CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	204	A
Examination	**Date**	**Pages**
Final	December 2014	2
Instructors		**Course Examiners**
A. Sierpinska, S. Shi,		E. Cohen
S. Vassileva, R. Mearns		

Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.
- ▷ All questions have equal value.

---

1. Use the Gauss-Jordan method to find all the solutions of the system:

$$\begin{array}{rcrcrcr} 2x_1 & + & 2x_2 & + & 2x_3 & = & 0 \\ -2x_1 & + & 5x_2 & + & 2x_3 & = & 1 \\ 8x_1 & + & x_2 & + & 4x_3 & = & -1 \end{array}$$

2. Determine the values of a for which the system has no solution, exactly 1 solution or infinitely many solutions:

$$\begin{array}{rcrcrcr} x & + & 2y & + & z & = & 2 \\ 2x & - & 2y & + & 3z & = & 1 \\ x & + & 2y & - & (a^2-3)z & = & a \end{array}$$

3. Find the inverse of $A = \begin{pmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{pmatrix}$, if it exists.

4. (a) Evaluate the determinant of $A = \begin{pmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}$.

   (b) Solve by Cramer's rule, when it applies:

$$\begin{array}{rcrcrcr} x_1 & & & + & 2x_3 & = & 6 \\ -3x_1 & + & 4x_2 & + & 6x_3 & = & 30 \\ -x_1 & - & 2x_2 & + & 3x_3 & = & 8 \end{array}$$

5. (a) Let $u = (1, 4, 2)$, $v = (1, 1, 0)$. Find the orthogonal projection of u on v.

   (b) Let $u_1 = (1, 1, 0), u_2 = (0, 1, 1), u_3 = (1, 0, 1)$. Find scalars c_1, c_2, c_3 such that $c_1u_1 + c_2u_2 + c_3u_3 = (1, 0, 0)$.

6. (a) Find the area of the triangle with vertices $(1, 1, 1)$, $(2, 0, 1)$, $(3, 1, 2)$. Find a vector orthogonal to the plane of the triangle.

   (b) (i) Find the distance between the point $(1, 5)$ and the line $2x = 5y - 1$.

   (ii) Find the equation of the plane containing the points $(1, 2, 1), (2, 1, 1), (1, 1, 2)$.

7. (a) Let $u = (-1, 0, 2)$, $v = (2, -1, 4)$, $w = (-1, 1, -6)$ are the vectors linearly dependent or independent?

   (b) Find the parametric equations of the line in $\mathbb{R}^3$ passing through $(1, 4, -5)$ and perpendicular to the plane $x - 3y + 2z = 4$.

8. Let $A = \begin{pmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & 2 & 0 & 6 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \\ t \\ u \end{pmatrix}$. Find a basis for the solution space of the homogeneous system $AX = 0$.

9. Find the standard matrices for the following 2 linear operators on $\mathbb{R}^2$:

   (a) a reflection about the line $y = x$.

   (b) a rotation counterclockwise of $30°$.

10. Let $A = \begin{pmatrix} -14 & 12 \\ -20 & 17 \end{pmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.