

# CHAPTER 31 Magnetism and Matter

## Answers to Understanding the Concepts Questions

1. The forces between an electron and a nucleus leave their center of mass stationary. But the nucleus is thousands or tens of thousands of times more massive than an electron, so the system's center of mass is essentially at the nucleus. The situation is much the same as Earth's orbit of the sun: the sun is so much more massive than Earth that for all practical purposes Earth orbits a stationary sun.
2. As the electron moves around the nucleus, the electric dipole moment of the system, which points from the electron to the nucleus, continuously and rapidly changes direction. The time-averaged value of the dipole moment is therefore very small, if not zero.
3. There are many ways to tell a paramagnet from a diamagnet. One way is to make the core of a solenoid out of the material and check to see if the magnetic field increases or decreases due to the presence of the core. If the field is enhanced by the core material, it must be paramagnetic; if the field is weakened by the core then it is diamagnetic.
4. As long as the surface over which we integrate the magnetic flux lies within a material of constant permeability  $\mu$ , the relation  $\vec{B} = \mu \vec{H}$  allows us to remove  $\mu$  from within the integral and express Gauss' law in terms of an integral over  $\vec{H}$ ; only if  $\mu$  varies over the surface integrated in the expression of Gauss' law do we need to keep  $\mu$  within the integral.
5. Yes. The induced emf in the sense coil is proportional to the number of turns in the coil.
6. This is so because of the difference in the magnetic properties that are desired for specific applications. Speaker coils and computer hard disks require a stable, permanent magnetic field; while transformers and computer read/write heads need a variable magnetic field that can be quickly adjusted.
7. Diamagnetism is a universal phenomenon, based on Faraday's law, so we expect it to be present in iron as well as in any other atom. However, it can be very difficult to detect if paramagnetism or ferromagnetism is present. High temperatures tend to decrease paramagnetic and ferromagnetic effects, since those effects depend on order. Thus one way to see diamagnetism is to search for it at high temperatures.
8. Aluminum is paramagnetic and are attracted by a magnet. Although the attraction is generally weak in comparison with ferromagnetic materials, it can be strong enough for the purpose of separating aluminum from the rest if a large magnet is used.
9. If we construct the latch of a magnetically hard material, the constant banging the material takes when the door is closed will not demagnetize it. Similarly, you wouldn't want your little brother to undo the effect of the latch by playing with external magnets nearby, and a magnetically hard material protects the latch from this possibility.
10. As is discussed in Question 6, a computer disk must be able to sustain a stable magnetic field since the information stored on the disk is represented by the magnetic field. This is why magnetically hard

materials must be employed.

11. Paramagnetism occurs when atoms or molecules with permanent magnetic moments align with an external magnetic field. The alignment becomes less effective as the temperature increases, because the constant jostling associated with finite temperatures increases as the temperature increases, and this jostling tends to destroy the alignment. Diamagnetism, based on Faraday's law of induction, is unaffected by the microscopic motion of thermal energy, and thus dominates over paramagnetism as the temperature increases.
12. Iron is ferromagnetic, with many magnetic domains that can be thought of as microscopic magnets. As a magnet is brought near an iron needle, the field of the magnet causes the magnetic domains of the iron needle to reorient, such that they produce a net field in the same direction as that of the magnet. The interaction of these two fields results in an attractive force between the magnet and the needle.
13. We can think of a paramagnetic material as a collection of magnetic dipoles generally pointing in the same direction. Similar to an electric dipole, which experiences a net electric force in a non-uniform external electric field pointing toward the direction of increasing electric field, A magnetic dipole, and hence a paramagnet, experiences a net magnetic force pointing toward the direction of increasing magnetic field. So it will be pulled into the region.
14. You can't. The rods will attract and repel as magnets even if one is initially unmagnetized, because the one that is initially magnetized will tend to magnetize the other. Moreover, Newton's third law holds in any case, and that law does not permit you to assign the source of the force to one or the other rod. If you had a bulb and a wire, you could make a loop and push each rod through the loop. The rod that is magnetized would induce a current that would light the bulb.
15. The orbital angular momentum of the electron is perpendicular to the plane of the orbit. To change this angular momentum, a torque perpendicular to the plane is required. Since a torque is always perpendicular to the force associated with it, the magnetic force that provides the torque must be in the plane of the orbit. Since a magnetic force is perpendicular to the magnetic field responsible for it, the field must be perpendicular to the plane of the orbit in order to change the orbital angular momentum of the electron.
16. If you had charge carriers of both signs, you could have the negative carriers circulate in one direction while the positive carriers circulate in the other in such a way that net angular momentum is zero. However, the current is all in the direction of the positive charge carrier movement, so there is a net current circulating and a net magnetic moment. If you have only one sign of charge to work with, it is not possible to have a circulating current (magnetic moment) without a net circulating momentum (angular momentum).
17. From the discussion in the textbook leading to Eq. (31-2), the definition of  $\vec{H}$ , it is clear that  $\vec{H}$  is due to the presence of ordinary, "free", current. An isolated permanent magnet is not associated with any free current; its magnetic field is solely due to its own magnetization. Therefore  $\vec{H} = 0$ .
18. The source of the original magnetic field that gave lodestones their permanent magnetic properties is Earth's magnetic field. This makes a correlation of the magnetization of lodestones of different age (in the sense of the time at which the rock was formed from, say, lava) an important geological tool. This tool traces the history of Earth's magnetic field, which is found to change rapidly at certain moments in Earth's history. These changes suggest significant abrupt changes at Earth's core, the region where Earth's magnetic field is generated.
19. The field of each bar magnet is not uniform; it is stronger near the two poles. A magnetic dipole generally

does experience a net force in a non-uniform magnetic field.

20. (a) The magnetic moment of a single current loop of area  $A$  is  $m = IA$ . In the case of a single charge  $q$  orbiting in a circle of radius  $R$  with speed  $v$  we have  $I = q/T = q/(2\pi R/v) = qv/(2\pi R)$ , and  $A = \pi R^2$ ; so  $m = \frac{1}{2} qvR$ . (b) A uniform disk rotating with angular speed  $\omega$  can be thought of as made of concentric current rings. Consider a ring of charge  $dq$ , between  $r$  and  $r + dr$ , with  $dq = \sigma dA$ . The magnetic moment of the ring is  $dm = \frac{1}{2} dq vR = \frac{1}{2} vR \sigma dA$ , where  $v = \omega r$  and  $A = \pi r^2$ . Integrate this over the entire disk to obtain  $m$ . (c) A uniform sphere can be thought of as made of parallel disks. First find  $dm$  for each disk according to the method described in part (b) above, and then integrate over all the disks.

**Solutions to Problems**

1. The magnetization is related to the external field by

$$\vec{M} = \chi_m \vec{H} = \chi_m \vec{B}_{\text{ext}} / \mu_0.$$

The magnetic dipole moment will be in the direction of the external field with magnitude

$$\begin{aligned} m &= MV = \chi_m B_{\text{ext}} V / \mu_0 \\ &= (8 \times 10^{-4})(1.0 \text{ T})\pi(1 \times 10^{-2} \text{ m})^2(5 \times 10^{-2} \text{ m}) / (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \\ &= \boxed{1.0 \times 10^{-2} \text{ A} \cdot \text{m}^2 \text{ in the direction of the magnetic field.}} \end{aligned}$$

2. The magnetic intensity is determined by the real current in the winding of the torus:

$$\begin{aligned} H &= B_{\text{ext}} / \mu_0 = \mu_0 n I / \mu_0 = n I \\ &= [(1600 \text{ turns}) / (0.34 \text{ m})](0.62 \text{ A}) = 2.9 \times 10^3 \text{ A/m.} \end{aligned}$$

We find the magnitude of the magnetic field from

$$B = \mu_0(1 + \chi_m)H = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1 + 2.8 \times 10^3)(2.9 \times 10^3 \text{ A/m}) = \boxed{10 \text{ T}}.$$

3. We find the magnetic susceptibility from

$$\begin{aligned} B &= (1 + \chi_m)B_{\text{ext}} = (1 + \chi_m)\mu_0 n I; \\ 1.907 \times 10^{-4} \text{ T} &= (1 + \chi_m)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(3400 \text{ turns/m})(0.450 \text{ A}), \text{ which gives } \chi_m = \boxed{992}. \end{aligned}$$

4. We find the current from

$$\begin{aligned} B &= (1 + \chi_m)B_{\text{ext}} = (1 + \chi_m)\mu_0 n I; \\ 3.5 \text{ T} &= [1 + (2.5 \times 10^4)](4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1300 \text{ turns/m})I, \text{ which gives } I = \boxed{0.086 \text{ A}}. \end{aligned}$$

5. We find the turn density from

$$\begin{aligned} H &= n I; \\ 30 \text{ A/m} &= n(6 \text{ A}), \text{ which gives } n = \boxed{5 \text{ turns/m}}. \end{aligned}$$

6. We find the magnetic moment by adding the magnetic moments of the turns:

$$\begin{aligned} m &= NIA = nLIA \\ &= (5 \text{ turns/m})(0.30 \text{ m})(6 \text{ A})\pi(2.5 \times 10^{-2} \text{ m})^2 = \boxed{1.8 \times 10^{-2} \text{ A} \cdot \text{m}^2}. \end{aligned}$$

7. The magnetization is the magnetic moment per unit volume:

$$\begin{aligned} m &= M/V = M / (0.20 V_E) = M / [0.20 (4\pi R_E^3 / 3)] \\ &= 3(10^{23} \text{ A} \cdot \text{m}^2) / [(0.20)4\pi (6.4 \times 10^6 \text{ m})^3] = \boxed{4.6 \times 10^2 \text{ A/m}}. \end{aligned}$$

8. The magnetic field inside the solenoid is

$$\begin{aligned} B &= (1 + \chi_m)\mu_0 n I, \text{ from which we solve for } \chi_m: \\ \chi_m &= B / \mu_0 n I - 1 = 0.20 \text{ T} / [(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(400/\text{m})(3.1 \text{ A})] - 1 = 127 \approx \boxed{1.3 \times 10^2}. \end{aligned}$$

9. The magnetic field inside the toroid is

$$\begin{aligned} B &= (1 + \chi_m)\mu_0 NI / 2\pi R, \text{ from which we solve for } I: \\ I &= 2\pi RB / [(1 + \chi_m)\mu_0 N] \\ &= 2\pi (0.087 \text{ m})(0.40 \text{ T}) / [(1 + 2.5 \times 10^4)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(487)] = \boxed{0.014 \text{ A}}. \end{aligned}$$

10. The magnetization of the toroid is

$$M = \chi_m H = \chi_m (NI / 2\pi R), \text{ so the magnetization per turn is}$$

$$M/N = \chi_m I / 2\pi R = (-2.0 \times 10^{-5})(8 \text{ A}) / [2\pi(0.20 \text{ m})] = \boxed{-1.3 \times 10^{-4} \text{ A/m}}.$$

11. (a) We find the magnetic field from

$$\vec{B} = \mu_0(1 + \chi_m) \vec{H} = (1 + \chi_m) \vec{B}_{\text{ext}},$$

so the change in the magnetic field for silver is

$$\Delta \vec{B} = \vec{B} - \vec{B}_0 = \chi_m \vec{B}_0 = \boxed{-(2.4 \times 10^{-5}) \vec{B}_0}.$$

- (b) For cupric oxide, we have

$$\Delta \vec{B} = \vec{B} - \vec{B}_0 = \chi_m \vec{B}_0 = \boxed{+(2.6 \times 10^{-4}) \vec{B}_0}.$$

12. The induced magnetization depends on the external field:

$$\begin{aligned} \vec{M} &= \chi_m \vec{H} = \chi_m \vec{B}_{\text{ext}} / \mu_0 \\ &= (-5.5 \times 10^{-6})(0.48 \text{ T}) / (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \\ &= \boxed{-0.21 \text{ A/m}} \quad (\text{opposite to the magnetic field}). \end{aligned}$$

13. We find the magnetic field from

$$\vec{B} = \mu_0(1 + \chi_m) \vec{H} = \mu \vec{H};$$

$$B = \mu n I;$$

$$0.16 \text{ T} = (1320)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(15 \times 10^2 \text{ turns/m})I, \text{ which gives}$$

$$I = \boxed{0.064 \text{ A}}.$$

14. We specify the orientation of an individual magnetic moment by the angle  $\theta_i$  from  $90^\circ$  to the external axis. The total magnetic moment along the axis of the 1 mol of atoms is

$$m = \sum m_0 \sin \theta_i = N m_0 (\sin \theta)_{\text{av}} = I A;$$

$$(6.02 \times 10^{23} \text{ atoms})(1.8 \times 10^{-23} \text{ A} \cdot \text{m}^2/\text{atom})(\sin \theta)_{\text{av}} = (0.3 \text{ A})(1 \times 10^{-4} \text{ m}^2),$$

which gives  $(\sin \theta)_{\text{av}} = 2.8 \times 10^{-6}$ .

Because the angle is small, we have

$$\theta_{\text{av}} = (\sin \theta)_{\text{av}} = 2.8 \times 10^{-6} \text{ rad} = \boxed{(1.6 \times 10^{-4})^\circ}.$$

15. (a) We let units help us find the number of electrons:

$$\begin{aligned} N &= (10 \text{ cm}^3)(7.87 \text{ g/cm}^3)[1/(55.8 \text{ g/mol})](6.02 \times 10^{23} \text{ atoms/mol})(26 \text{ electrons/atom}) \\ &= \boxed{2.2 \times 10^{25} \text{ electrons}}. \end{aligned}$$

- (b) For the magnetization of the iron we have

$$\begin{aligned} M &= (f_{\text{up}} - f_{\text{down}})m_0 N / V \\ &= (2 \times 10^{-7})(5 \times 10^{-24} \text{ A} \cdot \text{m}^2/\text{electron})(2.2 \times 10^{25} \text{ electrons}) / (10 \times 10^{-6} \text{ m}^3) \\ &= \boxed{2.2 \text{ A/m}}. \end{aligned}$$

16. The Coulomb force provides the centripetal acceleration:

$$(1/4\pi\epsilon_0)e^2/r^2 = m_e v^2/r, \quad \text{or} \quad (1/4\pi\epsilon_0)e^2/r = m_e v^2.$$

The total energy is the sum of the kinetic and potential energies:

$$E = \frac{1}{2} m_e v^2 - (1/4\pi\epsilon_0)e^2/r = \frac{1}{2} m_e v^2 - m_e v^2 = -\frac{1}{2} m_e v^2.$$

From these two equations, we get

$$v = (-2E/m_e)^{1/2}, \quad \text{and} \quad r = (1/4\pi\epsilon_0)e^2/m_e v^2 = -(1/4\pi\epsilon_0)e^2/2E.$$

The magnitude of the magnetic moment for the circular orbit is

$$\begin{aligned} m &= \frac{1}{2} e v r = \frac{1}{2} e (-2E/m_e)^{1/2} (1/4\pi\epsilon_0)e^2/2E = \frac{1}{2} (1/4\pi\epsilon_0)e^3/(-2m_e E)^{1/2} \\ &= \frac{1}{2} (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^3/[-2(9.1 \times 10^{-31} \text{ kg})(-13.5 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})]^{1/2} \\ &= \boxed{9.3 \times 10^{-24} \text{ A} \cdot \text{m}^2}. \end{aligned}$$

17. The current of the ring of total charge  $q$  due to its spin is

$I = q/T = \lambda(2\pi R)/(2\pi/\omega) = \lambda R\omega$ . Thus the magnetic moment of the ring is

$$m = IA = (\lambda R\omega)(\pi R^2) = \boxed{\lambda\pi R^3\omega}, \text{ along the axis of the ring.}$$

18. Consider a circular strip of the disk, located between  $r$  and  $r + dr$  from the center of the disk. The charge carried by the strip is

$dq = \sigma dA = \sigma(2\pi r dr)$ , and the current flowing in it is

$dI = dq/T = \sigma(2\pi r dr)/(2\pi/\omega) = \sigma\omega r dr$ . Since the area enclosed by this strip is  $A = \pi r^2$ , its

contribution to the magnetic moment of the disk is

$dm = A dI = (\pi r^2)\sigma\omega r dr$ . Integrate over the entire disk (with  $r$  going from 0 to  $R$ ):

$$m = \int_0^R \pi\sigma\omega r^3 dr = \frac{1}{4}\pi\sigma\omega R^4, \text{ along the axis of rotation.}$$

19. The magnetic moment of a circulating charge is

$$m = !evr_0.$$

For this to be equal to the Bohr magneton, we have

$$m_B = !evr_0;$$

$$9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2 = !(1.6 \times 10^{-19} \text{ C})v(2.8 \times 10^{-15} \text{ m}), \text{ which gives } v = 4.1 \times 10^{10} \text{ m/s},$$

which is greater than the speed of light by a factor of 100.

20.  $N$ , the number of circulating electrons depends on the density of atoms.

For  $N$  electrons moving at the drift speed  $v$ , the current is

$$I = -Ne/T = -Nev/2\pi R,$$

so the magnetic moment is  $m = IA = (-Nev/2\pi R)\pi R^2$ ,

and the angular momentum is  $L = Nm_e vR$ .

The gyromagnetic ratio is

$$g_L = m/L = [(-Nev/2\pi R)\pi R^2]/Nm_e vR = -e/2m_e,$$

which is independent of  $I$ ,  $R$ , and the density of atoms.

21. The external field in the torus, which is due to the current, is  $B_{\text{ext}} = \mu_0 nI$ .

The magnetic field in the core is

$$B = B_{\text{ext}} + \mu_0 M = \mu_0 nI + \mu_0 M = \mu_0(nI + M);$$

$$1.8 \text{ T} = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})[(800 \text{ turns/m})(5 \text{ A}) + M], \text{ which gives } M = \boxed{1.4 \times 10^6 \text{ A/m}}.$$

For the ratio, we have

$$\mu/\mu_0 = B/B_{\text{ext}} = B/\mu_0 nI$$

$$= (1.8 \text{ T})/(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(800 \text{ turns/m})(5 \text{ A}) = \boxed{3.6 \times 10^2}.$$

22. We find the maximum current from

$$B_{\text{max}} = (1 + \chi_m)\mu_0 nI_{\text{max}};$$

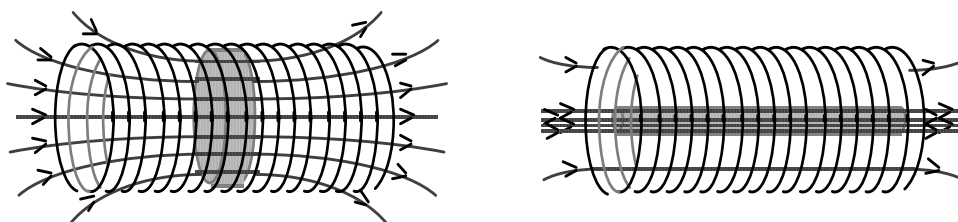
$$0.36 \text{ T} = [1 + (15 \times 10^3)](4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(14 \times 10^2 \text{ turns/m})I_{\text{max}}, \text{ which gives } I_{\text{max}} = \boxed{0.14 \text{ A}}.$$

- 23.** For the new value of  $B$  in the iron we have

$$B = (1 + \chi_m)B_{\text{ext}}$$

$$= (1 + \chi_m)(2.4 \times 10^{-3} \text{ T}) = \boxed{(2.4 \times 10^{-3} \text{ T})\chi_m}.$$

- 24.



25. The magnetic intensity inside the solenoid is  
 $H = nI = (400 \text{ turns/m})(0.5 \text{ A}) = 200 \text{ A/m}$ .

(a) The magnetic field inside the iron bar is

$$B = \mu H = (640)\mu_0 H \\ = (640)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(200 \text{ A/m}) = \boxed{0.16 \text{ T}}.$$

(b) The magnetic field inside the solenoid but outside the iron bar is

$$B_0 = \mu_0 H = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.0 \times 10^3 \text{ A/m}) = \boxed{2.5 \times 10^{-4} \text{ T}}.$$

26. We take the coil to be tightly wound. For the magnetic fields to be equal, we have

$$B_{\text{disk}} = B_{\text{coil}} = \mu_0 NI/2r; \\ 0.05 \text{ T} = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(80 \text{ turns})I/2(0.75 \times 10^{-2} \text{ m}), \text{ which gives } I = \boxed{7.5 \text{ A}}.$$

27. If we treat the coil as a tightly-wound coil, we have

$$H = NI/2R \\ = (50 \text{ turns})(0.16 \text{ A})/2(2.75 \times 10^{-2} \text{ m}) = \boxed{1.45 \times 10^2 \text{ A/m}}.$$

The magnetic field in the iron is

$$B = \mu_0(1 + \chi_m)H \\ = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})[1 + (4.8 \times 10^3)](1.45 \times 10^2 \text{ A/m}) = \boxed{0.88 \text{ T}}.$$

28. Because we take the magnetic field inside the torus to be constant, we have

$$B_{\text{ext}} = \mu_0 nI = \mu_0(N/2\pi R)I.$$

The magnetic field in the iron is

$$B = \mu H = (\mu/\mu_0)B_{\text{ext}} = 2800\mu_0 NI/2\pi R; \\ 1.5 \text{ T} = 2800(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1200)I/[2\pi(25 \times 10^{-2} \text{ m})], \text{ which gives } I = \boxed{0.56 \text{ A}}.$$

29. The maximum magnetization occurs when all the dipoles are aligned:

$$M_{\text{sat}} = (N/V)(2.2\mu_B) \\ = [(7.87 \text{ g/cm}^3)/(55.8 \text{ g/mol})](6.02 \times 10^{23} \text{ atoms/mol})(2.2)(9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2)(10^6 \text{ cm}^3/\text{m}^3) \\ = 1.73 \times 10^6 \text{ A/m}.$$

We estimate the maximum value of  $H$  by assuming that the susceptibility is constant up to this maximum magnetization:

$$M_{\text{sat}} = \chi_m H_{\text{max}}; \\ 1.73 \times 10^6 \text{ A/m} = (6000)H_{\text{max}}, \text{ which gives } H_{\text{max}} = \boxed{289 \text{ A/m}}.$$

30. The emf induced in the sense coil by the changing magnetic field of the torus is

$$\mathcal{E} = -d\Phi_B/dt = -d(NBA)/dt = -NA dB/dt.$$

The current produced by this emf is  $I = \mathcal{E}/R$ . Note that this current is not constant.

The change in magnetic field is

$$\Delta B = \int dB = -\int (\mathcal{E}/NA) dt = -(1/NA) \int \mathcal{E} dt \\ = -(1/NA) \int IR dt = -(R/NA) \int I dt.$$

Because the total charge that has passed through the coil is  $Q = \int I dt$ , we have

$$\Delta B = \boxed{-QR/NA}.$$

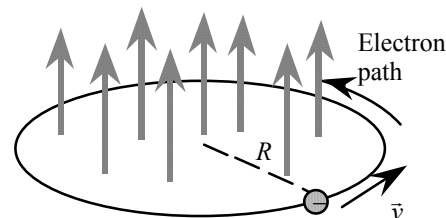
31. Using the result from Problem 24 for the magnitude of the change, we have

$$\Delta B = QR/NA;$$

$$B - 0 = (5 \times 10^{-3} \text{ C})(0.1 \Omega)/(40 \text{ turns})(0.02 \text{ m}^2), \text{ which}$$

gives

$$B = \boxed{6.3 \times 10^{-4} \text{ T}}.$$



32. We take + z-axis up and counterclockwise positive.

(a) From symmetry, we know that the electric field is circular.

We apply Faraday's law to the electron path:

$$\int \vec{E} \cdot d\vec{s} = E \int ds = -d\Phi_B/dt;$$

$$E2\pi R = -\pi R^2 dB/dt, \text{ which gives}$$

$$E = \boxed{-\frac{1}{2}R dB/dt \text{ (clockwise)}}.$$

(b) From Newton's second law, we have

$$F_{\text{tangential}} = m_e a;$$

$$-eE = -e(-\frac{1}{2}R dB/dt) = m_e dv/dt, \text{ which gives}$$

$$dv/dt = \boxed{(eR/2m_e) dB/dt}.$$

(c) We integrate to find the final speed:

$$\int_{v_i}^{v_f} dv = \int_0^{B_f} \frac{eR}{2m_e} dB;$$

$$v_f - v_i = \frac{eR}{2m_e} (B_f - 0), \text{ which gives}$$

$$v_f = \boxed{v_i + (eR/2m_e)B_f}.$$

(d) The orbital angular momentum of the electron is  $\vec{L} = m_e v R \hat{k}$ , so the change is

$$\Delta \vec{L} = \Delta(m_e v R \hat{k})$$

$$= m_e (v_f - v_i) R \hat{k} = m_e (eR/2m_e B_f) R \hat{k} = \boxed{+(eR^2/2)B_f \hat{k}}.$$

(e) The change in the orbital magnetic moment is

$$\Delta \vec{m} = -(e/2m_e) \Delta \vec{L} = -(e/2m_e) (eR^2/2) B_f \hat{k} = \boxed{-(e^2 R^2 / 4m_e) B_f \hat{k}}.$$

33. We take + z-axis up and counterclockwise positive, so the electron's velocity is negative.

From symmetry, we know that the electric field is circular.

With the positive direction counterclockwise, we apply Faraday's law to the electron path:

$$\int \vec{E} \cdot d\vec{s} = E \int ds = -d\Phi_B/dt;$$

$$E2\pi R = -\pi R^2 dB/dt, \text{ which gives } E = -\frac{1}{2}R dB/dt.$$

We have  $E < 0$  (clockwise).

From Newton's second law, we have

$$F_{\text{tangential}} = m_e a;$$

$$-eE = -e(-\frac{1}{2}R dB/dt) = m_e dv/dt, \text{ which gives } dv/dt = (eR/2m_e) dB/dt.$$

Because  $v_i < 0$ , with  $dv/dt > 0$ ; the electron slows down.

We integrate to find the final speed, with  $B_f$  the magnitude of the final magnetic field:



and Matter

$$\int_{v_i}^{v_f} dv = \int_0^{B_f} \frac{eR}{2m_e} dB;$$

$$v_f - v_i = \frac{eR}{2m_e} (B_f - 0), \text{ which gives}$$

$$v_f = v_i + (eR/2m_e)B_f.$$

Because  $v_i < 0$ , the electron's speed decreases.

The final magnetic field is  $\vec{B}_f = +B_f \hat{k}$ , and the orbital angular momentum of the electron is  $\vec{L} = m_e v R \hat{k}$ , so the change is

$$\begin{aligned} \Delta \vec{L} &= \Delta(m_e v R \hat{k}) \\ &= m_e (v_f - v_i) R \hat{k} = m_e (+eR/2m_e B_f) R \hat{k} = \boxed{+(eR^2/2)B_f \hat{k}}. \end{aligned}$$

The change in the orbital magnetic moment is

$$\Delta \vec{m} = -(e/2m_e) \Delta \vec{L} = -(e/2m_e) (eR^2/2) B_f \hat{k} = \boxed{-(e^2 R^2 / 4m_e) B_f \hat{k}}, \text{ the same result as in Problem 32.}$$

34. (a) Because the orbital angular momenta of the two electrons are in opposite directions, we have

$$\vec{m}_{\text{net}} = \vec{m}_1 + \vec{m}_2 = \boxed{0}.$$

- (b) From the results of Problems 32 and 33, we know that the electrons will have the same change in orbital magnetic moment:

$$\vec{m}_{\text{net}} = 2 \Delta \vec{m} = -2(e^2 R^2 / 4m_e) B_f \hat{k} = \boxed{-(e^2 R^2 / 2m_e) B_f \hat{k}}.$$

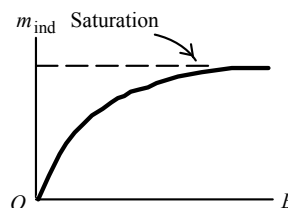
- (c) For the magnetization of a system with electron density  $\rho_e$  (the density of electron pairs is  $\rho_e/2$ ),

we have

$$\vec{M} = (\rho_e/2) \vec{m}_{\text{net}} = (\rho_e/2) (-(e^2 R^2 / 2m_e) B_f \hat{k}).$$

We find the magnetic susceptibility from

$$\chi_m = \mu_0 M / B = -(\mu_0 e^2 R^2 / 4m_e) \rho_e.$$



35. We find the density of electrons from

$$\begin{aligned} \rho_e &= (8.9 \text{ g/cm}^3)(1 \text{ mol}/63 \text{ g})(6.02 \times 10^{23} \text{ atoms/mol})(29 \text{ electrons/atom}) \\ &= 2.5 \times 10^{24} \text{ electron/cm}^3 = 2.5 \times 10^{30} \text{ electron/m}^3. \end{aligned}$$

We assume a radius of  $10^{-10}$  m. Because all of the electrons contribute the same change in orbital magnetic moment, we use the result from Problem 28 to find the magnetic susceptibility:

$$\begin{aligned} \chi_m &= -(\mu_0 e^2 R^2 / 4m_e) \rho_e \\ &= -[(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.6 \times 10^{-19} \text{ C})^2 (10^{-10} \text{ m})^2 / (4)(9.1 \times 10^{-31} \text{ kg})](2.5 \times 10^{30} \text{ electrons/m}^3) \\ &= \boxed{-2.2 \times 10^{-4}}. \end{aligned}$$

36. Initially the induced magnetic moment increases linearly. As the dipoles become aligned, the increase slows, and the induced magnetic moment approaches saturation.

37. The magnetic field produced by the free current is

$$B_{\text{ext}} = \mu_0 I / 2\pi r \text{ circular.}$$

The magnetic intensity is

$$\begin{aligned} H &= B_{\text{ext}} / \mu_0 = I / 2\pi r \text{ circular} \\ &= (10 \times 10^{-3} \text{ A}) / 2\pi r = \boxed{(1.6 \times 10^{-3} \text{ A}) / r \text{ circular}}. \end{aligned}$$

The magnetic field in the material is

$$B = \mu H = \mu_0(1 + \chi_m)H$$

$$= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})[1 + (2.6 \times 10^{-4})][(1.6 \times 10^{-3} \text{ A})/r] = \boxed{(2.0 \times 10^{-9} \text{ T} \cdot \text{m})/r \text{ circular}}$$

The change in temperature will change the susceptibility, which we find from Curie's law:

$$\chi_m T = \mu_0 C = \chi_m' T';$$

$$(2.6 \times 10^{-4})(300 \text{ K}) = \chi_m'(86 \text{ K}), \text{ which gives } \chi_m' = 9.1 \times 10^{-4}.$$

The change in the magnetic field is

$$\Delta B = \mu_0 \Delta \chi_m H$$

$$= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(9.1 \times 10^{-4} - 2.6 \times 10^{-4})[(1.6 \times 10^{-3} \text{ A})/r \text{ A/m}] = \boxed{(1.3 \times 10^{-12} \text{ T} \cdot \text{m})/r}.$$

38. We find the magnetic moment of the neutron from

$$m_n = g_S S = g_S (\hbar/2)$$

$$= -3.82(e/2M_n)(\hbar/2)$$

$$= -3.82[(1.60 \times 10^{-19} \text{ C})/2(1.67 \times 10^{-27} \text{ kg})](1.05 \times 10^{-34} \text{ J} \cdot \text{s})$$

$$= \boxed{-9.6 \times 10^{-27} \text{ A} \cdot \text{m}^2}.$$

39. We use the ideal gas law to find the density of protons:

$$n = N_A/V = N_A p/RT$$

$$= (6.02 \times 10^{23} \text{ protons/mol})(1.01 \times 10^5 \text{ Pa})/(8.314 \text{ J/mol} \cdot \text{K})(273 \text{ K}) = 2.68 \times 10^{25} \text{ protons/m}^3.$$

The magnetic moment of the proton is

$$m_p = g_S S = 5.58(e/2M_p)(\hbar/2)$$

$$= 5.58[(1.60 \times 10^{-19} \text{ C})/2(1.67 \times 10^{-27} \text{ kg})](1.05 \times 10^{-34} \text{ J} \cdot \text{s}) = 1.40 \times 10^{-26} \text{ A} \cdot \text{m}^2.$$

When all protons are aligned, we have

$$M = nm_p = (2.68 \times 10^{25} \text{ protons/m}^3)(1.40 \times 10^{-26} \text{ A} \cdot \text{m}^2) = \boxed{0.38 \text{ A/m}}.$$

We find the magnetic field inside the gas from

$$B = \mu_0 M = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.38 \text{ A/m}) = \boxed{4.7 \times 10^{-7} \text{ T}}.$$

40. We use the ideal gas law to find the density of  $^{17}\text{O}$  atoms:

$$n = N_A/V = N_A p/RT$$

$$= (6.02 \times 10^{23} \text{ atoms/mol})(1.01 \times 10^5 \text{ Pa})/(8.314 \text{ J/mol} \cdot \text{K})(273 \text{ K}) = 2.68 \times 10^{25} \text{ atoms/m}^3.$$

For the magnetization we have

$$M = (f_+ - f_-)nm$$

$$= (0.5005 - 0.4995)(2.68 \times 10^{25} \text{ molecules/m}^3)(-9.54 \times 10^{-27} \text{ A} \cdot \text{m}^2) = \boxed{-2.6 \times 10^{-4} \text{ A/m}}.$$

41. When we use  $\vec{B} = B\hat{k}$ , we have

$$d\vec{m}/dt = g_S \vec{m} \times \vec{B} = g_S(m_x \hat{i} + m_y \hat{j} + m_z \hat{k}) \times (B\hat{k});$$

$$(dm_x/dt)\hat{i} + (dm_y/dt)\hat{j} + (dm_z/dt)\hat{k} = g_S(m_y B \hat{i} - m_x B \hat{j}).$$

(a) From the  $\hat{k}$ -terms, we have

$$dm_z/dt = 0, \text{ so } m_z \text{ is a constant.}$$

(b) From the  $\hat{i}$ - and  $\hat{j}$ -terms, we have

$$dm_x/dt = g_S m_y B \quad \text{and} \quad dm_y/dt = -g_S m_x B.$$

If we differentiate  $m^2 = \vec{m} \cdot \vec{m}$ , we get

$$d(m_x^2 + m_y^2 + m_z^2)/dt = 2[(m_x dm_x/dt) + (m_y dm_y/dt) + (m_z dm_z/dt)]$$

$$= 2g_S[m_x(g_S m_y B) + m_y(-g_S m_x B) + 0] = 0,$$

so  $m_x^2 + m_y^2 + m_z^2$  is a constant.

This is consistent with the original equation, where the change in  $\vec{m}$  is perpendicular to  $\vec{m}$ .

(c) We differentiate the proposed solution,  $m_x = m_1 \cos(\omega t)$  and  $m_y = -m_1 \sin(\omega t)$ :

$$dm_x/dt = -m_1 \omega \sin(\omega t) = +\omega m_y;$$

$$dm_y/dt = -m_1 \omega \cos(\omega t) = -\omega m_x.$$

We see that these satisfy the  $\hat{i}$ - and  $\hat{j}$ -equations if  $\omega = g_S B$ .

42. We find the angular frequency from

$$\omega = g_S B = 5.58(e/2M_p)B.$$

For a magnetic field of  $50 \mu\text{T}$ , we have

$$\omega = 5.58[(1.60 \times 10^{-19} \text{ C})/2(1.67 \times 10^{-27} \text{ kg})](80 \times 10^{-6} \text{ T}) = \boxed{2.1 \times 10^4} \text{ rad/s} \quad (f = 3.3 \times 10^4 \text{ Hz}).$$

Because the relation is linear, for a change  $\Delta B$ , we have

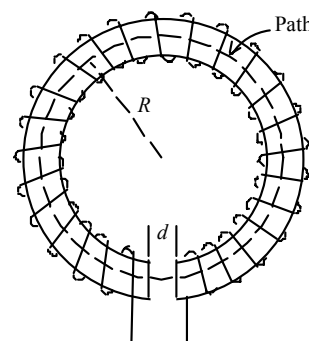
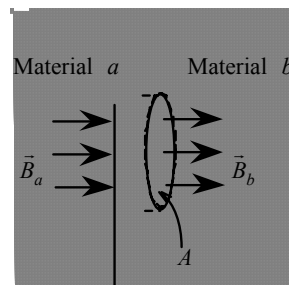
$$\begin{aligned} \Delta\omega &= 5.58(e/2M_p) \Delta B \\ &= 5.58[(1.60 \times 10^{-19} \text{ C})/2(1.67 \times 10^{-27} \text{ kg})](12 \times 10^{-9} \text{ T}) = \boxed{3.2} \text{ rad/s} \quad (\Delta f = 0.51 \text{ Hz}). \end{aligned}$$

43. For the frequency of precession of a proton, we have

$$\begin{aligned} f &= \omega/2\pi = g_S B/2\pi \\ &= 5.58(e/2M_p)(10^{-1} \text{ T})/2\pi \\ &= 5.58[(1.60 \times 10^{-19} \text{ C})/2(1.67 \times 10^{-27} \text{ kg})](10^{-1} \text{ T})/2\pi = \boxed{4.2 \times 10^6} \text{ Hz}. \end{aligned}$$

44. We use a small cylinder with its axis perpendicular to the interface as a Gaussian surface. Because the interface is perpendicular to the direction of the magnetic field, there is no flux through the sides of the cylinder. For Gauss's law we have

$$\begin{aligned} \oint \vec{B} \cdot d\vec{A} &= \iint_{\text{enda}} \vec{B} \cdot d\vec{A} + \iint_{\text{sides}} \vec{B} \cdot d\vec{A} + \iint_{\text{endb}} \vec{B} \cdot d\vec{A} = 0; \\ -B_a A + 0 + B_b A &= 0, \quad \text{so} \quad B_a = B_b. \end{aligned}$$



45. Because the gap of width  $d$  is perpendicular to the magnetic field, we know from the result of Problem 38 that the magnetic field is constant around the torus. For the magnetic intensities, we have

$$H_{\text{core}} = B/\mu, \quad \text{and} \quad H_{\text{gap}} = B/\mu_0.$$

We apply Ampere's law to a path which is a circle with radius  $R$ , the mean radius of the torus. Every turn of the coil passes through the area enclosed by this path, so we have

$$\begin{aligned} \oint \vec{H} \cdot d\vec{s} &= I_{\text{enclosed}}; \\ \int_{\text{core}} \vec{H} \cdot d\vec{s} + \int_{\text{gap}} \vec{H} \cdot d\vec{s} &= (B/\mu)(2\pi R - d) + (B/\mu_0)d = NI, \end{aligned}$$

which becomes

$$B = NI\mu_0 / [(2\pi R - d)/(\mu/\mu_0) + d].$$

For a 1 mm gap, we have

$$B = (500)(8 \text{ A})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) / \{[2\pi(0.30 \text{ m}) - 0.001 \text{ m}]/(1200) + 0.001 \text{ m}\} = \boxed{1.96 \text{ T}}.$$

For a 3 cm gap, we have

$$B = (500)(8 \text{ A})(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) / \{[2\pi(0.30 \text{ m}) - (0.03 \text{ m})]/(1200) + (0.03 \text{ m})\} = \boxed{0.16 \text{ T}}.$$

The magnetic field with no gap is 3.2 T, so we see that a small gap has a significant effect.

46. We find the density of electrons from

$$\rho_e = (3.5 \text{ g/cm}^3)(1 \text{ mol}/12 \text{ g})(6.02 \times 10^{23} \text{ atoms/mol})(6 \text{ electrons/atom}) \\ = 1.05 \times 10^{24} \text{ electrons/cm}^3 = 1.05 \times 10^{30} \text{ electrons/m}^3.$$

Using the result from Problem 28, we have

$$\chi_m = -(\mu_0 e^2 R^2 / 4m_e) \rho_e \\ = -[(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.6 \times 10^{-19} \text{ C})^2 (0.75 \times 10^{-10} \text{ m})^2 / \\ 4(9.1 \times 10^{-31} \text{ kg})](1.05 \times 10^{30} \text{ electrons/m}^3) \\ = \boxed{-5.2 \times 10^{-5} \quad (0.24 \chi_{m\text{Cu}})}.$$

47. We choose a small cylinder for a Gaussian surface. Its axis is along the axis of the torus, with one end in the gap and the other end in the ferromagnetic material. The magnetic field is parallel to the sides of the cylinder. We apply Gauss' law to the cylinder:

$$\oint \vec{B} \cdot d\vec{A} = \oint B \cdot dA = 0; \\ \iint_{\text{gap}} \vec{B} \cdot d\vec{A} + \iint_{\text{ferromagnetic}} \vec{B} \cdot d\vec{A} = 0; \\ + B_{\text{gap}} A - B_{\text{ferromagnetic}} A = 0, \text{ which gives } B_{\text{gap}} = B_{\text{ferromagnetic}}.$$

48. We need to find the external field produced by the two strips of current. Because the thickness of the strips and the separation are much smaller than the width, we assume that the field is the same as that between infinitely wide strips. This field was found in Problems 29-12 and 29-14, with the result

$$B_{\text{ext}} = \mu_0 h = \mu_0 I/w.$$

The magnetic field is

$$B = (\mu/\mu_0) B_{\text{ext}} = 650(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.0 \text{ A})/(0.025 \text{ m}) = \boxed{6.5 \times 10^{-2} \text{ T}}.$$

The magnetic intensity is

$$H = B_{\text{ext}}/\mu_0 = I/w = (2.0 \text{ A})/(0.025 \text{ m}) = \boxed{80 \text{ A/m}}.$$

49. The molar mass of cobalt is
- $M = 59 \text{ g/mol}$
- . The mass of each cobalt atom in grams is then
- $M/N_A$
- , where
- $N_A$
- is the Avogadro's number. The total mass per unit volume of cobalt is
- $\rho$
- , its density, so the number of cobalt atoms per unit volume is
- $N = \rho/(M/N_A)$
- . If the magnetic moments of all these atoms are aligned then we get the maximum possible magnetic moment per cubic centimeter:

$$m_{\text{max}} = (1.7 \mu_B)N = 1.7 \mu_B N_A \rho / M \\ = 1.7 (9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2)(6.022 \times 10^{23} / \text{mol})(8.7 \text{ g/cm}^3)/(59 \text{ g/mol}) \\ = \boxed{1.4 \text{ A} \cdot \text{m}^2 / \text{cm}^3},$$

i.e., the maximum magnetic moment per cubic centimeter of Co is  $1.4 \text{ A} \cdot \text{m}^2$ .

50. Denote the spherical shell region of between radii
- $r$
- and
- $a$
- with subscript 1 and the spherical region of radius
- $r$
- with subscript 2. We are given the expression for the magnetic moment of a
- uniformly*
- charged sphere in rotation. To unitize this result, imagine adding a certain amount of charge,
- $\Delta Q$
- , to the region between 0 and
- $r$
- to create a uniformly charged sphere of radius
- $a$
- , whose charge between
- $r$
- and
- $a$
- is
- $+Q/2$
- . To find
- $\Delta Q$
- , note that the charge density should be uniform throughout so

$$\Delta Q / Q_1 = V_2 / V_1; \\ \Delta Q / (+Q/2) = r^3 / (a^3 - r^3) = 1 / [(a/r)^3 - 1], \text{ or} \\ \Delta Q = +Q / \{2[(a/r)^3 - 1]\}.$$

The total charge of the larger, uniformly charged sphere of radius  $a$  (called region 3) would then be

$$Q_3 = Q_1 + \Delta Q = +Q/2 \{1 + 1/[(a/r)^3 - 1]\} = (+Q/2)[1/(1 - x^3)],$$

where  $x = r/a$ . The magnetic dipole moment of this uniformly charged sphere, of total charge  $Q_3$ , is

$$\mu_3 = Q_3 \omega a^2 / 5 = (Q \omega a^2 / 10) [1 / (1 - x^3)].$$

Subtract from this the dipole moment of the smaller sphere of radius  $r$  and charge  $\Delta Q$  and we obtain the dipole moment of the spherical shell between  $r$  and  $a$ :

$$\begin{aligned} \mu_1 &= \mu_3 - \Delta Q \omega r^2 / 5 = (Q \omega a^2 / 10) [1 / (1 - x^3)] - (Q \omega r^2 / 10) \{1 / [(a/r)^3 - 1]\} \\ &= (Q \omega a^2 / 10) (1 - x^5) / (1 - x^3). \end{aligned}$$

The dipole moment of the entire charge-neutral sphere in question is then

$$\mu_1 + \mu_2 = (Q \omega a^2 / 10) (1 - x^5) / (1 - x^3) + (-Q/2) \omega r^2 / 5 = 0.45 \mu = 0.45 (Q \omega a^2 / 5);$$

$$(1 - x^5) / (1 - x^3) - x^2 = 0.90;$$

$$9x^2 + 9x - 1 = 0; \text{ which gives } x = r/a \approx \boxed{0.10}.$$

51. If we approximate the magnetic field as that along the axis of the dipole, we have

$$B = (\mu_0 / 4\pi) (2m / R^3);$$

$$0.6 \times 10^{-4} \text{ T} = (10^{-7} \text{ T} \cdot \text{m} / \text{A}) 2m / (6.4 \times 10^6 \text{ m})^3, \text{ which gives } m = \boxed{8 \times 10^{22} \text{ A} \cdot \text{m}^2}.$$

The magnetization of the core is

$$M = m / V = (8 \times 10^{22} \text{ A} \cdot \text{m}^2) / [(4\pi/3)(3.2 \times 10^6 \text{ m})^3] = \boxed{6 \times 10^2 \text{ A/m}}.$$

If the magnetic moment is due to a circulating current, we have

$$m = IA;$$

$$8 \times 10^{22} \text{ A} \cdot \text{m}^2 = I\pi(3.2 \times 10^6 \text{ m})^2, \text{ which gives } I = \boxed{2 \times 10^9 \text{ A}}.$$

52. (a) For the magnetic intensity, we have

$$H = B_{\text{ext}} / \mu_0 = nI$$

$$= [450 \text{ turns} / 2\pi(12 \times 10^{-2} \text{ m})](0.8 \text{ A}) = 4.8 \times 10^2 \text{ A/m}, \text{ so}$$

$$\vec{H} = \boxed{4.8 \times 10^2 \text{ A/m circular}}.$$

- (b) For the magnetic field inside the silver, we have

$$B = \mu_0(1 + \chi_m)H = (4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A})[1 + (-2.4 \times 10^{-5})](4.8 \times 10^2 \text{ A/m}) = 6.0 \times 10^{-4} \text{ T}, \text{ so}$$

$$\vec{B} = \boxed{6.0 \times 10^{-4} \text{ T circular}}.$$

- (c) For the magnetization, we have

$$M = \chi_m H = (-2.4 \times 10^{-5})(4.8 \times 10^2 \text{ A/m}) = -1.1 \times 10^{-2} \text{ A/m}, \text{ so}$$

$$\vec{M} = \boxed{-1.1 \times 10^{-2} \text{ A/m circular}}.$$

- (d) The replacement with nickel does not affect the external field or the magnetic intensity:

$$\vec{H} = \boxed{4.8 \times 10^2 \text{ A/m circular}}.$$

For the magnetic field inside the nickel, we have

$$B = \mu_0(1 + \chi_m)H = (4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A})(1 + 95)(4.8 \times 10^2 \text{ A/m}) = 0.058 \text{ T}, \text{ so}$$

$$\vec{B} = \boxed{0.058 \text{ T circular}}.$$

For the magnetization, we have

$$M = \chi_m H = (95)(4.8 \times 10^2 \text{ A/m}) = 4.5 \times 10^4 \text{ A/m}, \text{ so}$$

$$\vec{M} = \boxed{4.5 \times 10^4 \text{ A/m circular}}.$$

53. For the charge moving in a ring of radius  $r_0$ , the effective current is

$$I = e/T = e/(2\pi/\omega) = e\omega/2\pi.$$

For the magnetic moment, we have

$$m_{\text{ring}} = IA = (e\omega/2\pi)\pi r_0^2 = !e\omega r_0^2.$$

The surface charge density on the disk is  $\sigma = e/\pi r_0^2$ . We consider the disk to be an infinite number of rings. For a differential element, we choose a ring of radius  $r$  and thickness  $dr$ , which has charge

$$dq = \sigma 2\pi r \, dr = (e/\pi r_0^2) 2\pi r \, dr = (2er/r_0^2) \, dr.$$

The effective current of the element is

$$dI = dq/T = [(2er/r_0^2) \, dr]/(2\pi/\omega) = (e\omega/\pi r_0^2) r \, dr.$$

We find the magnetic moment by integration:

$$\begin{aligned} m_{\text{disk}} &= \int dm = \int_0^{r_0} \left[ \left( \frac{e\omega}{\pi r_0^2} \right) r \, dr \right] \pi r^2 \\ &= \left( \frac{e\omega}{r_0^2} \right) \int_0^{r_0} (r^3 \, dr) = \left( \frac{e\omega}{r_0^2} \right) \left( \frac{r_0^4}{4} \right) = \frac{e\omega r_0^2}{4}. \end{aligned}$$

For the ratio of the magnetic moments of the two models, we have

$$m_{\text{ring}}/m_{\text{disk}} = (e\omega r_0^2)/(e\omega r_0^2/4) = \boxed{4}.$$

54. The combined mass of the heavier and lighter particles must be the mass of the neutron:

$$M + m = M_n.$$

The combined magnetic moments of the heavier and lighter particles must be the magnetic moment of the neutron:

$$e^+/2M - e^-/2m = -3.82(e/2M_n); \quad 1/M - 1/m = -3.82/2M_n.$$

When we use the mass equation, we get

$$1/(M_n - m) - 1/m = -3.82/2M_n,$$

which gives a quadratic equation for  $m$ :

$$1.91m^2 - 3.91M_n m + M_n^2 = 0.$$

The two solutions are  $m = 0.300M_n, 1.75M_n$ . Because  $m$  must be less than  $M_n$ , we have

$$\boxed{m = 0.300M_n = 0.50 \times 10^{-27} \text{ kg}}.$$

55. (a) We find  $C$  from the normalization condition:

$$N = \int dN = C \int_0^\pi 2\pi \sin \theta \, d\theta \, e^{(mB \cos \theta)/kT}.$$

We change variable to

$$z = (mB \cos \theta)/kT, \quad \text{with } dz = -(mB/kT) \sin \theta \, d\theta.$$

$$\begin{aligned} N &= C \int_{mB/kT}^{-mB/kT} -\frac{2\pi kT}{mB} \tilde{e} \, dz = -C \frac{2\pi kT}{mB} (\tilde{e}) \Big|_{mB/kT}^{-mB/kT} \\ &= -C \frac{2\pi kT}{mB} (e^{-mB/kT} - e^{mB/kT}), \text{ which gives} \\ C &= \left( \frac{NmB}{2\pi kT} \right) / (e^{mB/kT} - e^{-mB/kT}). \end{aligned}$$

- (b) The integrand used in part (a) represents  $dN$ , the number of systems at an angle  $\theta$ . We find  $\langle \cos \theta \rangle$  by putting  $\cos \theta$  into the integral, with the same change of variable:

$$\begin{aligned} \langle \cos \theta \rangle &= \frac{\int \cos \theta \, dN}{\int dN} = \frac{C}{N} \int_0^\pi (\cos \theta) 2\pi \sin \theta \, d\theta \, e^{(mB \cos \theta)/kT} \\ &= \frac{C}{N} \int_{mB/kT}^{-mB/kT} -2\pi \left( \frac{kT}{mB} \right)^2 z \tilde{e} \, dz = -\frac{C2\pi}{N} \left( \frac{kT}{mB} \right)^2 [\tilde{e}(z-1)] \Big|_{mB/kT}^{-mB/kT} \\ &= -\frac{C2\pi}{N} \left( \frac{kT}{mB} \right)^2 \left[ e^{-mB/kT} \left( -\frac{mB}{kT} - 1 \right) - e^{mB/kT} \left( \frac{mB}{kT} - 1 \right) \right]. \end{aligned}$$

When we use the result for  $C/N$  from part (a), we have

$$\langle \cos \theta \rangle = \frac{e^{mB/kT} + e^{-mB/kT}}{e^{mB/kT} - e^{-mB/kT}} - \frac{kT}{mB}.$$

- (c) This curve shows the saturation that occurs at large magnetic field. When the value of  $\langle \cos \theta \rangle \rightarrow 1$ ,  $\theta \rightarrow 0$  for all dipoles; the dipoles are aligned.

