10-Dec-09
(19:00-22:00)

(3)

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	All except EC
Examination	**Date**	**Pages**
Final	December 2009	3
Instructors		**Course Examiner**
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Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Only approved calculators are allowed.

MARKS

[11] 1. (i) Find $\lim_{x\to\infty} \frac{-3x^3+5x^2-7}{5-x^2} = \lim_{x\to\infty} \frac{-\frac{3x^3}{x^2}+\frac{5x^2}{x^2}-\frac{7}{x^2}}{\frac{5}{x^2}-\frac{x^2}{x^2}} = \lim_{x\to\infty} \frac{-3x+5-\frac{7}{x^2}}{\frac{5}{x^2}-1}$

$= \frac{-3(\infty)+5-0}{0-1} = +\infty$

(ii) Given $\lim_{x\to 6} f(x) = -5$ and $\lim_{x\to 6} g(x) = 4$, find

(a) $\lim_{x\to 6} [-3g(x)]$

$= -3\lim_{x\to 6} g(x)$

$= -3(4) = -12$

(b) $\lim_{x\to 6} \sqrt{g(x)}$

$\sqrt{\lim_{x\to 6} g(x)}$

$\sqrt{4} = 2$

(c) $\lim_{x\to 6} [g(x)/2f(x)]$

$\frac{\lim_{x\to 6} g(x)}{\lim_{x\to 6} 2f(x)} = \frac{4}{2\lim_{x\to 6} f(x)} = \frac{4}{2(-5)}$

$= \frac{-2}{5}$

(iii) Find the value of each of the following:

(a) $\lim_{x\to 4} \frac{x^2-16}{(x-4)}$

$\lim_{x\to 4} \frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}$

$= 8$

(b) $\lim_{x\to 4} \frac{x^2-3x+2}{(x-1)}$

$= \frac{4^2-3(4)+2}{4-1} = \frac{16-12+2}{3} = \frac{6}{3} = 2$

[5] 2. Using the definition of the derivative $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$, find the derivative if $f(x) = 3 - x^3$. $f(a) = 3 - a^3$

$f(a+h) = 3-(a+h)(a+h)(a+h) = 3-(a^3+3a^2h+3ah^2+h^3)$

$= 3-a^3-3a^2h-3ah^2-h^3$

$\lim_{h\to 0} \frac{\cancel{3}-\cancel{a^3}-3a^2h-3ah^2-h^3-\cancel{3}+\cancel{a^3}}{h} = \lim_{h\to 0} \frac{\cancel{h}(-3a^2-3ah-h^2)}{\cancel{h}}$

$= -3a^2-0-0 = -3a^2$

3. (a) If $f(x) = 2e - 5x^{16} - 3x^4$, find $f'(x)$. $= -80x^{15} - 12x^3$

(b) If $f(x) = \frac{x^2-9}{x^3+2x+5}$ find $f'(x)$.

(c) If $f(x) = \frac{x^2-3}{7\ln(x)+3}$ find $f'(x)$.

(d) If $y = e^{(5-x^2)}$, then $y' = ?$

$y' = e^{5-x^2}(-2x) = -2xe^{5-x^2}$

$= \frac{2x(x^3+2x+5)-(x^2-9)(3x^2+2)}{(x^3+2x+5)^2}$

$= \frac{2x^4+4x^2+10x-(3x^4+2x^2-27x^2-18)}{(x^3+2x+5)^2}$

$= \frac{-x^4+29x^2+10x+18}{(x^3+2x+5)^2}$

c) $f' = \frac{2x(7\ln x+3)-(x^2-3)(7/x)}{(7\ln x+3)^2} = \frac{14x\ln x+6x-(7x-\frac{21}{x})}{(7\ln x+3)^2} = \frac{14x\ln x-x+\frac{21}{x}}{(7\ln x+3)^2}$

3. (Continued) $\ln a^n = n\ln a$

(e) If $y = \ln(5x^3-5)^3$, then $\frac{dy}{dx} = ?$ $y = 3\ln(5x^3-5)$ $y' = \frac{3}{5x^3-5} \cdot 15x^2 = \frac{45x^2}{5(x^3-1)}$

$= \frac{9x^2}{x^3-1}$

(f) If $y = \frac{1}{\sqrt[5]{x-7}}$, then $\frac{dy}{dx} = ?$ $y = (x-7)^{-1/5}$ $\frac{dy}{dx} = -\frac{1}{5}(x-7)^{-6/5}$

(g) Find y' given $x^2y = e^{2y} + 7$.

$2xy + x^2y' = e^{2y} \cdot 2y'$

$x^2y' - 2e^{2y}y' = -2xy$

$y'(x^2-2e^{2y}) = -2xy$

$y' = \frac{-2xy}{x^2-2e^{2y}}$

[8] 4. Does the line tangent to the graph of $f(x) = e^x$ at $x = 1$ pass through the origin? Are there any other lines tangent to the graph of f that pass through the origin? Explain.

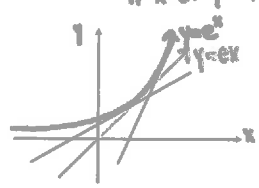
point: $f(1) = e^1 = e$ $(1, e)$ slope: $f'(x) = e^x$ $f'(1) = e$ b: $y = mx+b$

$e = e + b$

$b = 0$

tangent: $y = ex$

if $x = 0$: $y = e(0) = 0$ $\therefore$ it passes through the origin.



The graph shows that each tangent line has a unique x-intercept. $\therefore$ No other tangent passes through the origin.

[6] 5. A cube with a side of 12 centimeters is coated with ice 0.2 centimeter thick. Use differentials to estimate the volume of the ice. $V = x^3$ $dV = 3x^2\Delta x$

$= 3(12)^2(.4)$

$= 172.8\ cm^3$

Profit analysis. The total profit (in dollars) from the sale of x charcoal grills is

$$P(x) = 20x - 0.02x^2 - 320, \quad 0 \le x \le 1{,}000$$

$\bar{P} = 20 - 0.02x - \frac{320}{x}$

(a) Find the average profit per grill if 40 grills are produced. $\bar{P}(40) = 20 - .02(40) - \frac{320}{40} = 11.2$

(b) Find the marginal average profit at a production level of 40 grills, and interpret the results. $\bar{P}' = -0.02 + 320x^{-2}$ $\quad \bar{P}'(40) = -.02 + 320(40)^{-2}$ $= 0.18$

(c) Use the results from parts (a) and (b) to estimate the average profit per grill if 41 grills are produced. $\bar{P}(41) \approx 11.2 + 0.18 = \$11.38$

At a production level of 40 GRILLS the AVERAGE PROFIT is \$11.2 and is INCREASING at a rate of \$0.18 per additional GRILL produced

[9] 7. Suppose a point is moving along the graph of $x^2 + y^2 = 8$. When the point is at $(2, 2)$, its x coordinate is increasing at the rate of 0.3 units per second. How fast is the y coordinate changing at that moment?

S1: $x^2+y^2=8$ $\quad$ S2: $(2,2)$ $\quad$ S3: $2xx'+2yy'=0$ $\quad$ S4: $2(2)(.3)+2(2)y'=0$

$y' = -0.3\frac{u}{s}$

[2] 8. Compute the following:

(a) $\int (4x^2 - 7x)\, x\, dx = \int (4x^3 - 7x^2)\,dx = \frac{4x^4}{4} - \frac{7x^3}{3} + C = x^4 - \frac{7x^3}{3} + C$

(b) $\int \frac{1}{3+x^2}\, x\, dx$ $\quad u = 3+x^2$, $dx = \frac{du}{2x}$ $\quad \int \frac{1}{u} \cdot x \cdot \frac{du}{2x} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|3+x^2| + C$

(c) $\int (2x^3 + 7)^{18}\, x^2\, dx$ $\quad u = 2x^3+7$, $dx = \frac{du}{6x^2}$ $\quad = \int u^{18} x^2 \cdot \frac{du}{6x^2} = \frac{u^{19}}{19(6)} + C = \frac{(2x^3+7)^{19}}{114} + C$

(d) $\int \frac{x}{\sqrt{x+7}}\, dx$

(e) $\int e^{3-5x}\, dx$ $\quad u = 3-5x$, $dx = \frac{du}{-5}$ $\quad \int e^u \frac{du}{-5} = \frac{e^u}{-5} + C = \frac{e^{3-5x}}{-5} + C$

(f) $\int (x-3)^{-4}\, dx$ $\quad u = x-3$, $dx = du$ $\quad \int u^{-4}\,du = \frac{u^{-3}}{-3} + C = \frac{(x-3)^{-3}}{-3} + C$

d) $\int \frac{x}{\sqrt{x+7}}\, dx$ $\quad u = x+7$, $dx = du$, $x = u-7$ $\quad \int \frac{x}{u^{1/2}} \cdot du = \int \frac{u-7}{u^{1/2}}\,du = \int \left(\frac{u}{u^{1/2}} - \frac{7}{u^{1/2}}\right) du$

$= \int (u^{1/2} - 7u^{-1/2})\,du = \frac{2u^{3/2}}{3} - (2)7u^{1/2} + C = \frac{2(x+7)^{3/2}}{3} - 14(x+7)^{1/2} + C$

[7] 9. Evaluate the following integrals [accurate to 2 decimals].

(a) $\int_1^2 e^{x^2}\, x\, dx$ $\quad u = x^2$, $dx = \frac{du}{2x}$; $x=1 \to u=1$, $x=2 \to u=4$ $\quad \int_1^4 e^u x \cdot \frac{du}{2x} = \left[\frac{e^u}{2}\right]_1^4 = \frac{e^4}{2} - \frac{e^1}{2} = 25.94$

(b) $\int_0^4 (t^2 + 3)\, dt = \frac{t^3}{3} + 3t \Big|_0^4 = \frac{64}{3} + 12 = 33.33$

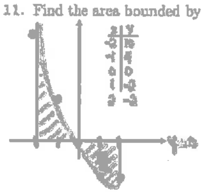
[4] 10. A note will pay \$25,000 at maturity 10 years from now. How much should you be willing to pay for the note now if money is worth 2%, compounded continuously?

$A = Pe^{rt}$ $\quad 25000 = Pe^{0.02(10)}$

$25000 = P(1.22)$

$P = \$20468.27$

[5] 11. Find the area bounded by the graphs of $y = x^2 - 3x$; $y = 0$, $-2 \le x \le 2$.



x	y
-2	10
-1	4
0	0
1	-2
2	-2

$A = \int_{-2}^{0} (x^2 - 3x)\,dx + \int_0^2 0 - (x^2 - 3x)\,dx$ $\quad -x^2+3x$

$= \frac{x^3}{3} - \frac{3x^2}{2}\Big|_{-2}^{0} + -\frac{x^3}{3} + \frac{3x^2}{2}\Big|_0^2$

$= 0 - \left(\frac{-8}{3} - 6\right) + \left(\frac{-8}{3} + 6\right) - (0)$

$= \frac{8}{3} + 6 - \frac{8}{3} + 6 = 12$

Find the interval(s) on which the graph of $f(x) = x^3 - 6x^2 + 9x + 1$ is concave upward, the interval(s) on which the graph of f is concave downward, and the inflection point(s).

$3x^2 - 12x + 9$

$6x - 12 = 0$

$x = 2$

$f(2) = 2^3 - 6(2)^2 + 9(2) + 1 = 3$ $\quad$ inflection point: $(2,3)$

	0	2	3
f''	$-$		$+$
f	$\cap$		$\cup$

concave down on: $x \in (-\infty, 2)$

concave up on: $x \in (2, \infty)$