SCHULICH School of Engineering

UNIVERSITY OF CALGARY

ENGG 407: NUMERICAL METHODS IN ENGINEERING

FINAL EXAMINATION

April 19, 2012 **3:30-6:30 pm**

Instructions:

1. Examination is open book and open lecture notes.
2. The exam consists of ***5 Problems*** and the total marks are 100. The mark allocation for each problem is also provided.
3. *To get full credit in the numerical solutions, please show all work, including the formulas, intermediate steps and derivations used, and explain clearly all steps in the procedure you followed. Partial credit will be given for incomplete solutions* ***only*** *when all steps have been clearly outlined.*
4. No laptops or calculators other than the ***Schulich*** sanctioned ones are allowed.
5. No wireless devices or earphones are allowed during exam.

Problem 1 (20 marks)

For the given matrix, compute

(a) the three eigenvalues;

(b) the (unit) eigenvector corresponding to the smallest eigenvalue;

(c) the condition number with the *1*-norm.

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 4 & 2 \\ 0 & -1 & 1 \end{pmatrix}$$

Problem 2 (25 marks)

Approximate the value of the integral $\int_0^2 xe^x\,dx$

(a) by Romberg integration, halving the initial interval three times; and

(b) by three-point Gaussian quadrature.

Round all your answers to 6 decimals.

Problem 3 (25 marks)

For the Initial-Value Problem: $y' = 2ty + t^2 e^{t^2}$, $y(0) = 0$, $0 \le t \le 1$,
which has exact solution of the form $y(t) = (t^3/3 + c)e^{t^2}$, $c = \text{constant}$,
derive the solution with $h = 0.2$ using

(a) the modified Euler method.

(b) the 3rd order Adams-Bashforth method. Use the exact solution to define the first three values required for the algorithm.

Is the solution you obtained for $y(1)$ in (b) more or less accurate than the one you obtained in (a)?

Problem 4 (20 marks)

For the Boundary-Value Problem:

$$y'' = -3y' + 2y + 2x + 3, \quad y(0) = 2, \quad y(1) = 1, \quad 0 \le x \le 1$$

(a) use the central difference formulas to discretize it and set up the tri-diagonal system of equations for $h = 0.2$.

(b) Then find the solution of the system (rounded to 6 decimals), given that $y(0.2) = 1.005311$.

Problem 5 (10 marks)

Convert the following Initial-Value Problem (IVP) ODEs to a system of simultaneous first-order IVP ODEs and change the corresponding initial conditions accordingly.

$$x'' - x + y + (3x')^2 - (y')^3 - 6y'' - 2t = 0$$

$$y''' - y'' + x' - e^x + t = 0$$

$$x(1) = 2, \; x'(1) = -4$$

$$y(1) = -2, \; y'(1) = 7, \; y''(1) = 6$$