

# CHAPTER 29    The Production and Properties of Magnetic Fields

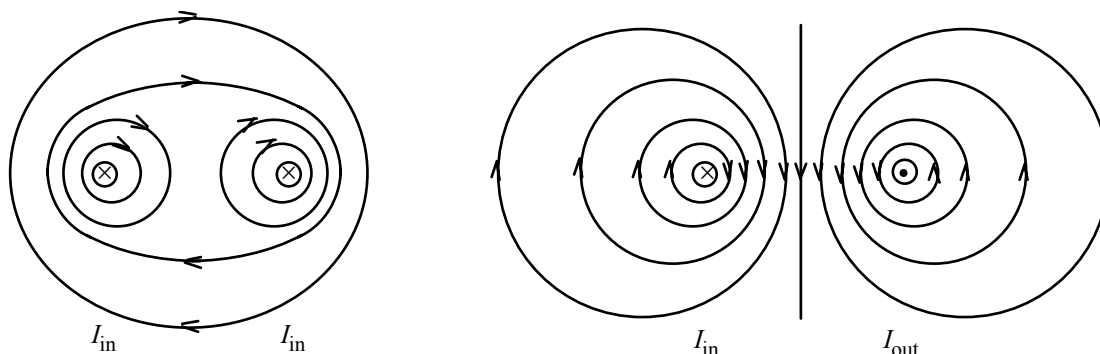
## Answers to Understanding the Concepts Questions

1. The compass needle aligns along the magnetic field lines. Thus in this case the compass needle will maintain an orientation that follows the circular path of the compass itself.
2. Yes. Each wire must be infinitely long for the definition to be exact. Note, however, that the magnetic force in question is the force *per unit length* of the infinitely long wires -- the total force between them is obviously infinity.
3. The magnetic field in the yoke of a U-shaped magnet “closes” the U-shape. The wire passing through the yoke lies roughly perpendicular to this magnetic field. When a current passes through the wire, the magnetic field exerts a force on it proportional to the vector product  $\vec{l} \times \vec{B}$ , where  $\vec{l}$  is parallel to the wire. If  $\vec{l} \times \vec{B}$  points upward, the wire will make an upward jump that will be quite sizeable if the force is large and the mass is small. If  $\vec{l} \times \vec{B}$  points downward, the wire will move downward -- assuming there is space for that to happen.
4. The magnetic field lines are still mostly trapped inside the torus, largely following its contour. If the shape of the torus is irregular, then the field outside (which is considerably weaker than that inside) would lose axial symmetry.
5. No, because the wires are electrically neutral.
6. Yes. It can be shown that, with a time-varying electric field, the conservation of charge requires the presence of the displacement current. The only case when the displacement current is not needed to satisfy the conservation of charge is when the current flow is steady, which occurs when the divergence of the current density is zero. Only in this special case can we obtain Ampere’s law without the displacement current term.
7. The source of a magnetic field is a current. As the current flows in a wire, each infinitesimal segment of the wire produces a magnetic field  $d\vec{B}$  that depends on the location, length and orientation of the segment. The total field is the sum of the contribution from each segment:  $\vec{B} = \int d\vec{B}$ . The Biot-Savart law gives an expression for  $d\vec{B}$ , from which we can then obtain  $\vec{B}$  through integration.
8. The magnetic field of a solenoid is proportional to the number of turns per unit length. To double the  $B$ -field we need to put on twice as many turns for the same length  $L$ . So we need to double the total length of the wire wound around the solenoid. Using the same type of wire, this doubling of the length would double the resistance of the wire.

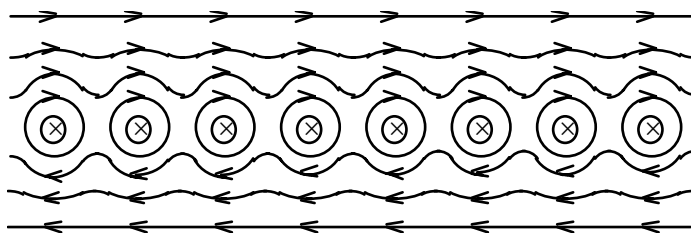
9. It is much easier to make precision measurements of forces than it is to count charges.
10. No. If the magnetic field lines are radially outward, we can then construct a Gaussian surface to capture these field lines, and the magnetic flux over the enclosed surface,  $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int |B| dA$ , would be positive, rather than zero. But this would only be possible if there was a net magnetic monopole enclosed by the Gaussian surface. We have yet to find any convincing experimental evidence suggesting the existence of magnetic monopoles.
11. The change can be summarized by the replacement of the permittivity of free space,  $\epsilon_0$ , by a permittivity associated with the material,  $\epsilon$ . In particular, the electric field will contain a factor  $1/\epsilon$  rather than  $1/\epsilon_0$ . This change in turn carries through in the discussion of the displacement current to the point where the displacement current derived in the text is multiplied by  $\epsilon/\epsilon_0$ .
12. The symmetry between the electric and magnetic fields would be complete. Not only would this make the laws of electricity and magnetism more aesthetically pleasing, which is in itself a valuable indicator of a successful theory because the laws of nature are, more often than not, beautiful; but there would also be practical consequences. Symmetrical to the behavior of an electric charge, a magnetic monopole, stationary or not, would produce a magnetic field, and the divergence of the magnetic field is therefore not necessarily zero but is rather proportional to the volume density of the monopole. As the monopole moves, it can also produce an electric field, and be subject to an electric force. If a large quantity of "free magnetic monopoles" were to exist in some materials, then a "magnetic motor", for example, could be constructed in which a coil carrying a magnetic monopole current can respond to an external electric field by spinning about its axis. The famous physicist P. A. M. Dirac even showed that, with but the existence of one magnetic monopole in the universe, the mystery of the quantization of electric charges (i.e., the fact that all electric charges are integer multiples of the fundamental charge,  $e$ ) can be understood.
13. If the current flows in the same direction, then the magnetic fields produced by both coils are in the same direction. There is a net magnetic field along the axis of the loops, and its direction is determined by the right-hand rule. If we look directly toward the loops and find the current flowing clockwise, for example, then the magnetic field at point P points away from us. If the current flows in opposite directions in the two loops, then the magnetic fields produced by the loops point in opposite directions and cancel out. The net field at point P is zero, if the magnitudes of the current in both loops are identical to each other.
14. The dimensions of electric field are [N/C]; the dimensions of magnetic field are [N/C] times the dimensions of inverse velocity, so that  $[E/B] = [v]$ , the dimensions of a velocity.
15. Statement (a) is correct. The currents in the adjacent turns in the coil are in the same direction so there is an attractive magnetic force between them, causing them to move closer each other, shortening the coil. Statement (c) is wrong since the current is due to the drifting motion of the free electrons, whose negative charges balance the positive charges carried by the ions which do not drift. The net charge of the wire is zero. Statements (b) and (c) contradict with (a) so they are obviously wrong.
16. Each turn of the helical coil carries a current flowing in the same direction as those in the other turns. Since two wires carrying currents that flow in the same direction attract each other, the coil tends to get shorter, so (a) is true. The coil remains charge-neutral, and it does produce a magnetic field.

**Solutions to Problems**

1.



2.



3. The two wires will repel each other. The force per meter on each wire is

$$\begin{aligned} F/L &= \mu_0 I_1 I_2 / 2\pi d \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(50 \times 10^3 \text{ A})(50 \times 10^3 \text{ A}) / [2\pi(30 \times 10^{-2} \text{ m})] = \boxed{1.7 \times 10^3 \text{ N/m}}. \end{aligned}$$

4. The magnetic field of a long wire depends on the distance from the wire:

$$B = (\mu_0 / 4\pi) 2I / r = (10^{-7} \text{ T} \cdot \text{m/A}) 2(17 \text{ A}) / (0.35 \text{ m}) = \boxed{9.7 \times 10^{-6} \text{ T circular}}.$$

5. If the two currents are in the same direction, they will be attracted with a force per unit length of

$$\begin{aligned} F/L &= \mu_0 I_1 I_2 / 2\pi d \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(5 \times 10^4 \text{ A})(100 \mu\text{A}) / [2\pi(4 \text{ m})] = \boxed{0.25 \mu\text{N/m attraction}}. \end{aligned}$$

6. Because the separation of the wires is much smaller than their length, we consider the two wires as two parallel long wires. The attractive force on each wire is

$$\begin{aligned} F &= \mu_0 I_1 I_2 L / 2\pi d \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(8 \times 10^{-6} \text{ A})(35 \times 10^{-6} \text{ A})(12 \times 10^{-2} \text{ m}) / [2\pi(1.2 \times 10^{-2} \text{ m})] \\ &= \boxed{5.6 \times 10^{-16} \text{ N/m attraction}}. \end{aligned}$$

7. We use the expression for the force between two wires to find the dimensions of  $\mu_0$ :

$$[\mu_0] = [F] [I]^{-2} = [MLT^{-2}] [C^{-2}T^2] = \boxed{[MLC^{-2}]} \quad (\text{SI unit: N/A}^2 \text{ or } \text{T} \cdot \text{m/A}).$$

We use the expression for the force between two charges to find the dimensions of  $\epsilon_0$ :

$$[\epsilon_0] = [Q]^2 [F]^{-1} [L]^{-2} = [C]^2 [M^{-1}L^{-1}T^2] [L^{-2}] = \boxed{[C^2M^{-1}L^{-3}T^2]} \quad (\text{SI unit: C}^2/\text{N} \cdot \text{m}^2).$$

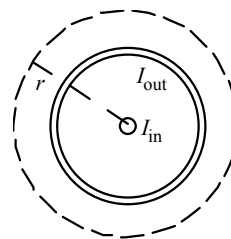
For the dimensions of the product, we have

$$[\mu_0 \epsilon_0] = [MLC^{-2}] [C^2M^{-1}L^{-3}T^2] = [T^2/L^2] = (1/\text{speed})^2.$$

The value of the speed is

$$\text{speed} = (1/\mu_0 \epsilon_0)^{1/2} = [1/(4\pi \times 10^{-7} \text{ N/A}^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)]^{1/2} = \boxed{3.00 \times 10^8 \text{ m/s}}.$$

8. From the cylindrical symmetry, we know that any field outside the cable will be circular, with a magnitude that depends only on  $r$ . We apply Ampere's law to a circular path of radius  $r$  as shown in the diagram:



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}} ;$$

$$B 2\pi r = \mu_0 (I - I) = 0, \text{ which gives } B = 0.$$

9. (a) The magnetic field of a wire with radius  $R$  is

$$B_{\text{inside}} = (\mu_0 I / 2\pi)(r/R^2), \quad B_{\text{outside}} = \mu_0 I / 2\pi r.$$

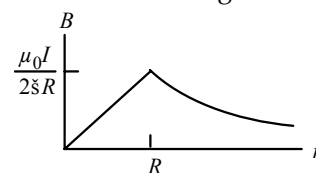
We see that the magnetic field inside increases as  $r$  increases and is maximal at  $r = R$ . The magnetic field outside decreases as  $r$  increases and is maximal at  $r = R$ . Thus the magnetic field is greatest at  $r = R$ .

- (d) (b) The maximum magnetic field is

$$B_{\text{max}} = \mu_0 I / 2\pi R \text{ at } r = R.$$

- (c) The minimum magnetic field is

$$B_{\text{min}} = 0 \text{ at } r = 0 \text{ and } r = \infty.$$



10. Because the current in the superconductor creates a field inside the wire, we estimate the maximum current from the maximum field, which is produced at the surface of the wire:

$$B = \mu_0 I / 2\pi R.$$

$$10 \text{ T} = (4\pi \times 10^{-7} \text{ N/A}^2) I / [2\pi(0.8 \times 10^{-3} \text{ m})],$$

$$\text{which gives } I = 4 \times 10^4 \text{ A}.$$

11. We choose

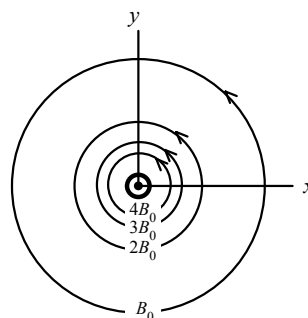
$$B_0 = \mu_0 I / 2\pi r_0.$$

For the other magnitudes, we have

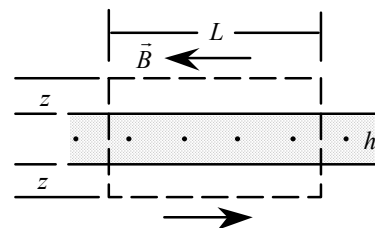
$$B_1 = 2B_0, \quad r_1 = r_0/2;$$

$$B_2 = 3B_0, \quad r_2 = r_0/3;$$

$$B_3 = 4B_0, \quad r_3 = r_0/4.$$



12. The sheet may be thought of as an infinite number of parallel wires. The figure shows a view looking directly at the current. If we consider a point above the sheet, the wire directly underneath produces a magnetic field parallel to the sheet. By considering a pair of wires symmetrically placed about the first one, we see that the net field will be parallel to the sheet. Below the sheet, the field will be in the opposite direction. We apply Ampere's law to the rectangular path shown in the diagram.



We apply Ampere's law to the rectangular path shown in the diagram.

For the sides perpendicular to the sheet,  $\vec{B}$  is perpendicular to  $d\vec{s}$ . For the sides parallel to the sheet,  $\vec{B}$  is parallel to  $d\vec{s}$  and constant in magnitude, because the upper and lower paths are equidistant from the sheet. We have

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}} ;$$

$$\int \vec{B} \cdot d\vec{s} + \int_{\text{lengths}} \vec{B} \cdot d\vec{s} = 0 + B \int_{\text{lengths}} ds = B 2L = \mu_0 h L.$$

This gives

$$B = \mu_0 h / 2 \text{ parallel to sheet and perpendicular to current (opposite directions on the two sides)}.$$

13. Assuming the ribbon is reasonably flat, we can use the result from Problem 12 for the field above a large sheet carrying a current:

$$B = \mu_0 h / 2$$

$$= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) [(120 \text{ A}) / (1 \text{ in.})(0.254 \text{ m/in.})] / 2 = \boxed{3.0 \times 10^{-3} \text{ T}}.$$

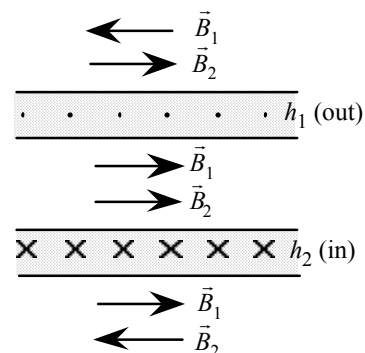
14. The magnetic field will be the sum of the two fields from the sheets. Because the magnitudes are the same, we have
- $$B = \mu_0 h \text{ parallel to the sheets in the region between the sheets;}$$
- $$B = 0 \text{ outside the sheets.}$$

If we reverse  $h_2$  to make the currents parallel, the direction of  $B_2$  will reverse, and we will have

$$B = \boxed{0 \text{ in the region between the sheets.}}$$

$$B = \boxed{\mu_0 h \text{ parallel to the sheets outside the sheets}}$$

(opposite directions on the two sides);

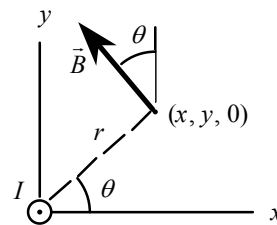


15. (a) The magnetic field of the wire is tangent to the circle of radius  $r = (x^2 + y^2)^{1/2}$ :

$$\vec{B} = \frac{\mu_0 I}{2\pi r} (-\sin \theta \hat{i} + \cos \theta \hat{j}) = \frac{\mu_0 I}{2\pi(x^2 + y^2)} (-y \hat{i} + x \hat{j}).$$

- (b) The total field from the two wires will be the sum of two expressions like the one in part (a), with  $x$  replaced by  $x - a$  or  $x + a$ :

$$\vec{B} = \frac{\mu_0 I}{2\pi} \left[ -\left\{ \frac{y}{(x-a)^2 + y^2} - \frac{y}{(x+a)^2 + y^2} \right\} \hat{i} + \left\{ \frac{x-a}{(x-a)^2 + y^2} - \frac{x+a}{(x+a)^2 + y^2} \right\} \hat{j} \right].$$



- (c) If one of the currents is reversed, both components of the field from that wire will reverse:

$$\vec{B} = \frac{\mu_0 I}{2\pi} \left[ \left\{ \frac{-y}{(x-a)^2 + y^2} - \frac{y}{(x+a)^2 + y^2} \right\} \hat{i} + \left\{ \frac{x-a}{(x-a)^2 + y^2} - \frac{x+a}{(x+a)^2 + y^2} \right\} \hat{j} \right].$$

16. From the cylindrical symmetry, we know that the magnetic field will be tangent to a circle centered on the cylindrical sheath and will depend only on the distance from the center. We use Ampere's law for a circular path of radius  $r$ .

For the path outside the sheath, we have

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}};$$

$$B_{\text{outside}} 2\pi r = \mu_0 J 2\pi R, \text{ which gives}$$

$$B_{\text{outside}} = \boxed{\mu_0 J R / r, \text{ tangent to the circle.}}$$

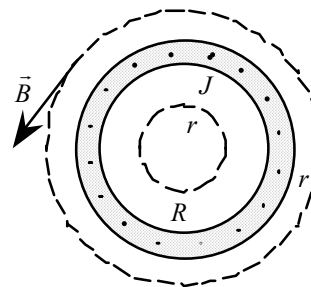
With the direction of current shown, the field is counterclockwise.

For the path inside the sheath, we have

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}};$$

$$B_{\text{inside}} 2\pi r = 0, \text{ which gives}$$

$$B_{\text{inside}} = \boxed{0}.$$



17. From the cylindrical symmetry, we know that the magnetic field will be tangent to a circle centered on the axis of the cylinders and will depend only on the distance from the axis. We use Ampere's law for a circular path of radius  $r$  midway between the inner and outer surfaces:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}};$$

$$B 2\pi r = \mu_0 I, \text{ which gives}$$

$$B = \mu_0 I / 2\pi r$$

$$= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(10 \text{ A}) / 2\pi(0.3 \times 10^{-2} \text{ m}) = \boxed{6.7 \times 10^{-4} \text{ T circular}}$$

18. We choose the  $x$ -axis along the line joining the wires, with the origin midway between the wires. Each wire will attract the other with a force that produces an acceleration. When the wires are separated by  $2x$ , for the wire at positive  $x$  we have

$$-(F/L) = (m/L)a;$$

$$-(\mu_0 I^2 / 2\pi 2x) = \lambda a, \text{ which becomes}$$

$$-(\mu_0 I^2 / 4\pi x) = \lambda a = \lambda dx/dt^2 = \lambda dv/dt = \lambda v dv/dx.$$

We rearrange to separate the variables:

$$-(\mu_0 I^2 dx) / 4\pi \lambda x = v dv,$$

and we integrate from  $x = D/2$  to  $x$ :

$$-\frac{\mu_0 I^2}{4\pi \lambda} \int_{D/2}^x \frac{dx'}{x'} = \int_0^v v' dv';$$

$$-\frac{\mu_0 I^2}{4\pi \lambda} \ln \left( \frac{x}{D/2} \right) = \frac{1}{2} v^2.$$

The motion of the wire is given by

$$dx/dt = [(\mu_0 I^2 / 2\pi \lambda) \ln(D/2x)]^{1/2}, \text{ with } x = D/2 \text{ at } t = 0.$$

The other wire will have a symmetrical motion toward the origin.

19. (a) We find the direction of the force due to the current in the wire from  $\vec{F} = q\vec{v} \times \vec{B}$ . From the diagram, we see that the force, and thus the deflection, will be away from the wire.
- (b) We assume that the time of passage and the magnetic force are small enough that the deflection is small. We can take the magnetic field at the electron beam to be constant, which gives a constant force. Because the force is away from the wire, the speed parallel to the wire is constant, and the time of passage is

$$\Delta t = L/v_0 = (1.0 \text{ m}) / (0.020)(3 \times 10^8 \text{ m/s}) = 1.67 \times 10^{-7} \text{ s}.$$

The magnetic field a distance  $d$  from the wire,  $B = \mu_0 I / 2\pi d$ , produces a force directed away from the wire:

$$F_{\perp} = qv_0 B = ev_0 \mu_0 I / 2\pi d.$$

The change in momentum created by the impulse of this force is

$$m \Delta v = F_{\perp} \Delta t = (ev_0 \mu_0 I / 2\pi d) \Delta t.$$

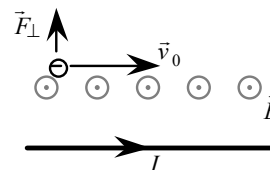
The average speed away from the wire is

$$v_{\perp} = (0 + \Delta v) / 2 = (ev_0 \mu_0 I / 4\pi m d) \Delta t, \text{ which produces a deflection}$$

$$\begin{aligned} s_{\perp} &= v_{\perp} \Delta t = (ev_0 \mu_0 I / 4\pi m d) (\Delta t)^2 = (\mu_0 / 4\pi) (ev_0 I / m d) (\Delta t)^2 \\ &= (10^{-7} \text{ T} \cdot \text{m/A}) [(1.6 \times 10^{-19} \text{ C})(6.0 \times 10^6 \text{ m/s})(0.20 \text{ A}) / \\ &\quad (9.1 \times 10^{-31} \text{ kg})(10.0 \times 10^{-2} \text{ m})] (1.67 \times 10^{-7} \text{ s})^2 = \boxed{5.9 \times 10^{-3} \text{ m}}. \end{aligned}$$

We see that our assumption of a small deflection is justified.

- (c) Because the magnetic force is always perpendicular to the velocity, the speed does not change, and



the beam will have the same energy.

20.



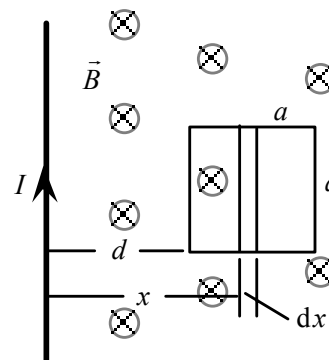
If there were magnetic monopoles, the internal field of the magnet would be in the opposite direction and would look like the electric field of an electric dipole.

21. At a distance  $x$  from the wire, the magnetic field is directed into the paper with magnitude

$$B = \mu_0 I / 2\pi x.$$

Because the field is not constant over the square, we find the magnetic flux by integration. We choose a differential element parallel to the wire at position  $x$  with area  $a \, dx$ :

$$\begin{aligned}\Phi_B &= \iint \vec{B} \cdot d\vec{A} = \iint B \, dA = \int_d^{a+d} \frac{\mu_0 I}{2\pi x} a \, dx \\ &= \frac{\mu_0 I a}{2\pi} \ln\left(\frac{a+d}{d}\right).\end{aligned}$$



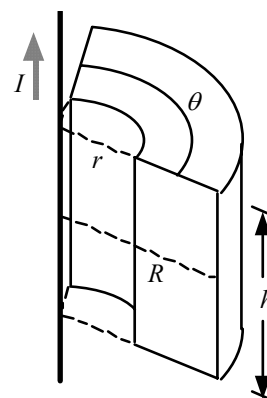
22. For a Gaussian surface, we choose a cylinder with ends of area  $A$  and its axis parallel to the  $x$ -axis. On the cylindrical surface,  $\vec{B}$  and  $d\vec{A}$  are perpendicular, so  $\vec{B} \cdot d\vec{A} = 0$ . For Gauss' law, we have

$$\begin{aligned}\Phi_B &= \oiint \vec{B} \cdot d\vec{A} = \iint_{x=x_1} (B\hat{i}) \cdot (-dA\hat{i}) + \iint_{x=x_2} (B\hat{i}) \cdot (+dA\hat{i}) \\ &= -B(x_1)A + B(x_2)A = 0, \text{ which gives} \\ B(x_2) &= B(x_1).\end{aligned}$$

Because  $x_1$  and  $x_2$  are arbitrary,  $B$  must be constant.

23. The magnetic field of the straight wire is circular, with magnitude depending on the radial distance from the wire. Over the top, bottom, and curved sides of the Gaussian surface,  $\vec{B}$  and  $d\vec{A}$  are perpendicular, so  $\vec{B} \cdot d\vec{A} = 0$ . The magnetic field is not constant over the rectangular surfaces. The field enters the front rectangle and leaves the rear rectangle. We choose a vertical differential element parallel to the wire at position  $r'$  with thickness  $dr'$ :

$$\Phi_B = \oiint \vec{B} \cdot d\vec{A} = \int_{\text{front}} \left(-\frac{\mu_0 I}{2\pi r'}\right) h \, dr' + \int_{\text{rear}} \left(+\frac{\mu_0 I}{2\pi r'}\right) h \, dr' = 0.$$





24. The area vectors on opposite sides of the parallelepiped point in opposite directions. Because the area vector for each rectangular surface is parallel to one of the coordinate axes, we use that component of  $\vec{B}$  to find the flux through the surface:

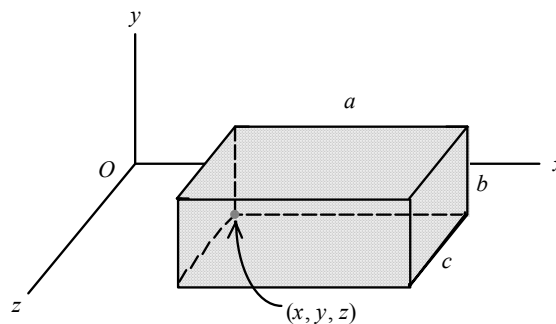
$$\begin{aligned}\Phi_B &= \oint \vec{B} \cdot d\vec{A} = 0; \\ bc[-B_x(x, y, z) + B_x(x + a, y, z)] + ac[-B_y(x, y, z) + B_y(x, y + b, z)] + ab[-B_z(x, y, z) + B_z(x, y, z + c)] \\ &= 0.\end{aligned}$$

When the dimensions are small, the definition of the partial derivative gives us

$$\begin{aligned}B_x(x + a, y, z) &= B_x(x, y, z) + a \left( \frac{\partial B}{\partial x} \right), \\ B_y(x, y + b, z) &= B_y(x, y, z) + b \left( \frac{\partial B}{\partial y} \right), \\ B_z(x, y, z + c) &= B_z(x, y, z) + c \left( \frac{\partial B}{\partial z} \right).\end{aligned}$$

When we use these equations, we get

$$(bca \frac{\partial B}{\partial x}) + (acb \frac{\partial B}{\partial y}) + (abc \frac{\partial B}{\partial z}) = 0, \quad \text{or} \quad \left( \frac{\partial B}{\partial x} \right) + \left( \frac{\partial B}{\partial y} \right) + \left( \frac{\partial B}{\partial z} \right) = 0.$$



25. We find the magnetic field at the center of the solenoid from

$$B = \mu_0 n I = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})[(320 \text{ turns})/(0.25 \text{ m})](3 \text{ A}) = \boxed{4.8 \times 10^{-3} \text{ T}}.$$

26. We find the magnetic field inside the solenoid from

$$B = \mu_0 n I = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(16 \times 10^4 \text{ turns/m})(12 \text{ A}) = \boxed{2.4 \text{ T along the axis}}.$$

27. If we assume that the cross-section of the toroid is small compared to the radius of the circle, we can use the expression for a solenoid:

$$\begin{aligned}B &= \mu_0 n I = \mu_0 (N/2\pi R) I \\ 5.1 \times 10^{-4} \text{ T} &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2500)(0.36 \text{ A})/2\pi R, \text{ which gives } R = \boxed{0.35 \text{ m}}.\end{aligned}$$

28. The magnetic field inside the torus is given by

$$B = \mu_0 N I / 2\pi r.$$

At the inner edge, we have

$$B_{\text{max}} = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(6400)(2.2 \text{ A})/[2\pi(0.30 \text{ m})] = 9.4 \times 10^{-3} \text{ T}.$$

At the outer edge, we have

$$B_{\text{min}} = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(6400)(2.2 \text{ A})/[2\pi(0.32 \text{ m})] = 8.8 \times 10^{-3} \text{ T}.$$

The range of the magnetic field is therefore  $\boxed{8.8 \times 10^{-3} \text{ T} < B < 9.4 \times 10^{-3} \text{ T}}.$

Because the range is small, we take the magnetic field at the center as the average:

$$\Delta B/B_{\text{center}} = [(0.3 \times 10^{-3} \text{ T})/(9.1 \times 10^{-3} \text{ T})]100\% = \boxed{3\% \text{ (decrease)}}.$$

29. The area of the disk is greater than the area of the solenoid. The magnetic field  $\vec{B}$  is only inside the solenoid along the axis of the solenoid and thus is parallel to the area  $\vec{A}$ . The flux is

$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = B a_{\text{solenoid}} = (0.15 \text{ T})(4 \times 10^{-4} \text{ m}^2) = \boxed{6.0 \times 10^{-5} \text{ Wb}}.$$

30. The magnetic field  $\vec{B}$  is along the axis of the solenoid and thus is parallel to the area  $\vec{A}$ .

The flux is

$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = B A_{\text{solenoid}} = \mu_0 n I \pi R^2.$$

31. The magnetic field inside the solenoid is

$$B = \mu_0 n I, \text{ where } n = N/L \text{ is the number of turns of wire per unit length. Solve for } I:$$

$$I = BL/\mu_0 N = (0.40 \times 10^{-4} \text{ T})(1.6 \text{ m})/[(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(800)] = 0.064 \text{ A} = \boxed{64 \text{ mA}}.$$

32. The magnetic field inside a toroid a distance  $R$  from the center is

$$B = \mu_0 NI/2\pi R = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1200)(25 \text{ A})/[2\pi(0.60 \text{ m})] = \boxed{1.0 \times 10^{-2} \text{ T}}.$$

33. (a) The radius of the circle is  $R = 20 \text{ m}/2\pi = 3.183 \text{ m}$ , so at the center of the circle

$$B = \mu_0 I/2R = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(22 \text{ A})/[2(3.183 \text{ m})] = \boxed{4.3 \times 10^{-6} \text{ T}}.$$

- (b) The thickness of each turn in the solenoid is just the diameter  $d$  of the wire, since it is tightly wound.

Thus the number of turns per unit length is

$$n = 1/d = 1/[(0.051 \text{ in.})(0.0254 \text{ m/in.})] = 772/\text{m}. \text{ The magnetic field inside the solenoid is then}$$

$$B = \mu_0 nI = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(772/\text{m})(22 \text{ A}) = \boxed{2.1 \times 10^{-2} \text{ T}}.$$

34. The magnetic field a distance  $r$  from the center of the toroid is

$$B = \mu_0 NI/2\pi r = C/r, \text{ where } C \text{ is a constant independent of } r.$$

Let the distance from the center of the toroid to the axis of its cross-section be  $R$ , and its minimum radius be  $R_{\min} = R - x$ , then its maximum radius is  $R_{\max} = R + x$ . We have

$$B_{\text{center}} = C/R; \quad B_{\min} = C/R_{\max} = C/(R + x); \quad B_{\max} = C/R_{\min} = C/(R - x).$$

Clearly, the greater the value of  $x$ , the greater the variation in  $B$ . If  $B$  is not to vary for more than 15%, then the corresponding maximum value of  $x$  must satisfy

$$\begin{aligned} \Delta B/B_{\text{center}} &= (B_{\max} - B_{\min})/B_{\text{center}} \\ &= [C/(R - x) - C/(R + x)]/(C/R) = 2xR/(R^2 - x^2) = 15\% = 0.15. \end{aligned}$$

The positive solution to this quadratic equation is  $x = 0.0746R$ . But we know that  $R = R_{\min} + x$ , so

$$x = 0.0746R = 0.0746(R_{\min} + x); \quad x = 0.0806R_{\min}. \text{ Thus}$$

$$R_{\max} = R + x = R_{\min} + 2x = R_{\min} [1 + 2(0.0806)] = (76 \text{ cm})(1.16) = \boxed{88 \text{ cm}}.$$

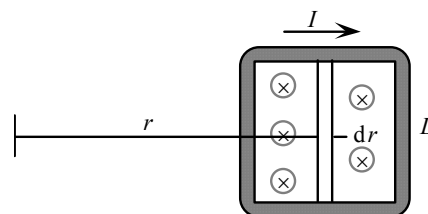
35. The magnetic field inside the toroidal solenoid is circular and varies with the distance from the center of the torus:

$$B = \mu_0 NI/2\pi r.$$

We find the flux by integration. We choose a differential element at radius  $r$  with area  $L dr$ :

$$\begin{aligned} \Phi_B &= \oint \vec{B} \cdot d\vec{A} = \int_R^{R+L} \frac{\mu_0 NI}{2\pi r} L dr \\ &= \frac{\mu_0 NIL}{2\pi} \int_R^{R+L} \frac{dr}{r}, \text{ which gives} \end{aligned}$$

$$\Phi_B = \boxed{(\mu_0 NIL/2\pi) \ln[(R + L)/R]}.$$



36. (a) Each turn has a length of 4.4 cm. The resistance of the 500 turns of wire is

$$R = \rho L/A = (1.72 \times 10^{-8} \Omega \cdot \text{m})(500)(0.044 \text{ m})/\pi(0.125 \times 10^{-3} \text{ m})^2 = 7.7 \Omega.$$

The current in the coil is

$$I = V/R = (1.5 \text{ V})/(7.7 \Omega) = 0.19 \text{ A}.$$

The magnetic field inside the torus is given by

$$B = \mu_0 NI/2\pi r.$$

For the maximum field at the inner edge we have

$$B_{\max} = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(500)(0.19 \text{ A})/2\pi(0.15 \text{ m}) = \boxed{1.3 \times 10^{-4} \text{ T}}.$$

For the minimum field at the outer edge we have

$$B_{\min} = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(500)(0.19 \text{ A})/2\pi(0.15 \text{ m} + 0.011 \text{ m}) = \boxed{1.2 \times 10^{-4} \text{ T}}.$$

- (b) From the result for Problem 31, we have

$$\Phi_B = (\mu_0 NIL/2\pi) \ln[(r + L)/r]$$

$$= [(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(500)(0.19 \text{ A})(0.011 \text{ m})/2\pi] \ln[(16.1 \text{ cm})/(15 \text{ cm})] = \boxed{1.5 \times 10^{-8} \text{ Wb}}.$$

- (c) The rate at which heat is produced in the winding is

$P = IV = (0.19 \text{ A})(1.5 \text{ V}) = \boxed{0.29 \text{ W}}$ . This is small enough that air cooling is probably sufficient.

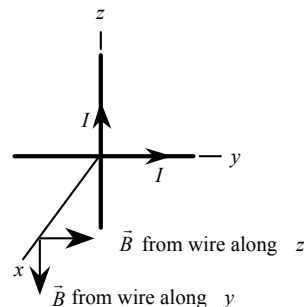
37. The magnetic field from each wire will be circular. At a distance
- $x$
- along the
- $x$
- axis, the magnitude of each field will be

$$B = \mu_0 I / 2\pi x.$$

The directions are shown in the figure, so we have

$$\vec{B} = (\mu_0 I / 2\pi x)(\hat{j} - \hat{k}), \text{ or}$$

$$B = \boxed{\sqrt{2}(\mu_0 I / 2\pi x)} \text{ in the } yz\text{-plane } 45^\circ \text{ below the } y\text{-axis}.$$



38. We find the magnetic dipole moment from

$$\mu = IA = (36 \times 10^{-3} \text{ A})(0.017 \text{ m})(0.150 \text{ m}) = \boxed{9.2 \times 10^{-5} \text{ A} \cdot \text{m}^2}.$$

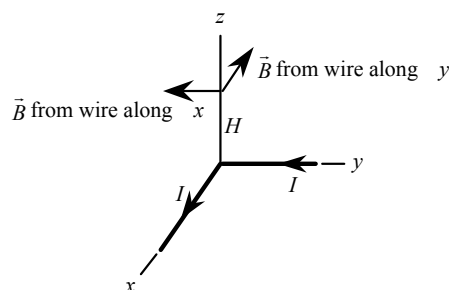
39. The magnitude of the field from each segment of the wire will be one-half the field from an infinite wire:

$$B = \mu_0 I / 4\pi H.$$

The directions are shown in the figure, so we have

$$\vec{B} = (\mu_0 I / 4\pi H)(-\hat{i} - \hat{j}), \text{ or}$$

$$B = \boxed{\sqrt{2}(\mu_0 I / 4\pi H)} \\ \text{in the } xy\text{-plane } 45^\circ \text{ from the } -x\text{-axis and the } -y\text{-axis}.$$



40. We place the wire along the
- $x$
- axis. We choose a differential element
- $d\vec{\ell}$
- that is parallel to the wire, so we have

$$d\vec{\ell} = d\ell \hat{i}.$$

The displacement from the element to the point where we want to find the magnetic field is  $\vec{r} = r\hat{i}$ .

We find the magnetic field of the differential element from the Biot-Savart law:

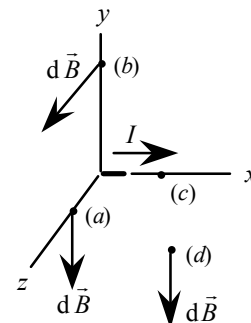
$$d\vec{B} = (\mu_0 / 4\pi) I (d\vec{\ell} \times \vec{r}) / r^3 = (\mu_0 / 4\pi) I [(d\ell \hat{i}) \times (r\hat{i})] / r^3 = \boxed{0}.$$

41. The differential element to be used in the Biot-Savart law is
- $d\vec{\ell} = d\ell \hat{i}$
- .

$$\begin{aligned} \text{(a)} \quad d\vec{B} &= (\mu_0 / 4\pi) I [d\vec{\ell} \times (\vec{r}_a / r_a^3)] = (\mu_0 / 4\pi) I [(d\ell \hat{i}) \times (r_a \hat{k} / r_a^3)] \\ &= [(\mu_0 / 4\pi) I dL / r_a^2] (-\hat{j}) = -(10^{-7} \text{ T} \cdot \text{m/A})(2 \text{ A}) dL / (3 \times 10^{-2} \text{ m})^2 \hat{j} \\ &= \boxed{-(2.2 \times 10^{-4} \text{ T/m}) dL \hat{j}}. \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad d\vec{B} &= (\mu_0 / 4\pi) I [d\vec{\ell} \times (\vec{r}_b / r_b^3)] = (\mu_0 / 4\pi) I [(d\ell \hat{i}) \times (r_b \hat{j} / r_b^3)] \\ &= [(\mu_0 / 4\pi) I dL / r_b^2] \hat{k} = -(10^{-7} \text{ T} \cdot \text{m/A})(2 \text{ A}) dL / (6 \times 10^{-2} \text{ m})^2 \hat{k} \\ &= \boxed{(0.56 \times 10^{-4} \text{ T/m}) dL \hat{k}}. \end{aligned}$$

$$\text{(c)} \quad d\vec{B} = (\mu_0 / 4\pi) I [d\vec{\ell} \times (\vec{r}_c / r_c^3)] = (\mu_0 / 4\pi) I [(d\ell \hat{i}) \times (r_c \hat{i} / r_c^3)] = \boxed{0}.$$



$$\begin{aligned}
 \text{(d) } d\vec{B} &= (\mu_0/4\pi)I[d\vec{\ell} \times (\vec{r}_d/r_d^3)] = (\mu_0/4\pi)I\{d\ell \hat{i} \times [(r_d/\sqrt{2})(\hat{i} + \hat{k})/r_d^3]\} \\
 &= [(\mu_0/4\pi)I dL/r_d^2\sqrt{2}](\hat{j}) = -(10^{-7} \text{ T} \cdot \text{m/A})(2 \text{ A}) dL/(6\sqrt{2} \times 10^{-2} \text{ m})^2\sqrt{2}\hat{j} \\
 &= \boxed{-(0.20 \times 10^{-4} \text{ T/m}) dL\hat{j}}.
 \end{aligned}$$

42. The rotating ring is equivalent to a circular current, which has a magnitude

$$I = Q/T = Qf.$$

The magnetic field at the center of a ring is

$$\begin{aligned}
 B &= \mu_0 I/2R = \mu_0 Qf/2R \\
 &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(8 \times 10^{-7} \text{ C})(120 \text{ rev/s})/2(3.5 \times 10^{-2} \text{ m}) = \boxed{1.7 \times 10^{-9} \text{ T along the axis}}.
 \end{aligned}$$

43. We can treat the disk as an infinite number of rings. We choose a differential element of radius  $r$  and thickness  $dr$ . The charge density on the disk is  $\sigma = Q/\pi R^2$ , so the current in the ring is

$$dI = \sigma dA/T = (Q/\pi R^2)f2\pi r dr = 2Qf r dr/R^2.$$

The field from all the rings is along the axis, so we integrate the magnitudes:

$$\begin{aligned}
 B &= \int |dB| = \int_{r=0}^R \frac{\mu_0 dI}{2r} = \int_0^R \frac{\mu_0 2Qf r dr}{2rR^2} = \frac{\mu_0 Qf}{R^2} \int_0^R dr = \frac{\mu_0 Qf}{R} \\
 &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2 \times 10^{-5} \text{ C})(200 \text{ rev/s})/(2.5 \times 10^{-2} \text{ m}) = \boxed{2.0 \times 10^{-7} \text{ T along the axis}}.
 \end{aligned}$$

44. The segment of the wire on the  $x$ -axis produces zero magnetic field at the origin, as you can check easily from the Biot-Savart law (as  $d\vec{\ell} \times \vec{r} = 0$ ). Furthermore, the two equal segments parallel to the  $y$ -axis produce no net magnetic field, since the current through them flow in opposite directions. That leaves only the 30-cm segment at  $y = 10$  cm. Its contribution to the  $B$ -field at the origin is found from Eq. (29-21), with  $L = 30$  cm and  $D = 10$  cm:

$$\begin{aligned}
 B &= \mu_0 IL/\{4\pi D[(L/2)^2 + D^2]^{1/2}\} \\
 &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(10 \text{ A})(0.30 \text{ m})/\{4\pi(0.10 \text{ m})[(0.15 \text{ m})^2 + (0.10 \text{ m})^2]\} \\
 &= \boxed{1.7 \times 10^{-5} \text{ T, in the negative } z\text{-direction}}.
 \end{aligned}$$

45. Each of the four segments of the rectangle produces a magnetic field in the same direction, perpendicular to the loop. So the magnitude of the total magnetic field is the sum of those of the four individual fields.

Following Eq. (29.21), with  $L = a$ ,  $D = b/2$  for the two segments of length  $a$  and  $L = b$ ,

$D = a/2$  for the other two segments of length  $b$ , we obtain

$$\begin{aligned}
 B &= 2\mu_0 Ia/\{4\pi(b/2)[(a/2)^2 + (b/2)^2]^{1/2}\} + 2\mu_0 Ib/\{4\pi(a/2)[(b/2)^2 + (a/2)^2]^{1/2}\} \\
 &= \boxed{2\mu_0 I(a^2 + b^2)^{1/2}/\pi ab}.
 \end{aligned}$$

If  $a = b$ , then the expression for  $B$  above reduces to

$$B = \boxed{2\sqrt{2}\mu_0 I/\pi a}. \quad \text{If } b \gg a \text{ then } B \approx \boxed{2\mu_0 I/\pi a}.$$

This expression is expected, as it is essentially the  $B$ -field midway between two infinitely long wires, a distance  $a$  apart, each carrying a current  $I$  opposite to each other. The field of each wire is  $\mu_0 I/2\pi(a/2) = \mu_0 I/\pi a$ , and the total field is twice as much.

46. The magnetic field from each of the sides will be directed out of the page. We can use the result of Example 29-6 for the magnitude of the field from one side:

$$B_{\text{side}} = (\mu_0 I/4\pi)L/D[(!L)^2 + D^2]^{1/2}.$$

At the center of the square, we have  $D = !L$ , so the total field is

$$B_{\text{square}} = 4(\mu_0 I/4\pi)L/!L[(!L)^2 + (!L)^2]^{1/2} = \boxed{4\mu_0 I/\pi L\sqrt{2} \text{ out of the page}}.$$

For the circular loop, we have

$$B_{\text{circle}} = \mu_0 I / 2R = \mu_0 I / 2(\pi L) = \mu_0 I / L.$$

The field of the square is

$$B_{\text{square}} = (4/\pi\sqrt{2})(\mu_0 I / L) = \boxed{0.900 B_{\text{circle}}}.$$

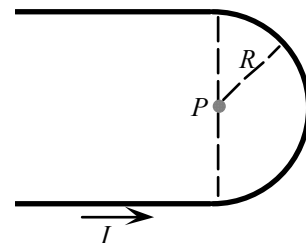
47. The magnetic field at  $P$  is the sum of three fields:

$$\vec{B}_P = \vec{B}_{\text{bottom}} + \vec{B}_{\text{semicircle}} + \vec{B}_{\text{top}},$$

all of which are directed out of the page, so the total field will be directed out of the page.

The field for the top and bottom wires is one-half the field for an infinite straight wire, and the field for the semicircle is one-half the field for a circular ring:

$$\begin{aligned} B_P &= \frac{1}{2}(\mu_0 I / 2\pi R) + \frac{1}{2}(\mu_0 I / 2R) + \frac{1}{2}(\mu_0 I / 2\pi R) \\ &= (\mu_0 / 4\pi)(I/R)(2 + \pi) = (10^{-7} \text{ T} \cdot \text{m/A})(2 + \pi) I/R \\ &= \boxed{(5.14 \times 10^{-7} \text{ T} \cdot \text{m/A}) I/R \text{ out of the page.}} \end{aligned}$$



48. The magnetic field at  $(x, y)$  is the sum of two fields:

$$\vec{B} = \vec{B}_{\text{left wire}} + \vec{B}_{\text{bottom wire}},$$

both of which are out of the page, so the total field will be directed out of the page, in the  $z$ -direction.

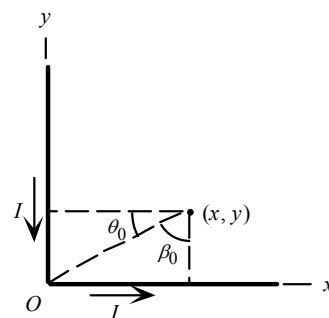
We use the method from Example 29-6 to set up the integral for each part of the wire, with the only change being in the limits of the integral. Using the intermediate result from Example 29-6 and the angles defined in the figure, we have

$$\begin{aligned} \vec{B}_{\text{left}} &= \frac{\mu_0 I \hat{k}}{4\pi x} \int_{-\pi/2}^{\theta_0} \cos \theta d\theta = \frac{\mu_0 I \hat{k}}{4\pi x} (\sin \theta_0 + 1) = \frac{\mu_0 I}{4\pi x} \left( 1 + \frac{y}{\sqrt{x^2 + y^2}} \right) \hat{k}; \\ \vec{B}_{\text{bottom}} &= \frac{\mu_0 I \hat{k}}{4\pi y} \int_{-\beta_0}^{\pi/2} \cos \beta d\beta = \frac{\mu_0 I \hat{k}}{4\pi y} (1 + \sin \beta_0) = \frac{\mu_0 I}{4\pi y} \left( 1 + \frac{x}{\sqrt{x^2 + y^2}} \right) \hat{k}. \end{aligned}$$

Note that the two results are the same, except for an interchange of  $x$  and  $y$ .

The total field is

$$\begin{aligned} \vec{B} &= \frac{\mu_0 I}{4\pi x} \left( 1 + \frac{y}{\sqrt{x^2 + y^2}} \right) \hat{k} + \frac{\mu_0 I}{4\pi y} \left( 1 + \frac{x}{\sqrt{x^2 + y^2}} \right) \hat{k} \\ &= \frac{\mu_0 I}{4\pi} \left[ \frac{1}{x} + \frac{1}{y} \left( \frac{y}{x} + \frac{x}{y} \right) \frac{1}{\sqrt{x^2 + y^2}} \right] \hat{k} = \frac{\mu_0 I}{4\pi} \left( \frac{x + y + \sqrt{x^2 + y^2}}{xy} \right) \hat{k}. \end{aligned}$$



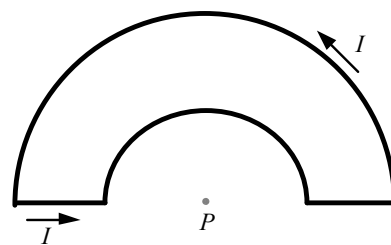
49. Because the point  $P$  is along the line of the two straight segments of the wire, there is no magnetic field from these segments.

The magnetic field at the point  $P$  is the sum of two fields:

$$\vec{B} = \vec{B}_{\text{inner semicircle}} + \vec{B}_{\text{outer semicircle}}.$$

Each field is half that of a circular loop, with the field of the inner semicircle into the page and that of the outer semicircle out of the page, so we subtract the two magnitudes:

$$\begin{aligned} B &= \frac{1}{2}(\mu_0 I / 2R_{\text{inner}}) - \frac{1}{2}(\mu_0 I / 2R_{\text{outer}}) \\ &= (\mu_0 I / 4)(1/R_{\text{inner}} - 1/R_{\text{outer}}) \\ &= (\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(12 \text{ A})[1/(0.05 \text{ m}) - 1/(0.08 \text{ m})], \text{ which gives} \\ B &= \boxed{2.8 \times 10^{-5} \text{ T into the page.}} \end{aligned}$$

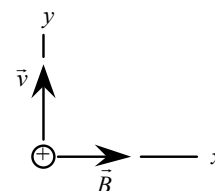


50. We use the result of Problem 46 for a square loop and multiply by the number of loops:

$$B = NB_{\text{square}} = 4\mu_0 IN / \pi L \sqrt{2}$$

$$= 4(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.7 \text{ A})(80 \text{ loops}) / [\pi(0.05 \text{ m})\sqrt{2}] = \boxed{1.3 \times 10^{-3} \text{ T}}$$

along the axis.



51. We choose the coordinate system shown in the figure.

The magnetic field of the dipole will be directed along the  $x$ -axis:

$$\vec{B} = (\mu_0 / 2\pi)(\mu / d^3) \hat{i}.$$

The force on the charge produces the acceleration of the charge:

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a};$$

$$q(v\hat{j}) \times [(\mu_0 / 2\pi)(\mu / d^3) \hat{i}] = qv(\mu_0 / 2\pi)(\mu / d^3)(-\hat{k}) = m\vec{a},$$

so the acceleration of the charge is

$$\vec{a} = -(\mu_0 / 2\pi)(qv\mu / md^3) \hat{k}.$$

Because the acceleration is perpendicular to the velocity, the charge will start to move in a circle with radius

$$R = mv / qB = mv / q(\mu_0 / 2\pi)(\mu / d^3) = \boxed{2\pi mv d^3 / q\mu\mu_0}.$$

52. The magnetic field of a circular current loop is

$$\text{at the center of the loop: } B_c = \mu_0 I / R;$$

$$\text{at a distance } x \text{ along the axis: } B_x = \mu_0 IR^2 / (R^2 + x^2)^{3/2}.$$

The ratio of the two fields is

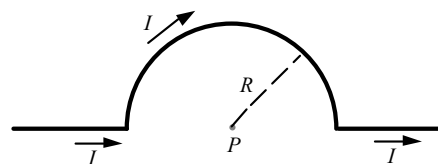
$$B_x / B_c = R^3 / (R^2 + x^2)^{3/2} = 1 / [1 + (x/R)^2]^{3/2}.$$

For a ratio of 0.90, we have

$$B_x / B_c = 0.5 = 1 / [1 + (x/R)^2]^{3/2}, \text{ which gives } x = \boxed{0.77R}.$$

For a ratio of 0.10, we have

$$B_x / B_c = 0.01 = 1 / [1 + (x/R)^2]^{3/2}, \text{ which gives } x = \boxed{4.5R}.$$

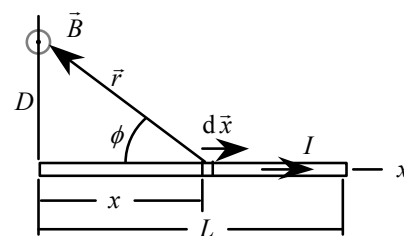


53. Because the point  $P$  is along the line of the two straight segments of the wire, there is no magnetic field from these segments. The magnetic field at the point  $P$  is the field of the semicircle:

$$B = B_{\text{semicircle}} = \mu_0 I / 2R$$

$$= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(8 \text{ A}) / [2(1.2 \times 10^{-2} \text{ m})]$$

$$= \boxed{2.1 \times 10^{-4} \text{ T into the page}}.$$



54. We place the wire along the  $x$ -axis. We choose the differential element  $dx$  and use the angle  $\phi$  indicated on the figure to specify the location of the point where we want to find the magnetic field.

From the figure, we see that  $\tan \phi = D/x$  and  $\sin \phi = D/r$ .

The angle  $\phi$  will vary from  $\pi/2$  to  $\phi_0 = \tan^{-1}(D/L)$ .

We relate the change in angle to the change in  $x$  from

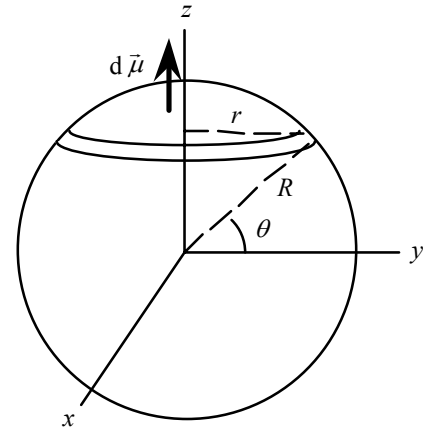
$$x = D / \tan \phi;$$

$$dx = -D[(\sec^2 \phi) / (\tan^2 \phi)] d\phi = -(D / \sin^2 \phi) d\phi.$$

The field from all the differential elements will be circular about the

wire, so the integration of  $d\vec{B}$  becomes a scalar integration of the magnitude:

$$\begin{aligned}
 B &= \int |\mathrm{d}\vec{B}| = \int_{x=0}^{x=L} \frac{\mu_0 I}{4\pi} \frac{|\mathrm{d}\vec{x} \times \vec{r}|}{r^3} = \frac{\mu_0 I}{4\pi} \int_{x=0}^{x=L} \frac{dx r \sin(\pi - \phi)}{r^3} \\
 &= \frac{\mu_0 I}{4\pi} \int_{\pi/2}^{\phi_0} \frac{[-D/\sin^2(\pi - \phi)] d\phi \sin \phi}{(D/\sin \phi)^2} = \frac{\mu_0 I}{4\pi D} (-\cos \phi) \Big|_{\pi/2}^{\phi_0}, \text{ so} \\
 B &= (\mu_0 I / 4\pi D) \cos \phi_0. \text{ With } \cos \phi_0 = L / (L^2 + D^2)^{1/2}, \text{ we get} \\
 B &= \boxed{(\mu_0 I / 4\pi D) [L / (L^2 + D^2)^{1/2}] \text{ circular}}.
 \end{aligned}$$



55. The charge density on the shell is  $\sigma = Q / 4\pi R^2$ . We choose a ring centered on the z-axis at an angle  $\theta$  from the xy-plane with thickness  $R d\theta$  for a differential element. The ring has a radius  $r = R \cos \theta$  and a charge

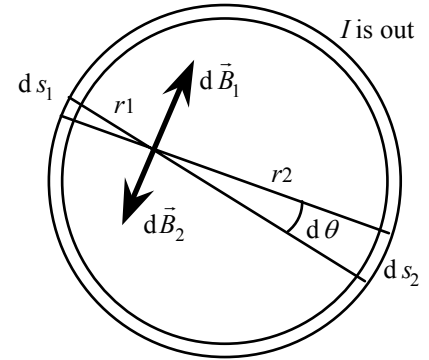
$$dq = \sigma dA = (Q / 4\pi R^2)(2\pi r R d\theta) = (Qr / 2R) d\theta.$$

Because the time for one revolution is  $T = 2\pi / \omega$ , the effective current of the ring is

$$dI = dq / T = (Qr / 2R)(\omega / 2\pi) = (Q\omega r / 4\pi R) d\theta.$$

We find the magnetic dipole moment by integration:

$$\begin{aligned}
 \vec{\mu} &= \int d\vec{\mu} = \int_{\theta=-\pi/2}^{\theta=+\pi/2} \pi r^2 dI \hat{k} \\
 &= \int_{\theta=-\pi/2}^{\theta=+\pi/2} \left( \frac{Q\omega r}{4\pi R} \right) \pi r^2 d\theta \hat{k} \\
 &= \left( \frac{Q\omega}{4R} \right) \int_{\theta=-\pi/2}^{\theta=+\pi/2} r^3 d\theta \hat{k} \\
 &= \left( \frac{Q\omega}{4R} \right) \int_{-\pi/2}^{+\pi/2} (R \cos \theta)^3 d\theta \hat{k} \\
 &= \left( \frac{Q\omega}{4R} \right) \left( \sin \theta - \frac{1}{3} \sin^3 \theta \right) \Big|_{-\pi/2}^{+\pi/2} \hat{k}, \text{ which gives} \\
 \vec{\mu} &= \boxed{\frac{1}{3} Q \omega R^2 \hat{k}}.
 \end{aligned}$$



56. We consider two diametrically opposite strips of the pipe as shown in the end view. Each strip subtends the same angle  $d\theta$  at a point  $P$  inside the pipe, so their widths are  $ds_1 = r_1 d\theta$  and  $ds_2 = r_2 d\theta$ . The currents in the two strips are

$$dI_1 = (I / 2\pi R) ds_1 = (Ir_1 / 2\pi R) d\theta,$$

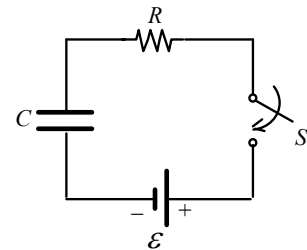
$$dI_2 = (I / 2\pi R) ds_2 = (Ir_2 / 2\pi R) d\theta.$$

Each strip is equivalent to an infinite straight wire, so the magnitudes of the two fields at  $P$  are

$$dB_1 = \mu_0 dI_1 / 2\pi r_1 = (\mu_0 I / 4\pi^2 R) d\theta,$$

$$dB_2 = \mu_0 dI_2 / 2\pi r_2 = (\mu_0 I / 4\pi^2 R) d\theta = dB_1.$$

Because the two fields are directed in opposite directions, the net field from the two strips is zero. The cylinder can be considered to consist of an infinite number of similar pairs, so the total field inside will be zero.



57. For the charge on the capacitor in the RC circuit, we have

$$Q = Q_0(1 - e^{-t/RC}) = C\mathcal{E}(1 - e^{-t/RC}).$$

The electric field between the plates is

$$E = \sigma / \epsilon_0 = Q / \epsilon_0 A, \text{ so the electric flux is}$$

$$\Phi_E = EA = Q / \epsilon_0.$$

We find the displacement current from

$$I_d = \epsilon_0 (d\Phi_E / dt) = dQ / dt = (C\mathcal{E} / RC) e^{-t/RC} = \boxed{(\mathcal{E} / R) e^{-t/RC}}.$$

58. When the switch has been closed for a long time, the charge on the capacitor is  $Q = Q_0 = C\mathcal{E}$ . After the switch is opened, there is no closed circuit. If we assume that there is no leakage, this charge remains constant, so the electric field remains constant. The displacement current is

$$I_d = \epsilon_0 (d\Phi_E / dt) = \boxed{0}.$$

59. The current through the closed path is the portion of the displacement current, which must have the magnitude of the charging current. We use Ampere's law:

$$\begin{aligned} \oint \vec{B} \cdot d\vec{\tau} &= \mu_0 I_{d, \text{enclosed}} = \mu_0 (A_{\text{enclosed}} / A_{\text{total}}) I \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}) [(5.0 \times 10^{-2} \text{ m}^2) / (0.25 \text{ m}^2)] (0.2 \text{ A}) = \boxed{5.0 \times 10^{-8} \text{ T} \cdot \text{m}}. \end{aligned}$$

60. (a) The potential across the capacitor plates is  $V = Q / C$ , so the rate of change of the voltage is

$$\begin{aligned} dV / dt &= (1 / C) dQ / dt = I / C \\ &= (15 \times 10^{-6} \text{ A}) / (3.5 \times 10^{-6} \text{ F}) = \boxed{4.3 \text{ V/s}}. \end{aligned}$$

- (b) The electric flux between the plates is

$$\Phi_E = EA = Q / \epsilon_0 = CV / \epsilon_0.$$

The rate of change of the flux is

$$\begin{aligned} d\Phi_E / dt &= (C / \epsilon_0) (dV / dt) \\ &= [(3.5 \times 10^{-6} \text{ F}) / (8.85 \times 10^{-12} \text{ F/m})] (4.3 \text{ V/s}) = \boxed{1.7 \times 10^6 \text{ V} \cdot \text{m/s}}. \end{aligned}$$

The displacement current is

$$\begin{aligned} I_d &= \epsilon_0 (d\Phi_E / dt) \\ &= (8.85 \times 10^{-12} \text{ F/m}) (1.7 \times 10^6 \text{ V} \cdot \text{m/s}) = \boxed{1.5 \times 10^{-5} \text{ A}}. \end{aligned}$$

61. If we assume a parallel-plate capacitor, the electric field in the capacitor is

$$E = V / d = (V_0 / d) \cos(\omega t), \text{ so the electric flux is}$$

$$\Phi_E = EA = (AV_0 / d) \cos(\omega t).$$

The displacement current is

$$I_d = \epsilon_0 A (dE / dt) = (\epsilon_0 AV_0 / d) [-\sin(\omega t)] \omega = \boxed{-CV_0 \omega \sin(\omega t)}.$$

Note that this is the charging current.

62. (a) If we assume a parallel-plate capacitor, the electric field in the capacitor is

$$E = V / d = (V_0 / d) \cos(\omega t), \text{ so the rate of change of the field is}$$

$$dE / dt = -(V_0 / d) \omega \sin(\omega t).$$

The maximum rate of change will occur when  $\sin(\omega t) = -1$ :

$$\begin{aligned} (dE / dt)_{\text{max}} &= V_0 \omega / d \\ &= (0.1 \text{ V}) (2 \times 10^4 \text{ rad/s}) / (1.5 \times 10^{-2} \text{ m}) = \boxed{1.33 \times 10^5 \text{ V/m} \cdot \text{s}}. \end{aligned}$$

- (b) The current leading to the capacitor is also the displacement current:

$$I = I_d = \epsilon_0 A (dE / dt) = (-\epsilon_0 A / d) V_0 \omega \sin(\omega t) = -CV_0 \omega \sin(\omega t).$$

The maximum value of the current is

$$\begin{aligned} I_{\text{max}} &= CV_0 \omega \\ &= (5 \times 10^{-9} \text{ F}) (0.1 \text{ V}) (2 \times 10^4 \text{ rad/s}) = 1.0 \times 10^{-5} \text{ A} = \boxed{10 \mu\text{A}}. \end{aligned}$$



63. From the charge density  $\sigma = \sigma_0(1 - t/t_0)$ , we find the rate at which the charge is changing:

$$d\sigma/dt = -\sigma_0/t_0.$$

The electric field at the surface is that of a point charge,  $Q = \sigma 4\pi R^2$ . We find the electric flux through a spherical surface just outside the conducting sphere:

$$\Phi_E = EA = (1/4\pi\epsilon_0)(\sigma 4\pi R^2/R^2)(4\pi R^2) = \sigma 4\pi R^2/\epsilon_0.$$

The displacement current is

$$I_d = \epsilon_0 d\Phi_E/dt = \epsilon_0(4\pi R^2/\epsilon_0) d\sigma/dt = \boxed{-4\pi R^2\sigma_0/t_0}.$$

The current in the wire is

$$I = dq/dt = A d\sigma/dt = \boxed{-4\pi R^2\sigma_0/t_0} = I_d.$$

64. Label the three wires as 1, 2, and 3, starting from the one on the left. The force per unit length exerted by wire 2 on wire 1 is

$$\begin{aligned} F_{12} &= \mu_0 I_1 I_2 / 2\pi d_{12} \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(200 \text{ A})(300 \text{ A}) / [2\pi(0.20 \text{ m})] \\ &= 0.060 \text{ N/m, to the left;} \end{aligned}$$

While that on wire 2 due to wire 3 is

$$\begin{aligned} F_{13} &= \mu_0 I_1 I_3 / 2\pi d_{13} \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(200 \text{ A})(100 \text{ A}) / [2\pi(0.40 \text{ m})] \\ &= 0.010 \text{ N/m, to the right.} \end{aligned}$$

The net force exerted on wire 1 is then

$$\begin{aligned} F_1 &= F_{12} - F_{13} \\ &= 0.060 \text{ N/m} - 0.010 \text{ N/m} \\ &= \boxed{0.050 \text{ N/m, to the left.}} \end{aligned}$$

Similarly, the net force on wire 2 due to wires 1 and 3 is

$$\begin{aligned} F_2 &= F_{21} - F_{23} \\ &= \mu_0 I_2 I_1 / 2\pi d_{21} - \mu_0 I_2 I_3 / 2\pi d_{23} \\ &= 0.060 \text{ N/m} - (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(300 \text{ A})(100 \text{ A}) / [2\pi(0.20 \text{ m})] \\ &= 0.060 \text{ N/m} - 0.030 \text{ N/m} \\ &= \boxed{0.030 \text{ N/m, to the right.}} \end{aligned}$$

Once these two forces are known there is no need to calculate the force on wire 3 directly using the formulas above. Just add  $F_1$  and  $F_2$  to obtain 0.020 N/m, to the left -- This is the net force exerted on wires 1 and 2 from wire 3. From Newton's third law, therefore, the net force on wire 3 from wires 1 and 2 must be

$$F_3 = \boxed{0.020 \text{ N/m, to the right.}}$$

65. The current configuration is equivalent to a pair of infinitely long, parallel wires plus a circular current loop. The magnetic field at point  $P$  due to each infinitely long wire is

$$B_1 = \mu_0 I / [2\pi(D/2)], \text{ pointing into the paper;}$$

while that due to the circular current loop is

$$B_2 = \mu_0 I / [2(D/2)], \text{ also pointing into the paper.}$$

The total magnetic field at point  $P$  is then

$$\begin{aligned} B &= 2B_1 + B_2 \\ &= 2\mu_0 I / [2\pi(D/2)] + \mu_0 I / [2(D/2)] \\ &= \boxed{(\mu_0 I / D)(2/\pi + 1), \text{ pointing into the paper.}} \end{aligned}$$

66. From Problem 12, each sheet creates a magnetic field at the other sheet, parallel to the sheet and

perpendicular to the current, with magnitude

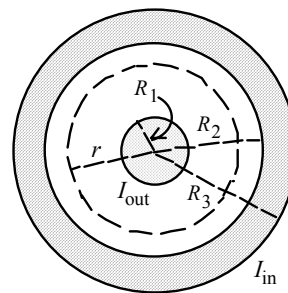
$$B = \mu_0 h.$$

This field will produce an attractive force on the sheet. To find the force on an area of the sheet with length  $L$  and width  $w$ , we choose a differential element of width  $dx$ . Because the length of the element is perpendicular to the field, we have

$$F = \int dF = \int LB \, dI = \int_0^w L \left( \frac{\mu_0 h}{2} \right) h \, dx = \frac{1}{2} \mu_0 h^2 Lw.$$

The force per unit area is

$$F/Lw = \frac{1}{2} \mu_0 h^2 \text{ (attractive)}.$$



67. We call the radius of the inner wire  $R_1$ , the inside radius of the outer wire  $R_2$ , and the outside radius of the outer wire  $R_3$ .

The current densities in the wires are

$$J_{\text{inner}} = I/\pi R_1^2 \quad \text{and} \quad J_{\text{outer}} = I/(\pi(R_3^2 - R_2^2)).$$

Because of the cylindrical symmetry, we know that the magnetic fields will be circular. In each case we apply Ampere's law to a circular path of radius  $r$ .

- (a) Inside the inner wire,  $r < R_1$ :

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$$

$$B2\pi r = \mu_0 J_{\text{inner}} \pi r^2 = \mu_0 I r^2 / \pi R_1^2, \text{ which gives}$$

$$B = \left( \mu_0 I / 2\pi R_1^2 \right) r \text{ circular CCW, } r < R_1.$$

- (b) Between the wires,  $R_1 < r < R_2$ :

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$$

$$B2\pi r = \mu_0 I, \text{ which gives}$$

$$B = \left( \mu_0 I / 2\pi r \right) \text{ circular CCW, } R_1 < r < R_2.$$

- (c) Inside the outer wire,  $R_2 < r < R_3$ :

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$$

$$B2\pi r = \mu_0 [I - J_{\text{outer}}(\pi r^2 - \pi R_2^2)]$$

$$= \mu_0 [I - I(\pi r^2 - \pi R_2^2) / (\pi(R_3^2 - R_2^2))] = \mu_0 I [1 - (r^2 - R_2^2) / (R_3^2 - R_2^2)], \text{ which gives}$$

$$B = \left( \mu_0 I / 2\pi r \right) (R_3^2 - r^2) / (R_3^2 - R_2^2) \text{ circular CCW, } R_2 < r < R_3.$$

- (d) Outside the outer wire,  $R_3 < r$ :

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$$

$$B2\pi r = \mu_0 (I - I), \text{ which gives } B = 0, R_3 < r.$$

68. The Coulomb attraction provides the centripetal force:

$$F = ma; \quad (1/4\pi\epsilon_0)(e^2/r^2) = mv^2/r.$$

The constraint on the angular momentum is

$$mvr = nh/2\pi.$$

We combine these two equations to find the radius of the orbit and the speed of the electron:

$$r = \epsilon_0 h^2 n^2 / \pi m e^2, \quad \text{and} \quad v = e^2 / 2\epsilon_0 h n.$$

The period of the orbit is

$$T = 2\pi r/v = 2\pi(\epsilon_0 \hbar^2 n^2 / \pi m e^2) / (e^2 / 2\epsilon_0 \hbar m) = 4\epsilon_0^2 \hbar^3 n^3 / m e^4.$$

The effective current of the electron is

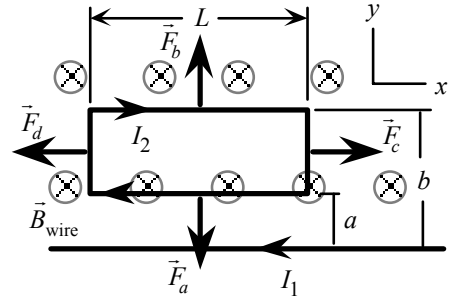
$$I = e/T = m e^5 / 4\epsilon_0^2 \hbar^3 n^3.$$

The magnetic field at the proton is that of a circular loop:

$$\begin{aligned} B &= \mu_0 I / 2r = \mu_0 (m e^5 / 4\epsilon_0^2 \hbar^3 n^3) / 2(\epsilon_0 \hbar^2 n^2 / \pi m e^2) \\ &= \pi \mu_0 m^2 e^7 / 8\epsilon_0^3 \hbar^5 n^5 \\ &= \pi(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(9.1 \times 10^{-31} \text{ kg})^2 (1.6 \times 10^{-19} \text{ C})^7 / \\ &\quad [8(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^5 n^5] \\ &= \boxed{12.4/n^5 \text{ T perpendicular to the orbit.}} \end{aligned}$$

The magnetic moment of the current loop is

$$\begin{aligned} \mu &= IA = I\pi r^2 = (m e^5 / 4\epsilon_0^2 \hbar^3 n^3) \pi (\epsilon_0 \hbar^2 n^2 / \pi m e^2)^2 = e \hbar m / 4\pi m \\ &= (1.6 \times 10^{-19} \text{ C})(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) n / 4\pi (9.1 \times 10^{-31} \text{ kg}) \\ &= \boxed{(9.3 \times 10^{-24}) n \text{ A} \cdot \text{m}^2 \text{ perpendicular to the orbit.}} \end{aligned}$$



69. Because the magnetic field produced by the long wire depends only on the distance from the wire  $y$ ,  $B = \mu_0 I_1 / 2\pi y$ , we find the force on the rectangular loop. The net force is

$$\vec{F} = \vec{F}_a + \vec{F}_b + \vec{F}_c + \vec{F}_d.$$

For segments  $c$  and  $d$  of the loop, the symmetry of the field and the opposite directions of the current  $I_2$  give

$$\vec{F}_c + \vec{F}_d = 0.$$

Because the wires and the magnetic field are perpendicular, we have

$$\begin{aligned} \vec{F} &= \vec{F}_a + \vec{F}_b = I_2 L (\mu_0 I_1 / 2\pi a) (-\hat{j}) + I_2 L (\mu_0 I_1 / 2\pi b) (+\hat{j}) \\ &= (\mu_0 / 4\pi) (2I_1 I_2 L) (1/b - 1/a) \hat{j} \\ &= (10^{-7} \text{ T} \cdot \text{m/A}) 2(10 \text{ A})(5 \text{ A})(0.20 \text{ m}) [1/(0.05 \text{ m}) - 1/(0.02 \text{ m})] \hat{j} \\ &= \boxed{-(6.0 \times 10^{-5} \text{ N}) \hat{j} \text{ (attraction)}}. \end{aligned}$$

70. If we assume that the turns are touching, the distance between their centers is 0.3 mm. Because this is much less than the radius of the solenoid, we can consider the two turns as infinite parallel conductors, each of length  $2\pi R$ . We find the force between the turns from

$$\begin{aligned} F &= \mu_0 I_1 I_2 2\pi R / 2\pi d = \mu_0 I^2 R / d \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(150 \text{ A})^2 (0.03 \text{ m}) / (0.8 \times 10^{-3} \text{ m}) \\ &= \boxed{1.1 \text{ N attraction}}. \end{aligned}$$

There may be thousands of turns, so significant forces may develop.

71. (a) For the attractive magnetic force per unit length, we have

$$\begin{aligned} F_B / L &= I(\mu_0 I / 2\pi d) = (\mu_0 / 2\pi)(I^2 / d) \\ &= (2 \times 10^{-7} \text{ A} \cdot \text{m}^2)(1 \text{ A})^2 / (1 \times 10^{-2} \text{ m}) = 2.0 \times 10^{-5} \text{ N/m}. \end{aligned}$$

The linear charge density is

$$\lambda = (10^{21} \text{ carriers/cm})(10^2 \text{ cm/m})(1.6 \times 10^{-19} \text{ C/carrier}) = 1.6 \times 10^4 \text{ C/m}.$$

The electric field a distance  $d$  from an infinite line of charge is  $E = (1/2\pi\epsilon_0)(\lambda/d)$ .

For the repulsive electric force per unit length, we have

$$\begin{aligned}
 F_E/L &= \lambda E = \lambda(1/2\pi\epsilon_0)(\lambda/d) = (1/2\pi\epsilon_0)(\lambda^2/d) \\
 &= 2(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^4 \text{ C/m})^2/(1 \times 10^{-2} \text{ m}) \\
 &= 4.6 \times 10^{20} \text{ N/m}.
 \end{aligned}$$

The ratio of the forces is

$$F_B/F_E = \boxed{4.3 \times 10^{-26}}.$$

- (b) For the forces to be equal, we have

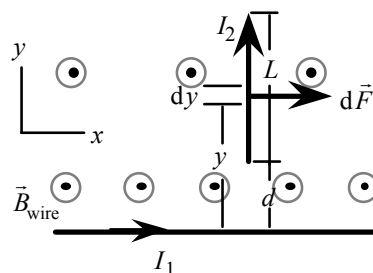
$$\lambda^2/\epsilon_0 = \mu_0 I^2, \text{ or } \lambda^2 = \epsilon_0 \mu_0 I^2.$$

If we use  $\epsilon_0 \mu_0 = 1/c^2$  and  $\lambda = ne$ , we get

$$\begin{aligned}
 n &= I/ce \\
 &= (1 \text{ A})/(3 \times 10^8 \text{ m/s})(1.6 \times 10^{-19} \text{ C/electron}) = 2.1 \times 10^{10} \text{ electrons/m} \\
 &= \boxed{2.1 \times 10^8 \text{ electrons/cm}}.
 \end{aligned}$$

- (c) For the fraction that is the excess, we have

$$f = (2.1 \times 10^8/\text{cm})/(10^{21}/\text{cm}) = \boxed{2.1 \times 10^{-13}}.$$



72. The magnetic field of the long wire at the second wire is out of the page, so the force on the second wire is in the direction of the current in the long wire, to the right. Because the magnetic field is not uniform, we choose a differential element of the second wire a distance  $y$  from the long wire to find the force and thus the torque:

$$\begin{aligned}
 d\vec{F} &= I_2 (dy \hat{j}) \times (\mu_0 I_1 / 2\pi y) \hat{k}, \\
 d\vec{\tau} &= (y \hat{j}) \times d\vec{F} \\
 &= y I_2 (\mu_0 I_1 / 2\pi y) dy [\hat{j} \times (\hat{j} \times \hat{k})] = I_2 (\mu_0 I_1 / 2\pi) dy (-\hat{k}).
 \end{aligned}$$

When we integrate over the length of the second wire, we get

$$\begin{aligned}
 \vec{\tau} &= -(\mu_0 I_1 I_2 / 2\pi) \hat{k} \int dy = -(\mu_0 I_1 I_2 / 2\pi) \hat{k} \Delta y = -(\mu_0 I_1 I_2 L / 2\pi) \hat{k}; \\
 &= \boxed{-(\mu_0 / 2\pi) I_1 I_2 L \hat{k} \text{ (into the page)}}.
 \end{aligned}$$

73. (a) We choose  $x = 0$  at the left coil. The magnetic fields on the axis from the coils are in the same direction, so we find the magnitude of the total field from

$$\begin{aligned}
 B(x) &= \frac{\mu_0 I}{2} \left\{ \frac{R^2}{(R^2 + x^2)^{3/2}} + \frac{R^2}{[R^2 + (R-x)^2]^{3/2}} \right\} \\
 &= \frac{\mu_0 I}{2R} \left\{ \frac{1}{[1 + (x/R)^2]^{3/2}} + \frac{1}{[2 - 2(x/R) + (x/R)^2]^{3/2}} \right\}.
 \end{aligned}$$

$$\text{At } x = 0, \text{ we have } B(0) = (\mu_0 I / 2R)[1 + (1/2^{3/2})] = \boxed{0.677 \mu_0 I / R}.$$

At  $x = R/4$ , we have

$$B(R/4) = (\mu_0 I / 2R) \{1/[1 + (1/4)^2]^{3/2}\} + \{1/[2 - 2(1/4) + (1/4)^2]^{3/2}\} = \boxed{0.713 \mu_0 I / R}.$$

At  $x = R/2$ , we have

$$B(R/2) = (\mu_0 I / 2R) \{1/[1 + (1/2)^2]^{3/2}\} + \{1/[2 - 2(1/2) + (1/2)^2]^{3/2}\} = \boxed{0.716 \mu_0 I / R}.$$

- (b) When we differentiate the expression for  $B$ , we get

$$\begin{aligned}
\frac{dB}{dx} &= \frac{\mu_0 I}{2R} \left\{ \frac{(-3/2)2(1/R)(x/R)}{[1 + (x/R)^2]^{5/2}} + \frac{(-3/2)2[(-1/R) + (1/R)(x/R)]}{[2 - 2(x/R) + (x/R)^2]^{5/2}} \right\} \\
&= -\frac{3\mu_0 I}{2R^2} \left\{ \frac{(x/R)}{[1 + (x/R)^2]^{5/2}} + \frac{(x/R) - 1}{[2 - 2(x/R) + (x/R)^2]^{5/2}} \right\}; \\
\frac{d^2 B}{dx^2} &= -\frac{3\mu_0 I}{2R^2} \left\{ \frac{\frac{1/R}{[1 + (x/R)^2]^{5/2}} + \frac{(-5/2)2(1/R)(x/R)^2}{[1 + (x/R)^2]^{7/2}}}{\frac{1/R}{[2 - 2(x/R) + (x/R)^2]^{5/2}} + \frac{[(x/R) - 1](-5/2)2[(-1/R) + (1/R)(x/R)]}{[2 - 2(x/R) + (x/R)^2]^{7/2}}} \right\} \\
&= -\frac{3\mu_0 I}{2R^3} \left\{ \frac{1}{[1 + (x/R)^2]^{5/2}} - \frac{5(x/R)^2}{[1 + (x/R)^2]^{7/2}} + \frac{1}{[2 - 2(x/R) + (x/R)^2]^{5/2}} - \frac{5[(x/R) - 1]^2}{[2 - 2(x/R) + (x/R)^2]^{7/2}} \right\}.
\end{aligned}$$

(Continued in the next page)

At  $x = R/2$ , we have

$$\begin{aligned}
\frac{dB}{dx} &= -\frac{3\mu_0 I}{2R^2} \left\{ \frac{1/2}{[1 + (1/2)^2]^{5/2}} + \frac{(1/2) - 1}{[2 - 2(1/2) + (1/2)^2]^{5/2}} \right\} = -\frac{3\mu_0 I}{2R^2} \left\{ \frac{1/2}{(5/4)^{5/2}} + \frac{-(1/2)}{(5/4)^{5/2}} \right\} = 0; \\
\frac{d^2 B}{dx^2} &= -\frac{3\mu_0 I}{2R^3} \left\{ \frac{1}{[1 + (1/2)^2]^{5/2}} - \frac{5(1/2)^2}{[1 + (1/2)^2]^{7/2}} + \frac{1}{[2 - 2(1/2) + (1/2)^2]^{5/2}} - \frac{5[(1/2) - 1]^2}{[2 - 2(1/2) + (1/2)^2]^{7/2}} \right\} \\
&= -\frac{3\mu_0 I}{2R^3} \left[ \frac{1}{(5/4)^{5/2}} - \frac{5/4}{(5/4)^{7/2}} + \frac{1}{(5/4)^{5/2}} - \frac{5/4}{(5/4)^{7/2}} \right] = 0.
\end{aligned}$$

74. (a) We use the magnetic field at the center of the Helmholtz coils from the solution for Problem 73 to determine the necessary current:

$$B_{R/2} = 0.716 \mu_0 NI/R;$$

$$5 \times 10^{-5} \text{ T} = 0.716(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(50)I/(0.50 \text{ m}), \text{ which gives}$$

$$I = \boxed{0.56 \text{ A}}.$$

- (b) We place the equipment with a length of 10 cm symmetrically about the midpoint: from 20 cm ( $x = 2R/5$ ) to 30 cm ( $x = 3R/5$ ). Using the formula for  $B(x)$  derived in Problem 73, the fractional change between the field at one end and the midpoint is

$$\Delta B/B = (B_{R/2} - B_{2R/5})/B_{R/2} = (0.71554 - 0.71546)/0.71554 = 0.0001.$$

If we adjust the current so the field is slightly greater than Earth's field in the center and slightly less at the ends of the equipment, the uncompensated field will be one-half of the difference:

$$B_{\text{uncompensated}} = (0.00005)(5 \times 10^{-5} \text{ T}) = \boxed{3 \times 10^{-9} \text{ T}}.$$

75. We choose a radius of  $r = 10$  cm for the trajectory. This requires a tube with a diameter of  $\approx 30$  cm. If the accelerating potential of the electron gun is  $V$ , we find the speed of the electron from

$$\frac{1}{2}mv^2 = eV, \text{ or } v = (2eV/m)^{1/2}.$$

From Example 29-2, we have

$$e/m = v/Br = (2eV/m)^{1/2}/Br, \text{ which becomes}$$

$$V = (e/2m)r^2B^2 = [(1.6 \times 10^{-19} \text{ C})/2(9.1 \times 10^{-31} \text{ kg})](0.10 \text{ m})^2B^2 = (8.8 \times 10^8 \text{ V/T}^2)B^2.$$

Some possible combinations are

$B, \text{T}$	$V, \text{V}$
$5 \times 10^{-4}$	220
$1 \times 10^{-3}$	880
$1.5 \times 10^{-3}$	2000
$2 \times 10^{-3}$	3500
$5 \times 10^{-3}$	22000

We choose a convenient potential of  $V = 2000$  V, so  $B = 1.5 \times 10^{-3}$  T, which is  $\approx 30 B_{\text{earth}}$ .

We create the field with Helmholtz coils with a radius of 20 cm to accommodate the tube.

From the solution for Problem 65 we have

$$B \approx 0.71 \mu_0 NI/R;$$

$$1.5 \times 10^{-3} \text{ T} = 0.71(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})NI/(0.20 \text{ m}), \text{ which gives } \boxed{NI = 336 \text{ A} \cdot \text{turns}}.$$

If we wind 400 turns on the coils, the current is

$$I = (336 \text{ A} \cdot \text{turns})/(400 \text{ turns}) = 0.84 \text{ A}.$$

The concern is that the current not be so high that the heat generation is a problem.

76. For the magnitude of the magnetic field  $\vec{B} = \beta(y\hat{i} - x\hat{j})$ , we have

$$|\vec{B}| = \beta(y^2 + x^2)^{1/2} = \beta r,$$

which means that the magnitude of the field depends on only the distance from the origin.

If we find the scalar product of  $\vec{r}$  and  $\vec{B}$ , we have

$$\vec{r} \cdot \vec{B} = (x\hat{i} + y\hat{j}) \cdot \beta(y\hat{i} - x\hat{j}) = 0,$$

which means that  $\vec{r}$  and  $\vec{B}$  are perpendicular; the field is circular, centered on the origin.

From the symmetry of the field, we know that the current is in the  $-z$ -direction, with a current density that is a function of  $r$  only. We apply Ampere's law to a circular path a distance  $r$  from the origin:

$$\oint \vec{E} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}};$$

$$-\oint \beta r ds = -\beta r 2\pi r = -\mu_0 \int_0^r J(r') 2\pi r' dr'.$$

For the integral on the right-hand side to give  $r^2$ , we see that the current density must be constant:

$$\beta 2\pi r^2 = \mu_0 J \pi r^2, \text{ which gives } J = 2\beta/\mu_0.$$

The magnetic field is that produced inside a wire with a uniform current. If the wire has radius  $R$ , then

$$I = \boxed{2\beta \pi R^2/\mu_0 \text{ in the } -z\text{-direction}}.$$