is

$(1.10)^3$-1=33.10%

PE (10%) $_{total}$ = -1744.481[1+ (P/A, 33.10%, 7)]
= -6.302.63

Project A2:
PE (10%) $_{cycle}$ = -1,800-300(P/A, 10%, 8) +500(P/F, 10%, 8)
=-3167.22
PE (10%) $_{total}$ = -3167.22[1+ (P/F, 10%, 3) + (P/F, 10%, 16)]
=-5334.03

Project A2 is preferred.

(C)

PE (10%) $_{A1}$ = -1744.48
PE (10%) $_{A2}$ = -1,800-300(P/A, 10%, 3) +S (P/F, 10%, 3)
-2546.06+0.7513S
Let PE (10%) $_{A1}$=PE (10%) A2 and solve for S
S=1,067

**Question (8)**

Annual Cash Flow Analysis (Capital investment cost)
The capital cost of an asset is the cost to purchase and install it, and then disposes of it at the end of its life. A positive salvage value at the end of the asset's life is treated as a negative cost. Note that capital costs explicitly exclude operating and maintenance cost (O&M costs). When we write any equation for cost, a negative cash flow becomes a positive cost. This is because it is the custom is not to say, for example, that a new car costs - $30,000. We say the car costs $30,000.
EAC for asset capital cost = annualized cost to purchase, install and later dispose of an asset.
Therefore, EAC for asset capital cost = P $(A/P,i\%,N)$ - S $(A/F,i\%,N)$
Using the identity $(A/F,i\%,N) = (A/P,i\%,N) - i$ we obtain:
EAC for asset capital cost = $(P - S)\ (A/P,i\%,N) + Si$
Notation:
EAC = equivalent uniform annual cost. O&M cost = operating and maintenance cost
P = asset initial cost. S = asset salvage value end of life

An asset with an initial cost of $12,000, including installation, has an estimated salvage value of $2,000 at the end of its estimated 5-year life. Using an MARR of 15%, what is the equivalent uniform annual cost of owning this asset, not including O&M costs?
$P$ = $12,000, $S$ = $2,000, $N$ = 5 years
Hint: If the MARR were zero, then the annualized capital cost would be [ $(P - S) / N$ ] or $2,000 per year. With an MARR greater than zero, however, the annualized cost will be greater than $2,000 because the money tied up in this asset for five years could presumably be earning interest at the MARR rate.

Solution:
EAC for asset capital cost = (P - S) (A/P,i%,n) + Si
= (12,000 - 2,000)(A/P,15%,5) + (2,000) ( 0.15 )
= (10,000) (0.2983) + 300
= $2,983 + $300 = $3,283 per year