

# CHAPTER 26 Currents in Materials

## Answers to Understanding the Concepts Questions

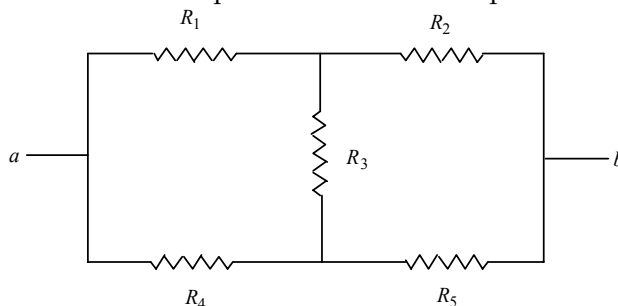
1. The current is a measurement of the rate at which charge passes. Since charge is conserved, the rate at which electrons pass various points along a beam is the same no matter how the individual electrons in it may have been accelerated; otherwise you would say that some charge has been lost or gained along the way. In the part of the beam that has been sped up, the electrons have become more widely spaced. In this way the rate of passage remains the same.
2. Power dissipation occurs when electrical energy is converted into thermal energy. Microscopically, this is due to the collision between the drifting electrons with the lattice. Between successive collisions, an electron is accelerated by the electric field and builds up a certain drift velocity. As a result of the collision, the electron has lost all the “memory” of that drift velocity and has to start anew with zero average velocity. The electrons give up part of their kinetic energies through collisions with the lattice particles, which as a result oscillate faster on average, increasing the temperature. This is how electrical energy is turned into heat. The microscopic picture does agree with the observed linear relationship between voltage and current, as  $\vec{J} = \sigma \vec{E}$ , where  $\sigma$  is the conductivity [see Eq. (26-25)], which leads to  $V = IR$ .
3. The resistance is inversely proportional to the area, and for a fixed current the power dissipated,  $I^2R$  is therefore inversely proportional to the area. The thinner wire will get hotter.
4. Consider a certain segment of a wire, of length  $L$ , cross-sectional area  $A$ , and resistivity  $\rho$ . The power consumed is  $P = I^2R = I^2(\rho L/A)$ , and the thermal energy generated over a time interval  $t$  is then  $Pt = I^2(\rho L/A)t$ , which we equate to the energy needed to raise the temperature of the wire by  $\Delta T$ :  $Pt = I^2(\rho L/A)t = cm\Delta T = c(\rho'AL)\Delta T$ , where  $\rho'$  is the density of the wire and  $c$  is its specific heat. Solve for  $\Delta T$ :  

$$\Delta T = I^2(\rho L/A)t/[c(\rho'AL)] = (I^2t/A^2)(\rho/c\rho').$$
Thus
$$\Delta T_{\text{Al}}/\Delta T_{\text{Brass}} = (\rho_{\text{Al}}/c_{\text{Al}}\rho'_{\text{Al}})/(\rho_{\text{Brass}}/c_{\text{Brass}}\rho'_{\text{Brass}}) = (\rho_{\text{Al}}/\rho_{\text{Brass}})(\rho'_{\text{Brass}}/\rho'_{\text{Al}})(c_{\text{Brass}}/c_{\text{Al}})$$

$$= (2.82/7)(8.9/2.7)(0.092/0.215) \approx 0.6 < 1,$$
so  $\Delta T_{\text{Al}} < \Delta T_{\text{Brass}}$ . The brass wire would get hotter.
5. It follows from Eq. (26-9) that for constant current and area the drift velocity only depends inversely on the density of free electrons. This is a characteristic of the material making up the conductor.
6. Imagine that the cylinder is made of  $N$  segments of equal length and equal cross-sectional area. Each segment has the same resistance and they are in series. When a certain current flows through the cylinder, each segment, with the same resistance, must have the same voltage drop (as  $V = IR$ ). Thus the voltage difference applied across the entire cylinder is divided evenly over each identical segment, meaning that the voltage drops linearly over the length of the cylinder.
7. After the faucet is turned on, the water particles have to actually move from the faucet through the entire hose before they can flow out of the hose, and that accounts for the time delay. In a wire, there are free electrons at every segment. The moment the power is switched on, an electric field is established, and every free electron instantly experiences the force of the electric field and start to drift. You don't have to wait for an electron to traverse from one end of the wire to the other end before a current is established. (In fact, the electrons drift incredibly slowly due to their frequent collisions with

the lattice, at only several millimeters per hour for a typical current -- so if you needed to wait for the electrons to move through the wire, you'd have a real long wait!)

8. In the free electron model, the temperature dependence of the resistivity [Eq. (26-25)] has to do with the time between collisions between electrons and the obstacle -- fixed ions -- that give rise to the drag force on the electrons. At very low temperatures the time between random collisions would increase; however, the accelerating field would still be present and there would still be a current. Thus the free electron model as it stands, with the accelerations due to the field as a small perturbation on the random motion, would have to be replaced with a "pinball" model, in which the only motion is due to the field and the electrons must "navigate," through multiple collisions, the forest of fixed ions. There would still be conduction even at  $T = 0$  in this picture, whereas the free electron model would predict none.
9. In principle, yes it does. As  $T$  increases, not only does the resistivity change, but also the length as well as the cross-sectional area of the wire increase. All these contribute to the temperature-dependency of electrical resistance. We do not, however, expect the dimensional change to be a major factor, since the fractional change in length and cross-sectional area is usually so small over a reasonable range of temperature variation.
10. When a switch is thrown and charge flows, it is because there is an electric field in the wire. Free charge in the wire -- mainly electrons -- will move, but the wire itself remains neutral, because as many charges as leave a segment of wire from one end enter it from the other end.
11. The current in the wire is given by  $I = JA = n_e e v A$ . As the electrons crowd to one side of the wire  $n_e$  increases, while the effective value of  $A$  decreases by the same factor. Thus  $I$  remains the same, as does the resistance of the wire.
12. There is no electric field inside a conductor when there is equilibrium. When charges are flowing due to a continuously applied potential we do not have equilibrium, and charges can flow inside the conductor.
13. According to Eq. (26-25),  $\rho = m / n_e e^2 \tau$ . Here  $m = 9.1 \times 10^{-31}$  kg,  $e = 1.6 \times 10^{-19}$  C,  $n_e \approx 10^{29} / \text{m}^3$  (see Example 26-3), and  $\tau \approx 10^{-14}$  s. These data indeed yield  $\rho \approx 10^{-8} \Omega \cdot \text{m}$ .
14. No. Here is an example in which the decomposition is generally not possible:



15. When the power  $P = VI$  becomes too large, there is too much power dissipated for the heat to be conducted away in time, and melting of the resistor material occurs. Since generally the potential  $V$  is held fixed, the current becomes too large if the resistance is too small.
16. The resistance  $R$  of the filament, along with the voltage  $V$  applied across it, determines the power consumption of the light bulb:  $P = V^2/R$ . Here  $R = \rho L/A$ , so  $P = (V^2/\rho)(A/L)$ . For a certain power rating, the ratio  $A/L$  must therefore be preserved. When choosing a certain diameter  $d$  (and, therefore, the cross-sectional area  $A = \pi d^2/4$ ) of the filament, we must make sure that the corresponding length  $L$  of the filament is reasonable so that it can be coil up to fit inside the light bulb.

17. The resistance is proportional to the length of the resistor. This means that if two wires are tied together, the resistance is the sum of the resistances of the individual wires. This is consistent with the rule  $R_{\text{eq}} = R_1 + R_2$  for two resistors in series.
18. The current in the wire is given by  $I = JA = n_e e v A$ , which is proportional to the product of  $vA$ . Even though  $v_2 < v_1$ , we have  $A_1 > A_2$ . Moreover,  $v_1 A_1 = v_2 A_2$ , due to the continuity of flow of the charge carriers, analogous to the equation of continuity for the flow of incompressible fluids. Thus  $I_1 = I_2$ .
19. The total potential difference between A and B is fixed, i.e.,  $V_{AB} = V_{AC} + V_{CB} = \text{constant}$ . If the switch is closed then the overall resistance  $R_{\text{eq}}$  between A and B is reduced, and the current flowing through bulb 1,  $I = V_{AB}/R_{\text{eq}}$ , must increase. So bulb 1 will be brighter with the switch closed. As for bulb 2, when the switch is closed  $V_{AC}$ , the potential difference across bulb 1, increases (due to the increase in the current that flows through it), so  $V_{CB}$  must decrease. Thus bulb 2 becomes dimmer when the switch is closed.
20. As the diameter changes by a factor of 1/10 the cross-sectional area  $A$  changes by a factor of 1/100. Meanwhile, the volume of the wire remains fixed, so as the area changes by a factor of 1/100 the length  $L$  must change by a factor of 100. Overall, the resistance, which is proportional to  $L/A$ , must increase by a factor of  $100/(1/100) = 10\,000$ , or  $10^4$ .
21. The resistors are connected in parallel. If one more resistor is added in parallel, the equivalent resistance is reduced, and the current,  $I = V_{AB}/R_{\text{eq}}$ , would increase.
22. For a network of resistors consisting of several parallel branches, the equivalent resistance is lower than that of the branch with the least resistance. So, to minimize  $R_{\text{eq}}$ , we can put the 1- $\Omega$  resistor in the lower branch and the other two in series in the upper branch. Then  $R_{\text{eq}} < 1\,\Omega$ . If we wish to maximize  $R_{\text{eq}}$ , then put the largest (4- $\Omega$ ) resistor in the lower branch. You can easily verify these results with a straightforward calculation:  $R_{\text{eq}} = [R_1^{-1} + (R_2 + R_3)^{-1}]^{-1} = R_1(R_2 + R_3)/(R_1 + R_2 + R_3)$ , where  $R_1$  is the resistance of the single resistor in the lower branch.
23. The term “high wattage” refers to the large amount of power dissipated in the bulb. More power is dissipated when the resistance is larger, and this is done in a light bulb by making the filament long and thin. A curled up filament allows for a longer filament in a small space.

**Solutions to Problems**

1. We find the average current density from

$$J_{\text{av}} = I/A = (0.46 \text{ A})/[\pi(2.2 \times 10^{-3} \text{ m})^2/4] = \boxed{1.2 \times 10^5 \text{ A/m}^2}.$$

The charge that passes a fixed point is

$$q = It = (0.46 \text{ A})(1 \text{ s}) = \boxed{0.46 \text{ C}}.$$

2. We find the density of carriers from

$$J_{\text{av}} = I/A = n_q q v_d$$

$$(1.2 \text{ A})/(4.2 \times 10^{-5} \text{ m}^2) = n_q(1.6 \times 10^{-19} \text{ C})(0.32 \times 10^{-5} \text{ m/s}), \text{ which gives } n_q = \boxed{5.6 \times 10^{28} \text{ carriers/m}^3}.$$

3. We find the drift speed from

$$J_{\text{av}} = I/A = n_q q v_d$$

$$(100 \text{ A})/(36 \times 10^{-6} \text{ m}^2) = (5.7 \times 10^{28} \text{ carriers/m}^3)(1.6 \times 10^{-19} \text{ C})v_d, \text{ which gives } v_d = 2.0 \times 10^{-4} \text{ m/s}.$$

The time to travel the length of the cable is

$$t = L/v_d = (2 \text{ m})/(2.0 \times 10^{-4} \text{ m/s}) = \boxed{1.0 \times 10^4 \text{ s (2.8 h)}}.$$

4. We find the current from

$$I = JA;$$

$$I_1 = (3 \times 10^5 \text{ A/m}^2)(0.02 \times 10^{-6} \text{ m}^2) = 6 \times 10^{-3} \text{ A} = \boxed{6.0 \text{ mA}}.$$

$$I_2 = (13 \times 10^4 \text{ A/m}^2)(0.2 \times 10^{-6} \text{ m}^2) = 2.6 \times 10^{-2} \text{ A} = \boxed{26 \text{ mA}}.$$

$$I_3 = (15 \times 10^4 \text{ A/m}^2)(2 \times 10^{-6} \text{ m}^2) = \boxed{0.30 \text{ A}}.$$

5. We find the number of electrons from the charge that passes the point:

$$N = Q/e = It/e = (0.092 \text{ A})(1 \text{ s})/(1.6 \times 10^{-19} \text{ C}) = \boxed{5.8 \times 10^{17} \text{ electrons}}.$$

6. For the current density, we have

$$J = n_q q v_d$$

$$7.2 \times 10^2 \text{ A/m}^2 = (3.5 \times 10^{24} \text{ carriers/m}^3)(1.6 \times 10^{-19} \text{ C})v_d, \text{ which gives } v_d = \boxed{1.3 \times 10^{-3} \text{ m/s}}.$$

7. For the current density, we have

$$J = I/A = n_q q v_d$$

$$(1.2 \text{ A})/[\pi(1.8 \times 10^{-3} \text{ m})^2] = (8.5 \times 10^{28} \text{ electrons/m}^3)(1.6 \times 10^{-19} \text{ C/electron})v_d, \text{ which gives}$$

$$v_d = \boxed{8.7 \times 10^{-6} \text{ m/s}}.$$

For the second wire, the only change is the area, so we have

$$v_{d2} = v_{d1} R_1^2/R_2^2 = (8.7 \times 10^{-6} \text{ m/s})(1.8 \text{ mm})^2/(1.2 \text{ mm})^2 = \boxed{2.0 \times 10^{-5} \text{ m/s}}.$$

8. For the current density, we have

$$J = I/A = n_e q v_d$$

$$(6.1 \times 10^{-3} \text{ A})/(0.50 \times 10^{-4} \text{ m}^2) = n_e(1.6 \times 10^{-19} \text{ C})(3.5 \times 10^7 \text{ m/s}), \text{ which gives}$$

$$n_e = \boxed{2.2 \times 10^{13} \text{ electrons/m}^3}.$$

9. We find the number of electrons from the charge that passes the point:

$$N = Q/e = It/e = (200 \times 10^{-3} \text{ A})(1 \text{ h})(3600 \text{ s/h})/(1.6 \times 10^{-19} \text{ C}) = \boxed{4.5 \times 10^{21} \text{ electrons}}.$$

We find the number of electrons in a 1-m length of the beam from the time to travel 1 m:

$$t = L/v;$$

$$N = It/e = IL/ev = (200 \times 10^{-3} \text{ A})(1 \text{ m})/(1.6 \times 10^{-19} \text{ C})(3 \times 10^8 \text{ m/s}) = \boxed{4.2 \times 10^9 \text{ electrons}}.$$

10. Because the current density is constant, we find the current from

$$I_s = \iint \vec{J} \cdot d\vec{A} = \vec{J} \cdot \vec{A};$$

$$I_x = [(A\hat{i} + B\hat{j} + C\hat{k}) \text{ mA/cm}^2] \cdot (1 \text{ cm}^2)\hat{i} = \boxed{A \text{ mA}}.$$

$$I_y = [(A\hat{i} + B\hat{j} + C\hat{k}) \text{ mA/cm}^2] \cdot (1 \text{ cm}^2)\hat{j} = \boxed{B \text{ mA}}.$$

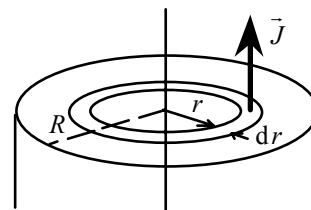
$$I_z = [(A\hat{i} + B\hat{j} + C\hat{k}) \text{ mA/cm}^2] \cdot (1 \text{ cm}^2)\hat{k} = \boxed{C \text{ mA}}.$$

11. Because the current density is a function of the distance from the axis, we choose a circular ring for the differential area and integrate to find the current:

$$I = \iint_{\text{area}} \vec{J} \cdot d\vec{A} = \iint_{\text{area}} J dA = \int_0^R J_0 \left(1 - \frac{r^2}{R^2}\right) 2\pi r dr$$

$$= 2\pi J_0 \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$= 2\pi J_0 \left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right) \Big|_0^R = \frac{\pi}{2} J_0 R^2.$$



12. We take the  $x$ -axis to the right. If  $v_e$  is the speed of the electrons, we have

$$\vec{J} = \vec{J}_{\text{electrons}} + \vec{J}_{\text{ions}} = n(-e)(-v_e)\hat{i} + ne(1.5 \times 10^{-3} v_e)\hat{i} = nev_e(1 + 1.5 \times 10^{-3})\hat{i} = (1.0015nev_e)\hat{i},$$

so the net current density is  $\boxed{1.005nev_e \text{ to the right}}.$

13. Because a mol of NaCl contributes an Avogadro's number of positive ions and an equal number of negative ions, we find the density for each carrier from

$$n_+ = n_- = n = (0.1 \text{ mol/L})(6.02 \times 10^{23} \text{ ions/mol})(10^3 \text{ L/m}^3) = 6.02 \times 10^{25} \text{ ions/m}^3.$$

Because both types of carriers are present, we have

$$J = n_+ q_+ v_+ + n_- q_- v_- = nev_+ + n(-e)v_- = ne[v_+ - (-1.5v_+)] = 2.5nev_+;$$

$$40 \text{ A/m}^2 = (6.02 \times 10^{25} \text{ ions/m}^3)(1.6 \times 10^{-19} \text{ C/ion})(2.5v_+), \text{ which gives}$$

$$v_+ = \boxed{1.7 \times 10^{-6} \text{ m/s}}, \quad v_- = \boxed{-2.5 \times 10^{-6} \text{ m/s}}.$$

14. For the current density, we have

$$J = I/A = n_q q v_d;$$

$$(100 \text{ A})/\pi(2 \times 10^{-3} \text{ m})^2 = (8.5 \times 10^{22} \text{ electrons/cm}^3)(10^2 \text{ cm/m})^3(1.6 \times 10^{-19} \text{ C/electron})v_d,$$

which gives  $v_d = \boxed{5.9 \times 10^{-4} \text{ m/s}}.$

If the diameter were doubled,  $A$  would increase by a factor of 4, so  $v_d$  would  $\boxed{\text{decrease by a factor of 4}}.$

15. The time for the charged particle to circle the accelerator is  $T = 2\pi R/v$ . So the current is

$$I = q/T = qv/2\pi R = (1.6 \times 10^{-19} \text{ C})(3 \times 10^8 \text{ m/s})/[2\pi(2.5 \times 10^3 \text{ m})] = \boxed{3.1 \times 10^{-15} \text{ A}}.$$

16. Because each particle contributes the same current, we have

$$N = I_{\text{total}}/I = (42 \times 10^{-3} \text{ A})/(3.1 \times 10^{-15} \text{ A}) = \boxed{1.4 \times 10^{13}}.$$

- 17.** We find the current from  $I = Q/t = (10 \times 10^3 \text{ C})/(3.6 \times 10^3 \text{ s}) = \boxed{2.8 \text{ A}}.$  The current density is

$$J = I/A = (2.8 \text{ A})/(50 \times 10^{-6} \text{ m}^2) = \boxed{5.6 \times 10^4 \text{ A/m}^2}.$$

We find the free electron density from

$$n_e = \rho N_A / M = (2.7 \text{ g/cm}^3)(10^2 \text{ cm/m})^3(6.02 \times 10^{23} \text{ atm/mol})/(27 \text{ g/mol}) = 6.0 \times 10^{28} \text{ electrons/m}^3.$$

We find the drift speed from

$$v_d = J/n_e e = (5.6 \times 10^4 \text{ A/m}^2)/(6.0 \times 10^{28} \text{ electrons/m}^3)(1.6 \times 10^{-19} \text{ C/electron}) = \boxed{5.8 \times 10^{-6} \text{ m/s}}.$$

18. We find the free-electron density from

$$\begin{aligned} n_e &= \rho N_A / M \\ &= (19.3 \times 10^3 \text{ kg/m}^3)(6.02 \times 10^{23} \text{ atm/mol}) / [(197 \text{ g/mol})(10^{-3} \text{ kg/g})] \\ &= 5.90 \times 10^{28} \text{ electrons/m}^3. \end{aligned}$$

We find the drift speed from

$$\begin{aligned} v_d &= J / n_e e = I / A n_e e \\ &= (0.3 \text{ A}) / [\pi(0.5 \times 10^{-3} \text{ m})^2 (5.90 \times 10^{28} \text{ electrons/m}^3)(1.60 \times 10^{-19} \text{ C/electron})] \\ &= \boxed{4.0 \times 10^{-5} \text{ m/s}}. \end{aligned}$$

19. The total current must be the same on each side of the junction:

$$\begin{aligned} I_{\text{total}} &= 2I_1 = 3I_2; \\ 2(3 \text{ A}) &= 3I_2, \text{ which gives } I_2 = 2 \text{ A in each of the smaller wires.} \end{aligned}$$

We find the drift speed in the larger wires from

$$\begin{aligned} v_{\text{in}} &= J / n_e e = I_1 / A_1 n_e e \\ &= (3 \text{ A}) / [\pi(0.1 \text{ cm})^2 (7 \times 10^{22} \text{ electrons/cm}^3)(1.60 \times 10^{-19} \text{ C/electron})] \\ &= 8.5 \times 10^{-3} \text{ cm/s} = \boxed{8.5 \times 10^{-5} \text{ m/s}}. \end{aligned}$$

We find the drift speed in the smaller wires from

$$\begin{aligned} v_{\text{out}} &= J / n_e e = I_2 / A_2 n_e e \\ &= (2 \text{ A}) / [\pi(0.05 \text{ cm})^2 (7 \times 10^{22} \text{ electrons/cm}^3)(1.60 \times 10^{-19} \text{ C/electron})] \\ &= 2.3 \times 10^{-2} \text{ cm/s} = \boxed{2.3 \times 10^{-4} \text{ m/s}}. \end{aligned}$$

The combined area of the smaller wires is less than the combined area of the larger wires. Charge conservation is equivalent to mass conservation in water flow, so the smaller area requires a greater speed.

20. As a function of  $r$ , the drift speed is

$$v = v_0(1 - r/R).$$

This variable drift speed means the current density is a function of  $r$ . We find the total current by selecting a ring of radius  $r$  and thickness  $dr$ , then we add (integrate) the contributions from all of the rings:

$$\begin{aligned} I &= \int \vec{J} \cdot d\vec{A} = \int_0^R n_q q v 2\pi r dr = 2\pi n_q q v_0 \int_0^R \left(1 - \frac{r}{R}\right) r dr \\ &= 2\pi n_q q v_0 \left( \frac{r^2}{2} - \frac{r^3}{3R} \right) \Big|_0^R = 2\pi n_q q v_0 \left( \frac{R^2}{2} - \frac{R^3}{3R} \right) = \frac{1}{3} \pi n_q q v_0 R^2. \end{aligned}$$

For a constant drift speed of  $v_0$ , the current is

$$I' = n_q q (v_0) \pi R^2 = \pi n_q q v_0 R^2.$$

The ratio of currents is

$$I/I' = \frac{1}{3} = \boxed{\frac{1}{3}}.$$

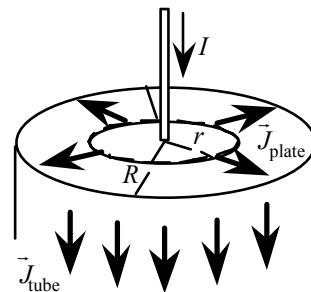
Even though the drift speed varies linearly, this ratio is not 1, since there is more area at the lower speeds.

21. With  $d \ll R$ , from symmetry, the current density in the tube is constant:

$$J_{\text{tube}} = I/A = \boxed{I/2\pi R d \text{ along the tube}}.$$

The current density in the top plate is radial and depends on the distance from the axis  $r$ :

$$J_{\text{plate}} = I/A = \boxed{I/2\pi r d \text{ radial}}.$$



22. The variable charge density and drift speed means the current density is a function of  $r$ . We find the total current by selecting a ring of radius  $r$  and thickness  $dr$ , then add (integrate) the contributions from all of the rings:

$$\begin{aligned} I &= \int \vec{J} \cdot d\vec{A} = \int_0^R nqv \, 2\pi r \, dr = 2\pi q \int_0^R (n_0 - n'r)(v_0 - v'r^2)r \, dr \\ &= 2\pi q \int_0^R (n_0 v_0 r - n' v_0 r^2 - n_0 v' r^3 + n' v' r^4) \, dr \\ &= 2\pi q \left( n_0 v_0 \frac{r^2}{2} - n' v_0 \frac{r^3}{3} - n_0 v' \frac{r^4}{4} + n' v' \frac{r^5}{5} \right) \Big|_0^R, \text{ which gives} \\ I &= \boxed{\pi q R^2 (n_0 v_0 - 2n' v_0 R/3 - n_0 v' R^2/2 + 2n' v' R^3/5)}. \end{aligned}$$

23. Because the material is the same, we have

$$R_2/R_1 = (L_2/A_2)/(L_1/A_1) = (L_2/L_1)(D_1/D_2)^2 = (!)(2)^2 = 2; \quad \boxed{R_2 = 2R_1}.$$

24. Because the length is the same, we have

$$\begin{aligned} R_{\text{gold}}/R_{\text{silver}} &= (\rho_{\text{gold}}/A_{\text{gold}})/(\rho_{\text{silver}}/A_{\text{silver}}) = (\rho_{\text{gold}}/\rho_{\text{silver}})(D_{\text{silver}}/D_{\text{gold}})^2 \\ 1 &= (1/1.5)(D_{\text{silver}}/D_{\text{gold}})^2, \text{ which gives } \boxed{D_{\text{silver}}/D_{\text{gold}} = 1.22}. \end{aligned}$$

25. (a) We find the resistance from

$$R = \rho L/A = (2.82 \times 10^{-8} \, \Omega \cdot \text{m})(528 \, \text{m})/(0.12 \times 10^{-4} \, \text{m}^2) = \boxed{1.2 \, \Omega}.$$

- (b) We form the ratio of resistances:

$$\begin{aligned} R_2/R_1 &= (\rho_2/\rho_1)(L_2/L_1)(A_1/A_2); \\ 1 &= [(1.72 \times 10^{-8} \, \Omega \cdot \text{m})/(2.82 \times 10^{-8} \, \Omega \cdot \text{m})](1)[(0.12 \times 10^{-4} \, \text{m}^2)/\pi r_2^2], \text{ which gives} \\ r_2 &= 1.5 \times 10^{-3} \, \text{m} = \boxed{0.15 \, \text{cm}}. \end{aligned}$$

26. (a) We find the resistance from

$$\begin{aligned} R &= \rho L/A \\ &= (1.72 \times 10^{-8} \, \Omega \cdot \text{m})(100 \, \text{ft})(0.305 \, \text{m/ft})/\{\pi[(0.0403 \, \text{in})(2.54 \times 10^{-2} \, \text{m/in})]^2\} = \boxed{0.637 \, \Omega}. \end{aligned}$$

- (b) We find the length from

$$\begin{aligned} L_2 &= R_2(L/R) \\ &= (7.5 \, \Omega)(100 \, \text{ft})/(0.637 \, \Omega) = \boxed{1.18 \times 10^3 \, \text{ft} \, (359 \, \text{m})}. \end{aligned}$$

27. We find the resistance from

$$R = \rho L/A = (1.72 \times 10^{-8} \, \Omega \cdot \text{m})(10 \, \text{m})/[\pi(0.2588 \times 10^{-2} \, \text{m})^2] = \boxed{3.27 \times 10^{-2} \, \Omega}.$$

28. We find the resistance from

$$R = \rho L/A = (3.5 \times 10^{-5} \, \Omega \cdot \text{m})(20.0 \times 10^{-2} \, \text{m})/[\pi(5.0 \times 10^{-3} \, \text{m})^2] = \boxed{0.36 \, \Omega}.$$

- We find the current from

$$I = V/R = (380 \, \text{V})/(0.36 \, \Omega) = \boxed{1.1 \times 10^3 \, \text{A}}.$$

29. We find the resistance from

$$R_{20} = \rho_{20} L/A = (1.72 \times 10^{-8} \, \Omega \cdot \text{m})(2 \, \text{m})/(36 \times 10^{-6} \, \text{m}) = \boxed{9.6 \times 10^{-4} \, \Omega}.$$

- The increase in resistance is

$$\Delta R = R_{20} \alpha (T - 20^\circ \text{C}) = (9.6 \times 10^{-4} \, \Omega)(0.0039/\text{C}^\circ)(80^\circ \text{C}) = \boxed{3.0 \times 10^{-4} \, \Omega}.$$

30. We find the radius from

$$\begin{aligned} R &= \rho L/A = R = \rho L/\pi r^2; \\ 10 \, \Omega &= (1.72 \times 10^{-8} \, \Omega \cdot \text{m})(175 \times 10^3 \, \text{m})/\pi r^2, \text{ which gives } r = 9.8 \times 10^{-3} \, \text{m} = \boxed{9.8 \, \text{mm}}. \end{aligned}$$

31. The resistance of the wire of length  $L$  and cross-sectional area  $A$  is  $R = \rho L/A$ . But according to Ohm's law  $R = V/I$ , so  $\rho L/A = V/I$ , which we solve for  $L$ :

$$L = AV/\rho I = (0.20 \times 10^{-6} \text{ m}^2)(120 \text{ V})/[5.6 \times 10^{-8} \text{ } \Omega \cdot \text{m}](15 \text{ A}) = \boxed{29 \text{ m}}.$$

32. The current density is given by

$$J_{\text{av}} = I/A = n_q q v_d; \text{ or } v_d = J_{\text{av}}/n_q q. \text{ Thus}$$

$$(v_d)_{\text{Cu}}/(v_d)_{\text{Al}} = (J_{\text{av}}/n_q q)_{\text{Cu}}/(J_{\text{av}}/n_q q)_{\text{Al}} = (n_q)_{\text{Al}}/(n_q)_{\text{Cu}} = (18 \times 10^{28} \text{ m}^{-3})/(9 \times 10^{28} \text{ m}^{-3}) = \boxed{2}.$$

33. The power consumed in a resistor is

$$P = IV = I^2 R = V^2/R.$$

With a fixed potential difference, we have

$$(P_2 - P_1)/P_1 = (1/R_2 - 1/R_1)/(1/R_1) = (R_1 - R_2)/R_2.$$

If we assume that the temperature coefficient does not change with temperature, we get

$$\begin{aligned} (P_2 - P_1)/P_1 &= \{R_{20}[1 + \alpha(T_1 - 20^\circ\text{C})] - R_{20}[1 + \alpha(T_2 - 20^\circ\text{C})]\}/R_{20}[1 + \alpha(T_2 - 20^\circ\text{C})] \\ &= \alpha(T_1 - T_2)/[1 + \alpha(T_2 - 20^\circ\text{C})] = (0.0045/^\circ\text{C})(-400^\circ\text{C})/[1 + (0.0045/^\circ\text{C})(1180^\circ\text{C})] \\ &= \boxed{-0.27}. \end{aligned}$$

Because the resistance has increased, the power consumption has decreased.

34. We find the length of the equivalent single wire from

$$R = V/I = \rho L/A;$$

$$(1.5 \text{ V})/(0.14 \text{ A}) = (1.7 \times 10^{-8} \text{ } \Omega \cdot \text{m})L/[\pi(0.24 \times 10^{-3} \text{ m})^2], \text{ which gives } L = 28.5 \text{ m}.$$

The distance to the short is  $d = L/2 = \boxed{14.3 \text{ m}}.$

35. We find the current from

$$R = V/I = \rho L/A = \rho_0(1 + \alpha \Delta T)L/A.$$

Table 26-2 gives the resistivity at  $20^\circ\text{C}$ . At  $25^\circ\text{C}$ , we have

$$(50 \text{ V})/I_{25} = (100 \times 10^{-8} \text{ } \Omega \cdot \text{m})[1 + (4 \times 10^{-4}/^\circ\text{C})(5^\circ\text{C})](0.50 \text{ m})/[\pi(0.5 \times 10^{-3} \text{ m})^2],$$

which gives  $I_{25} = \boxed{19.6 \text{ A}}.$

At  $400^\circ\text{C}$ , we have

$$(50 \text{ V})/I_{400} = (100 \times 10^{-8} \text{ } \Omega \cdot \text{m})[1 + (4 \times 10^{-4}/^\circ\text{C})(380^\circ\text{C})](0.50 \text{ m})/[\pi(0.5 \times 10^{-3} \text{ m})^2],$$

which gives  $I_{400} = \boxed{17.1 \text{ A}}.$

36. If we ignore dimension changes, with  $I$  constant, we have

$$V_2/V_1 = R_2/R_1 = \rho_2/\rho_1 = \rho_{20}[1 + \alpha(T - 20^\circ\text{C})]/\rho_{20} = 1 + \alpha(T - 20^\circ\text{C});$$

$$(8.7 \text{ mV})/(8.5 \text{ mV}) = 1 + (0.0039/^\circ\text{C})(T - 20^\circ\text{C}), \text{ which gives } T = \boxed{26^\circ\text{C}}.$$

37. From the expression for the resistance,  $R = \rho L/A$ , we form the ratio

$$R_{\text{Al}}/R_{\text{Cu}} = (\rho_{\text{Al}}/\rho_{\text{Cu}})(L_{\text{Al}}/L_{\text{Cu}})(A_{\text{Cu}}/A_{\text{Al}}) = (\rho_{\text{Al}}/\rho_{\text{Cu}})(L_{\text{Al}}/L_{\text{Cu}})(r_{\text{Cu}}/r_{\text{Al}})^2;$$

$$1 = [(2.8 \times 10^{-8} \text{ } \Omega \cdot \text{m})/(1.7 \times 10^{-8} \text{ } \Omega \cdot \text{m})](L/5L)(r_{\text{Cu}}/r_{\text{Al}})^2, \text{ which gives } \boxed{r_{\text{Cu}}/r_{\text{Al}} = 1.74}.$$

38. We find the resistivity from

$$R = V/I = \rho L/A;$$

$$(12.0 \text{ V})/(1.07 \text{ A}) = \rho(100 \text{ m})/(0.5 \text{ mm}^2)(10^{-3} \text{ m/mm})^2, \text{ which gives } \rho = 5.6 \times 10^{-8} \text{ } \Omega \cdot \text{m}.$$

When we look at the values listed in Table 26-2, we see that the material is tungsten.

39. We find the length of the wire from

$$R = \rho L/A;$$

$$1.35 \text{ } \Omega = (1.72 \times 10^{-8} \text{ } \Omega \cdot \text{m})L/[(1.5 \text{ mm}^2)(10^{-3} \text{ m/mm})^2], \text{ which gives } L = 118 \text{ m}.$$

The number of turns around the spool is



$$N = L/\pi D = (118 \text{ m})/[\pi(0.20 \text{ m})] = \boxed{188 \text{ turns}}.$$

40. We express the voltage in terms of the current and radius:

$$V = IR = I\rho L/A = I\rho L/\pi r^2.$$

If we use this for the two wires and take the ratio, we have

$$V_1/V_2 = (I_1\rho L/I_2\rho L)(\pi r_2^2/\pi r_1^2) = (I_1/I_2)(r_2/r_1)^2.$$

We see that  $r_2/r_1$  will be minimum when  $V_2/V_1$  has its maximum value of 1.5:

$$1/1.8 = (1/2)(r_2/r_1)_{\min}^2, \text{ which gives } (r_2/r_1)_{\min} = \boxed{1.05}.$$

41. We find the resistance from

$$\begin{aligned} R_{\text{Al}} &= \rho_{\text{Al}}L/A_{\text{Al}} \\ &= (2.8 \times 10^{-8} \Omega \cdot \text{m})(80 \text{ m})/\pi(0.12 \times 10^{-2} \text{ m})^2 = \boxed{2.0 \Omega}. \end{aligned}$$

The mass of the wire is

$$\begin{aligned} m_{\text{Al}} &= \rho_{m,\text{Al}}A_{\text{Al}}L \\ &= (2.7 \times 10^3 \text{ kg/m}^3)[\pi(0.12 \times 10^{-2} \text{ m})^2](80 \text{ m}) = \boxed{0.24 \text{ kg}}. \end{aligned}$$

We find the area of the copper wire from

$$\begin{aligned} R_{\text{Cu}} &= \rho_{\text{Cu}}L/A_{\text{Cu}}; \\ 2.0 \Omega &= (1.7 \times 10^{-8} \Omega \cdot \text{m})(80 \text{ m})/A_{\text{Cu}}, \text{ which gives } A_{\text{Cu}} = 6.8 \times 10^{-7} \text{ m}^2. \end{aligned}$$

The mass of the copper wire is

$$m_{\text{Cu}} = \rho_{m,\text{Cu}}A_{\text{Cu}}L = (8.9 \times 10^3 \text{ kg/m}^3)(6.8 \times 10^{-7} \text{ m}^2)(80 \text{ m}) = \boxed{0.48 \text{ kg}}.$$

42. For the resistance, we have

$$\begin{aligned} R &= \rho L/A = \rho L/\pi r^2; \\ 2 \Omega &= (1.72 \times 10^{-8} \Omega \cdot \text{m})L/\pi r^2. \end{aligned}$$

The mass of the wire is

$$\begin{aligned} m &= \rho_m AL; \\ 1.5 \text{ kg} &= (8.9 \times 10^3 \text{ kg/m}^3)\pi r^2 L. \end{aligned}$$

This gives us two equations with two unknowns,  $L$  and  $r$ . When we solve them, we get

$$r = 6.2 \times 10^{-4} \text{ m} = \boxed{0.62 \text{ mm}}, \text{ and } L = \boxed{1.4 \times 10^2 \text{ m}}.$$

43. For the resistance, we have

$$\begin{aligned} R &= \rho L/A; \\ 5 \Omega &= (1.59 \times 10^{-8} \Omega \cdot \text{m})(10^3 \text{ m})/A, \text{ which gives } A = 3.18 \times 10^{-6} \text{ m}^2. \end{aligned}$$

We find the mass of the wire from

$$m = \rho_m AL = (10.5 \times 10^3 \text{ kg/m}^3)(3.18 \times 10^{-6} \text{ m}^2)(10^3 \text{ m}) = \boxed{33.4 \text{ kg}}.$$

44. For the resistance, we have

$$\begin{aligned} R &= \rho L/A = \rho L/\pi(r_{\text{outside}}^2 - r_{\text{inside}}^2); \\ 3.5 \Omega &= (1.72 \times 10^{-8} \Omega \cdot \text{m})L/\pi[(2.75 \times 10^{-2} \text{ m})^2 - (2.45 \times 10^{-2} \text{ m})^2], \text{ which gives } L = \boxed{1.0 \times 10^5 \text{ m}}. \end{aligned}$$

45. For the resistance, we have

$$\begin{aligned} R_1 &= \rho L/A_1 = \rho L/\pi(r_{\text{outside}}^2 - r_{\text{inside}}^2) \\ &= (1.72 \times 10^{-8} \Omega \cdot \text{m})(1 \text{ m})/\pi[(0.2 \times 10^{-2} \text{ m})^2 - (0.1 \times 10^{-2} \text{ m})^2] = \boxed{1.82 \times 10^{-3} \Omega}. \end{aligned}$$

For the solid wire, we have

$$R_2 = \rho L/A_2 = \rho L/\pi r^2.$$

If we divide the two equations, we get

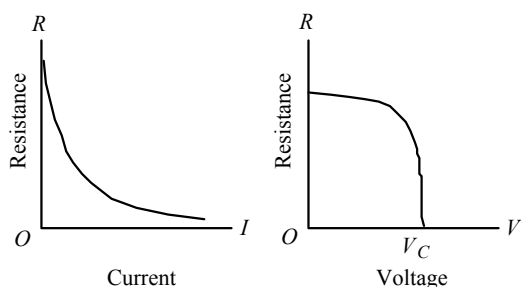
$$\begin{aligned} R_2/R_1 &= 1 = r^2/(r_{\text{outside}}^2 - r_{\text{inside}}^2), \text{ which becomes} \\ r^2 &= r_{\text{outside}}^2 - r_{\text{inside}}^2 = (0.2 \text{ cm})^2 - (0.1 \text{ cm})^2, \text{ which gives } r = \boxed{0.17 \text{ cm}}. \end{aligned}$$

The ratio of masses is

$$m_2/m_1 = \rho_m LA_2/\rho_m LA_1 = A_2/A_1 = r^2/(r_{\text{outside}}^2 - r_{\text{inside}}^2) = 1.$$

The masses are the same.

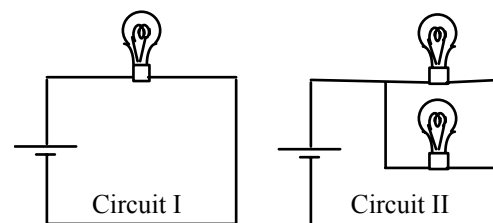
46.



At  $V_C$ ,  $R \rightarrow 0$ ;  $V_C$  is the breakdown voltage.

47. When two identical lightbulbs are connected in series the current flowing through them is the same, and so is the power consumption. They should be identical in brightness. If a third lightbulb is connected in series with the second one, then the current flowing through the first lightbulb must split equally for the second and the third lightbulb, so the current in bulb 1 is 2 times as much as those in bulb 2 and 3. Since the power consumption is proportional to  $I^2$ , bulb 1 consumes 4 times as much power as bulbs 2 and 3, and is therefore 4 times as bright as the other two lightbulbs.

48. (a) Because the bulbs in Circuit II are in parallel, the potential difference across each bulb is the same as in Circuit I. Each bulb will have the same brightness, which will be the brightness of the bulb in Circuit I. (Note that more power will come from the battery.)  
 (b) If one bulb is removed from Circuit II, there will still be a closed circuit for the other bulb, which will be the same as Circuit I, so the brightness of the remaining bulb will not change. It does not matter which bulb is removed.



49. The voltage across the two-resistor combination is

$V = I(R + R_x)$ , so  $I = V/(R + R_x)$ . Thus the voltage drop across the resistor  $X$  is

$$V_x = IR_x = VR_x/(R + R_x); \text{ so}$$

$$8 \text{ V} = VR_x/(10 \Omega + R_x) \text{ and } 12 \text{ V} = VR_x/(5 \Omega + R_x); \text{ from which we get } R_x = \boxed{5 \Omega}, \quad V = \boxed{24 \text{ V}}.$$

50. The effective resistance  $R$  of the three-resistor combination satisfies

$$1/R = 1/R_1 + 1/R_2 + 1/R_3.$$

With  $R = 4 \Omega$ ,  $R_1 = 20 \Omega$ , and  $R_2 = 12 \Omega$ , we find the value of the third resistor as

$$R_3 = (1/R - 1/R_1 - 1/R_2)^{-1} = [1/(4 \Omega) - 1/(12 \Omega) - 1/(20 \Omega)]^{-1} = \boxed{8.6 \Omega}.$$

- 51.** The equivalent resistance of the five resistors is

$$R_{\text{eq}} = \sum R_i = 5R_1 = 5(18 \Omega) = \boxed{90 \Omega}.$$

The current in each resistor is the same:

$$I = V/R_{\text{eq}} = (16 \text{ V})/(90 \Omega) = \boxed{0.18 \text{ A}}.$$

The total power dissipated is

$$P = I^2 R_{\text{eq}} = (0.18 \text{ A})^2 (90 \Omega) = \boxed{2.8 \text{ W}}.$$

52. The equivalent resistance of the two resistors is

$$R_{\text{eq}} = \sum R_i = 2R_1 = 2(60 \, \Omega) = 120 \, \Omega.$$

The current in each resistor is the same:

$$I = V/R_{\text{eq}} = (120 \, \text{V})/(120 \, \Omega) = 1.0 \, \text{A}.$$

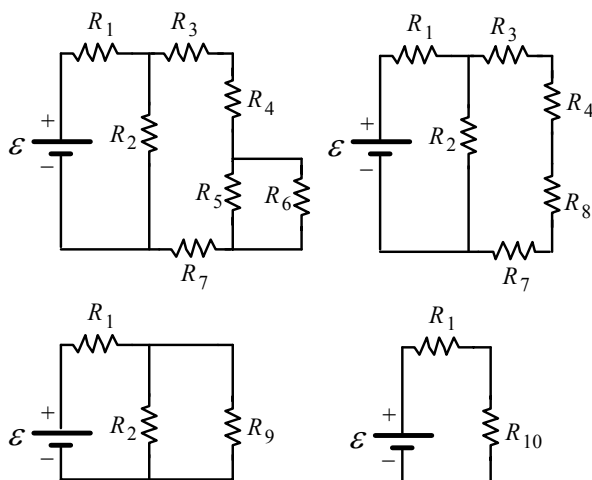
The total power dissipated is

$$P = I^2 R_{\text{eq}} = (1.0 \, \text{A})^2 (120 \, \Omega) = \boxed{120 \, \text{W}}.$$

53. We find the equivalent resistance of the two resistors from

$$\begin{aligned} 1/R_{\text{eq}} &= 1/R_1 + 1/R_2 = 1/R_1 + 1/(2R_1) = \\ 3/(2R_1) &= 3/[2(150 \, \Omega)], \text{ which gives } R_{\text{eq}} = \boxed{100} \\ &\quad \boxed{\Omega}. \end{aligned}$$

54.



We combine  $R_5$  and  $R_6$ , which are in parallel:

$$1/R_8 = 1/R_5 + 1/R_6 = 1/(5 \, \Omega) + 1/(5 \, \Omega), \text{ which gives } R_8 = 2.5 \, \Omega.$$

We combine  $R_3$ ,  $R_4$ ,  $R_8$ , and  $R_7$ , which are in series:

$$\begin{aligned} R_9 &= R_3 + R_4 + R_8 + R_7 \\ &= 2 \, \Omega + 1.5 \, \Omega + 2.5 \, \Omega + 2 \, \Omega = 8 \, \Omega. \end{aligned}$$

We combine  $R_2$  and  $R_9$ , which are in parallel:

$$1/R_{10} = 1/R_2 + 1/R_9 = 1/(5 \, \Omega) + 1/(8 \, \Omega),$$

which gives  $R_{10} = 3.1 \, \Omega$ .

We combine  $R_1$  and  $R_{10}$ , which are in series:

$$R_{\text{eq}} = R_1 + R_{10} = 3 \, \Omega + 3.1 \, \Omega = \boxed{6.1 \, \Omega}.$$

- 55.** The current in the 24- $\Omega$  resistor is

$$I_1 = V_{AB}/(24 \, \Omega) = (16 \, \text{V})/(24 \, \Omega) = \boxed{0.67 \, \text{A}}.$$

Since the equivalent resistance of the upper branch of the circuit is

$$R = 8 \, \Omega + (12 \, \Omega)(6 \, \Omega)/(12 \, \Omega + 6 \, \Omega) = 12 \, \Omega, \text{ the current in the } 8\text{-}\Omega \text{ resistor is}$$

$$I_2 = V_{AB}/(12 \, \Omega) = (16 \, \text{V})/(12 \, \Omega) = \boxed{1.3 \, \text{A}}.$$

This current is split between the two remaining resistors, with

$$I_3 = [6 \, \Omega / (6 \, \Omega + 12 \, \Omega)] I_2 = \approx (1.33 \, \text{A}) = \boxed{0.44 \, \text{A}} \text{ in the } 12\text{-}\Omega \text{ resistor and}$$

$$I_4 = I_2 - I_3 = 1.33 \text{ A} - 0.44 \text{ A} = \boxed{0.89 \text{ A}} \text{ in the } 6\text{-}\Omega \text{ resistor.}$$

56. In Problem 55 we calculated the current flowing through each resistor. Then we may use  $P = I^2 R$  to find the power dissipated on each of them:

$$P = I^2 R = (0.667 \text{ A})^2 (24 \Omega) = \boxed{11 \text{ W}} \quad (\text{for the } 24\text{-}\Omega \text{ resistor});$$

$$P = I^2 R = (1.33 \text{ A})^2 (8 \Omega) = \boxed{14 \text{ W}} \quad (\text{for the } 8\text{-}\Omega \text{ resistor});$$

$$P = I^2 R = (0.889 \text{ A})^2 (6 \Omega) = \boxed{4.7 \text{ W}} \quad (\text{for the } 6\text{-}\Omega \text{ resistor});$$

$$P = I^2 R = (0.444 \text{ A})^2 (12 \Omega) = \boxed{2.4 \text{ W}} \quad (\text{for the } 12\text{-}\Omega \text{ resistor}).$$

57. Call the lower branch with the single  $12\text{-}\Omega$  resistor branch 1 and the other one (which contains all the other resistors) branch 2. First, we find  $R_2$ , the equivalent resistance of branch 2. Combine the  $2\text{-}\Omega$  and  $4\text{-}\Omega$  resistors in parallel to obtain  $(2 \Omega)(4 \Omega)/(2 \Omega + 4 \Omega) = 1.33 \Omega$ , which we add to the  $8\text{-}\Omega$  resistor and put the resultant  $9.33\text{-}\Omega$  resistor in parallel with the  $24\text{-}\Omega$  resistor:  $(9.33 \Omega)(24 \Omega)/(9.33 \Omega + 24 \Omega) = 6.72 \Omega$ . Add this to the  $6\text{-}\Omega$  resistor to obtain  $R_2 = 12.72 \Omega$ .

We now combine  $R_1 (= 12 \Omega)$  and  $R_2$  in parallel to obtain the equivalent resistance between A and B:

$$R_{\text{eq}} = R_1 R_2 / (R_1 + R_2) = (12.72 \Omega)(12 \Omega) / (12.72 \Omega + 12 \Omega) = 6.18 \Omega.$$

The voltage across AB, i.e., that across the  $12\text{-}\Omega$  resistor, is then

$$V_{12 \Omega} = I_{\text{AB}} R_{\text{eq}} = (20 \text{ A})(6.18 \Omega) = \boxed{124 \text{ V}}.$$

The current in the  $12\text{-}\Omega$  resistor is

$$I_{12 \Omega} = V_{\text{AB}} / (12 \Omega) = (124 \text{ V}) / (12 \Omega) = \boxed{10.3 \text{ A}},$$

And the current through the  $6\text{-}\Omega$  resistor is then

$$I_{6 \Omega} = 20 \text{ A} - 10.3 \text{ A} = \boxed{9.7 \text{ A}}, \text{ which requires a voltage of}$$

$$V_{6 \Omega} = I_{6 \Omega} (6 \Omega) = (9.7 \text{ A})(6 \Omega) = \boxed{58 \text{ V}}.$$

The voltage difference across the  $24\text{-}\Omega$  resistor is now

$$V_{24 \Omega} = 123.6 \text{ V} - 58.2 \text{ V} = \boxed{65.4 \text{ V}}, \text{ which drives a current of}$$

$$I_{24 \Omega} = (65.4 \text{ V}) / (24 \Omega) = \boxed{2.7 \text{ A}}, \text{ leaving the current in the } 8\text{-}\Omega \text{ resistor as}$$

$$I_{8 \Omega} = 9.7 \text{ A} - 2.7 \text{ A} = \boxed{7.0 \text{ A}}, \text{ which requires a voltage of}$$

$$V_{8 \Omega} = I_{8 \Omega} (8 \Omega) = (7.0 \text{ A})(8 \Omega) = \boxed{56 \text{ V}}.$$

The voltage applied on the two remaining resistors ( $4\text{-}\Omega$  and  $2\text{-}\Omega$ ) is then

$$V_{4 \Omega} = V_{2 \Omega} = 123.5 \text{ V} - 58.2 \text{ V} - 56.0 \text{ V} = \boxed{9.3 \text{ V}}, \text{ which drives a}$$

current of

$$I_{4 \Omega} = (9.3 \text{ V}) / (4 \Omega) = \boxed{2.3 \text{ A}} \text{ in the } 4\text{-}\Omega \text{ resistor and}$$

$$I_{2 \Omega} = (9.3 \text{ V}) / (2 \Omega) = \boxed{4.7 \text{ A}} \text{ in the } 2\text{-}\Omega \text{ resistor.}$$

58. The two parallel branches has an equivalent resistance of  $R(R + x)/[R + (R + x)]$ , so for the equivalent resistance of the entire load we have

$$R_{\text{eq}} = 3R + R(R + x)/(2R + x) = x;$$

$$x^2 - 2Rx - 7R^2 = 0;$$

$$x = \boxed{(1 + \sqrt{8})R \approx 3.83 R}.$$

59. (a) We combine  $R_2$  and  $R_3$ , which are in parallel:

$$1/R_5 = 1/R_2 + 1/R_3 = 1/(75 \Omega) + 1/(60 \Omega),$$

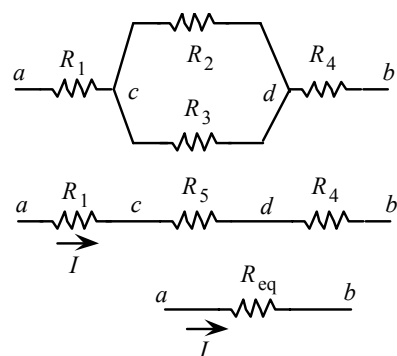
which gives  $R_5 = 33.3 \Omega$ .

We combine  $R_1$ ,  $R_5$ , and  $R_4$ , which are in series:

$$R_{\text{eq}} = R_1 + R_5 + R_4 = 33 + 33.3 \Omega + 25 \Omega,$$

which gives  $R_{\text{eq}} = \boxed{91.3 \Omega}$ .

- (b) We find the current from



$$I = V_{ab}/R_{eq} = (12 \text{ V})/(91.3 \Omega) = 0.131 \text{ A}.$$

The potential difference across the 75- $\Omega$  resistor is  $V_{cd}$ , which we find from

$$V_{cd} = IR_5 = (0.131 \text{ A})(33.3 \Omega) = \boxed{4.38 \text{ V}}.$$

- (c) From part (b), the current through the 33- $\Omega$  resistor is  $I = \boxed{0.131 \text{ A}}.$

60. We find the drift speed from

$$v_d = eE\tau/m \\ = (1.6 \times 10^{-19} \text{ C})(2.0 \times 10^{-3} \text{ V/m})(2.4 \times 10^{-14} \text{ s})/(9.1 \times 10^{-31} \text{ kg}) = \boxed{8.4 \times 10^{-6} \text{ m/s}}.$$

61. We estimate the mean free path as the average distance traveled between collisions:

$$\lambda = v_{av}\tau = (2.7 \times 10^6 \text{ m/s})(2.4 \times 10^{-14} \text{ s}) = \boxed{6.5 \times 10^{-8} \text{ m}}.$$

62. From Eq. 19-46 for the mean free path, we have

$$\lambda = 1/(n\sigma\sqrt{2}); \\ 3.7 \times 10^{-8} \text{ m} = 1/[(8.5 \times 10^{28} / \text{m}^3)\sigma\sqrt{2}], \text{ which gives } \sigma = \boxed{2.2 \times 10^{-22} \text{ m}^2}.$$

63. In terms of the average time between collisions, the resistivity is

$$\rho = m/ne^2\tau.$$

The average time between collisions depends on the drift speed:

$$\tau = mv_d/eE.$$

When we combine these, we have

$$\rho = meE/ne^2mv_d = E/nev_d.$$

For the given  $r$ -dependence of  $v_d$ , we have

$$\rho = E/[nev_0(1 - r/R)] = \boxed{\rho_0/(1 - r/R)}, \text{ where } \rho_0 = E/nev_0.$$

64. From kinetic theory, we have the average kinetic energy of the electron:

$$K = \frac{1}{2}kT,$$

which we use as the energy necessary to cross the energy gap. For the given elements, we have

$$\text{Si: } (1.1 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = \frac{1}{2}(1.38 \times 10^{-23} \text{ J/K})T_{\text{Si}}, \text{ which gives } T_{\text{Si}} = \boxed{8.5 \times 10^3 \text{ K}}.$$

$$\text{Ge: } (0.7 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = \frac{1}{2}(1.38 \times 10^{-23} \text{ J/K})T_{\text{Ge}}, \text{ which gives } T_{\text{Ge}} = \boxed{5.4 \times 10^3 \text{ K}}.$$

$$\text{C: } (6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = \frac{1}{2}(1.38 \times 10^{-23} \text{ J/K})T_{\text{C}}, \text{ which gives } T_{\text{C}} = \boxed{4.6 \times 10^4 \text{ K}}.$$

65. For an ohmic resistor, we have

$$P = IV = V^2/R, \text{ or } R = V^2/P = (12 \text{ V})^2/(65 \text{ W}) = \boxed{2.2 \Omega}.$$

66. For an ohmic resistor, we have

$$P = IV = V^2/R, \text{ or } V^2 = PR = (1.5 \text{ W})(1000 \Omega), \text{ which gives } V = \boxed{39 \text{ V}}.$$

67.  $\text{Cost} = (\text{rate})Pt = (10 \text{ ¢/kWh})(100 \text{ W})(1 \text{ h})(10^{-3} \text{ kW/W}) = \boxed{1 \text{ ¢}}.$

68. For an ohmic resistor, we have

$$P = IV = I^2R, \text{ or } I = (P/R)^{1/2}.$$

For the various resistors, we have

$$P = 1/8 \text{ W: } I = [(1/8 \text{ W})/(100 \Omega)]^{1/2} = 0.035 \text{ A} = \boxed{35 \text{ mA}};$$

$$P = 1/4 \text{ W: } I = [(1/4 \text{ W})/(100 \Omega)]^{1/2} = 0.050 \text{ A} = \boxed{50 \text{ mA}};$$

$$P = 1/2 \text{ W: } I = [(1/2 \text{ W})/(100 \Omega)]^{1/2} = 0.071 \text{ A} = \boxed{71 \text{ mA}};$$

$$P = 1 \text{ W: } I = [(1 \text{ W})/(100 \Omega)]^{1/2} = \boxed{0.10 \text{ A}};$$

$$P = 2 \text{ W: } I = [(2 \text{ W})/(100 \Omega)]^{1/2} = \boxed{0.14 \text{ A}}.$$

69. For an ohmic resistor, we have

$$P = IV = I^2 R, \text{ or } I = (P/R)^{1/2}.$$

$$(a) \quad I = [(5 \text{ W})/(160 \, \Omega)]^{1/2} = \boxed{0.18 \text{ A}}.$$

$$(b) \quad I = [(3 \text{ W})/(2.5 \times 10^3 \, \Omega)]^{1/2} = 3.5 \times 10^{-2} \text{ A} = \boxed{35 \text{ mA}}.$$

$$70. \quad P = IV = (100 \times 10^{-6} \text{ A})(8 \times 10^6 \text{ V}) = \boxed{800 \text{ W}}.$$

71. For an ohmic resistor, we have

$$P = IV = V^2/R, \text{ or } V^2 = PR,$$

so the maximum allowed operating voltage is

$$V_{\max} = \boxed{(PR)^{1/2}}.$$

$$72. \quad \text{Cost} = (\text{rate})IV = (7 \text{ ¢/kWh})(10 \text{ A})(120 \text{ V})(10^{-3} \text{ kW/W}) = \boxed{8.4 \text{ ¢/h}}.$$

73. We find the energy dissipated as heat from

$$U = Pt = I^2 Rt = (100 \text{ A})^2(9 \times 10^{-4} \, \Omega)(20 \text{ s}) = \boxed{1.9 \times 10^2 \text{ J}}.$$

74. For an ohmic resistor, we have

$$P = IV = V^2/R, \text{ or } V^2 = PR = P\rho L/A;$$

$$(110 \text{ V})^2 = (1250 \text{ W})(10^{-6} \, \Omega \cdot \text{m})L/(0.2 \times 10^{-6} \text{ m}^2), \text{ which gives } L = \boxed{1.9 \text{ m}}.$$

75. For an ohmic resistor, we have

$$P = IV = V^2/R, \text{ or } V^2 = PR = P\rho L/A;$$

$$(12 \text{ V})^2 = (0.8 \times 10^3 \text{ W})(1.72 \times 10^{-8} \, \Omega \cdot \text{m})L/(8 \times 10^{-6} \text{ m}^2), \text{ which gives } L = \boxed{84 \text{ m}}.$$

76. (a) For an ohmic resistor, we have

$$P = IV, \text{ so the maximum power is}$$

$$P_{\max} = I_{\max} V = (15 \text{ A})(110 \text{ V}) = 1.65 \times 10^3 \text{ W} = \boxed{1.65 \text{ kW}}.$$

- (b) We find the maximum number of light bulbs from

$$N = P_{\max}/P_{\text{bulb}} = (1.65 \times 10^3 \text{ W})/(75 \text{ W}) = 22 \rightarrow \boxed{22 \text{ bulbs}}.$$

77. The energy taken out of the battery is

$$U = Pt = IVt = (50 \times 10^{-3} \text{ A})(6 \text{ V})(18 \text{ h})(3600 \text{ s/h}) = \boxed{1.94 \times 10^4 \text{ J} \quad (5.4 \text{ kWh})}.$$

78. Assuming a constant resistance, we have

$$P = IV = V^2/R, \text{ or}$$

$$P_2/P_1 = (V_2/V_1)^2(R_1/R_2) = (V_2/V_1)^2;$$

$$P_2/(500 \text{ W}) = [(105 \text{ V})/(115 \text{ V})]^2, \text{ which gives } P_2 = \boxed{417 \text{ W}}.$$

79. The power dissipated on a wire of surface area
- $A$
- and surface temperature
- $T$
- is

$$P = \sigma T^4 A, \text{ where } A = \pi dl, \text{ with } l \text{ its length and } d \text{ its cross-sectional diameter.}$$

The current  $I$  is related to the power  $P$  and the resistance  $R$  of the wire as

$$P = I^2 R, \text{ so}$$

$$I = (P/R)^{1/2} = [\sigma T^4 (\pi dl) / (\rho l / \pi d^2)]^{1/2} = \text{constant} \times (T^2 d^{3/2}),$$

which suggest that  $I$  depends more strongly on  $T$  (to the 2<sup>nd</sup> power) than on  $d$  (to the 3/2-th power). So it would be preferable to change the temperature.

80. Because the powers add and the resistors are identical, we have

$$P = I^2 R_{\text{eq}} = I^2 (3R) = 3I^2 R = \boxed{3P_0}.$$

81. If we estimate that it takes 3 min to boil a 0.50-L pot, we have

$$mc \Delta T = IVt;$$

$$(500 \text{ cm}^3)(1 \text{ g/cm}^3)(1 \text{ cal/g} \cdot ^\circ\text{C})(100^\circ\text{C})(4.185 \text{ J/cal}) = (4\text{A})V(5 \text{ min})(60 \text{ s/min}),$$

which gives  $V = 170 \text{ V}$ .

We find the resistance from

$$R = V/I = 170 \text{ V}/4 \text{ A} = 43 \Omega.$$

82. We find the total energy used in the month from

$$U = \$25.33/(\$0.08/\text{kWh}) = 317 \text{ kWh}.$$

The average current during the month was

$$I = P/V = U/Vt, \text{ so the charge that passed through the meter was}$$

$$Q = It = U/V, \text{ and the number of electrons was}$$

$$N = Q/e = U/Ve = (317 \text{ kWh})(10^3 \text{ W/kW})(3600 \text{ s/h})/(120 \text{ V})(1.6 \times 10^{-19} \text{ C}) = 5.94 \times 10^{25} \text{ electrons}.$$

83. Because the volume is constant, we have

$$A_2 L_2 = A_1 L_1.$$

For a fixed voltage, the power dissipation is

$$P = V^2/R = V^2 A / \rho L.$$

If we apply this to the two wires and divide the two expressions, we get

$$\begin{aligned} P_2/P_1 &= (V^2 A_2 / \rho L_2) / (V^2 A_1 / \rho L_1) \\ &= (A_2 / A_1)(L_1 / L_2) = (L_1 / L_2)^2 = (1/2)^2 = 1/4. \end{aligned}$$

The power decreases by 3/4.

84. (a) We find the number of protons from

$$N = Q/e = It/e$$

$$= (5 \times 10^{-6} \text{ A})(1 \text{ h})(3600 \text{ s/h}) / (1.6 \times 10^{-19} \text{ C}) =$$

$$1.1 \times 10^{17} \text{ protons}.$$

- (b) Because each proton has an energy of 4 MeV, the total energy is

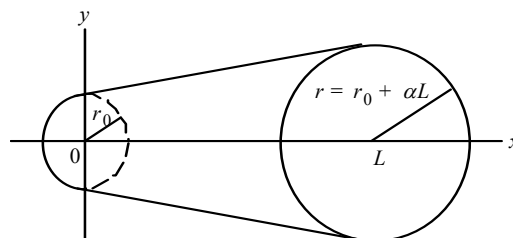
$$U_{\text{total}} = NU$$

$$= (1.1 \times 10^{17} \text{ protons})(4 \times 10^6 \text{ eV/proton})(1.6 \times 10^{-19} \text{ J/eV}) = 7.2 \times 10^4 \text{ J}.$$

- (c) We find the power of the beam from

$$P = U_{\text{total}}/t$$

$$= (7.2 \times 10^4 \text{ J}) / (1 \text{ h})(3600 \text{ s/h}) = 20 \text{ W}.$$



85. To find the resistance of the cylinder, we choose a vertical slice at a distance  $x$  from the origin, with radius  $r = r_0 + \alpha x$  and thickness  $dx$ . We find the resistance by integrating over these slices:

$$\begin{aligned} R &= \int \frac{\rho dL}{A} = \int_0^L \frac{\rho dx}{\pi r^2} = \frac{\rho}{\pi} \int_0^L \frac{dx}{(r_0 + \alpha x)^2} \\ &= -\frac{\rho}{\pi \alpha} \frac{1}{r_0 + \alpha x} \Big|_0^L = -\frac{\rho}{\pi \alpha} \left( \frac{1}{r_0 + \alpha L} - \frac{1}{r_0} \right) = \frac{\rho L}{\pi r_0 (r_0 + \alpha L)}. \end{aligned}$$

86. (a) When the bulbs are connected in series, the equivalent resistance is

$$R_{\text{series}} = \sum R_i = 10R_{\text{bulb}}.$$

The power consumption is

$$P = V_{ab}^2 / R_{\text{eq}};$$

$$50 \text{ W} = (120 \text{ V})^2 / (10R_{\text{bulb}}), \text{ which gives } R_{\text{bulb}} = \boxed{28.8 \Omega}$$

$\Omega$ .

(b) In part (a), the power consumption of each bulb is 5 W, which is the maximum power rating, so the voltage across each bulb, 12 V, must be the maximum allowed. With the limiting resistor connected in series with the parallel bulb combination, we find the equivalent resistance of the ten bulbs from

$1/R_{\text{parallel}} = \sum (1/R_i) = 10/R_{\text{bulb}} = 10/(28.8 \Omega)$ , which gives  $R_{\text{parallel}} = 2.88 \Omega$ .

For the maximum consumption, the voltage across each bulb, and thus  $R_{\text{parallel}}$ , is 12 V, so we have

$$V_{\text{parallel}} = I_{\text{total}} R_{\text{parallel}};$$

$$12 \text{ V} = I_{\text{total}} (2.88 \Omega), \text{ which gives } I_{\text{total}} = 4.17 \text{ A}.$$

With the series resistor, we have

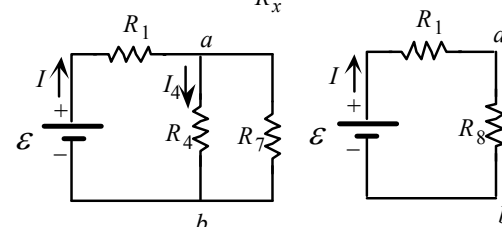
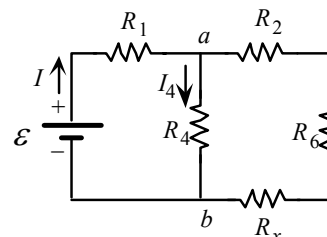
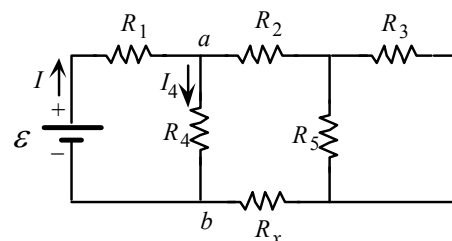
$$V_{ab} = I_{\text{total}} (R_{\text{parallel}} + R_2);$$

$$120 \text{ V} = (4.17 \text{ A})(2.88 \Omega + R_2), \text{ which gives } R_2 = \boxed{25.9 \Omega}$$

in series.

The power loss in the added resistor is

$$P_2 = I_{\text{total}}^2 R = (4.17 \text{ A})^2 (25.9 \Omega) = \boxed{4.5 \times 10^2 \text{ W}}.$$



87. We can reduce the circuit to a single loop by successively combining parallel and series combinations.

We combine  $R_3$  and  $R_5$ , which are in parallel:

$$1/R_6 = 1/R_3 + 1/R_5 = 1/(3 \Omega) + 1/(6 \Omega),$$

which gives  $R_6 = 2 \Omega$ .

We combine  $R_2$ ,  $R_6$ , and  $R_x$ , which are in series:

$$R_7 = R_2 + R_6 + R_x = 2 \Omega + 2 \Omega + R_x = 4 \Omega + R_x.$$

We combine  $R_4$  and  $R_7$ , which are in parallel:

$$1/R_8 = 1/R_4 + 1/R_7 = 1/(4 \Omega) + 1/(4 \Omega + R_x),$$

which gives  $R_8 = (16 \Omega + 4R_x)/(8 \Omega + R_x)$ .

We find the current in the single loop from

$$\begin{aligned} I &= \mathcal{E} / (R_1 + R_8) \\ &= (24 \text{ V}) / [1 \Omega + (16 \Omega + 4R_x) / (8 \Omega + R_x)] \\ &= \boxed{[24(8 \Omega + R_x) / (24 \Omega + 5R_x)] \text{ A}}. \end{aligned}$$

We use the voltage across  $R_8$  and across  $R_4$ :

$$\begin{aligned} V_{ab} &= IR_8 = I_4 R_4; \\ [24 \text{ A} (8 \Omega + R_x) / (24 \Omega + 5R_x)] \times \\ &\quad [(16 \Omega + 4R_x) / (8 \Omega + R_x)] = I_4 (4 \Omega), \end{aligned}$$

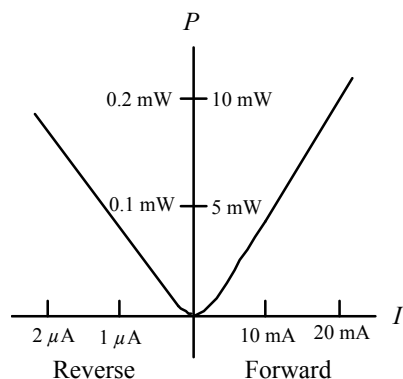
which gives

$$I_4 = [24 (4 \Omega + R_x) / (24 \Omega + 5R_x)] \text{ A}.$$

The power dissipated is

$$\begin{aligned} P_4 &= I_4^2 R_4 \\ &= [24(4 \Omega + R_x) / (24 \Omega + 5R_x)]^2 (4 \Omega) \end{aligned}$$





$$= \boxed{[48(4 \Omega + R_x)/(24 \Omega + 5R_x)]^2 \text{ W}}.$$

88. The power dissipated in the bus bar is

$$P = I^2 R = I^2 \rho L / A = I^2 \rho_0 (1 + \alpha \Delta T) L / A;$$

$$0.2 \text{ W} = (100 \text{ A})^2 (1.72 \times 10^{-8} \Omega \cdot \text{m}) [1 + (0.0039 / ^\circ\text{C})(300 \text{ K} - 20 \text{ K})] (0.25 \text{ m}) / A,$$

$$\text{which gives } A = \boxed{4.5 \times 10^{-4} \text{ m}^2}, \quad 1.5 \text{ cm} \times 3.0 \text{ cm}.$$

89. The power delivered by the generator is

$$P = IV = (75 \text{ A})(12 \text{ V}) = 9.0 \times 10^2 \text{ W} = \boxed{0.9 \text{ kW}}.$$

For the generator to supply the energy required to raise the temperature of the water, we have

$$mc \Delta T = Pt_1;$$

$$(10^{-3} \text{ m}^3)(10^2 \text{ cm/m})^3 (1 \text{ g/cm}^3)(1 \text{ cal/g} \cdot ^\circ\text{C})(7.5^\circ\text{C})(4.185 \text{ J/cal}) = (9.0 \times 10^2 \text{ W})t_1,$$

which gives  $t_1 = \boxed{35 \text{ s}}$ .

The mass of 0.5 L of water is  $m = (0.1 \times 10^3 \text{ cm}^3)(1 \text{ g/cm}^3) = 500 \text{ g}$ . For the water to boil away, we have

$$mc \Delta T + mL_v = Pt_2;$$

$$(500 \text{ g})(1 \text{ cal/g} \cdot ^\circ\text{C})(75^\circ\text{C})(4.185 \text{ J/cal}) + (500 \text{ g})[(1 \text{ mol})/(18 \text{ g})](41 \times 10^3 \text{ J/mol}) = (9.0 \times 10^2 \text{ W})t_2,$$

which gives  $t_2 = 1.4 \times 10^3 \text{ s} = \boxed{23 \text{ min}}$ .

90.

We find the power from  $P = IV$ , estimating values from the plot. Note the change in scales on the two sides of the plot.

In the ideal diode, either the potential or the current is zero, so the power is zero.

91. (a) If the resistivity of the wire is  $\rho_0$  at  $T_0$ , the resistance of the wire is

$$R = \rho L / A = (\rho_0 L / A)(1 + \alpha \Delta T) = \boxed{(\rho_0 L / A)(1 + \alpha k t^2)}.$$

For a constant potential, the current is

$$I = V / R = VA / \rho_0 L (1 + \alpha k t^2) = I_0 / (1 + \alpha k t^2).$$

- (b) The power dissipated in the wire is

$$P = IV = \boxed{VI_0 / (1 + \alpha k t^2)}.$$

- (c) The rate at which the dissipated power changes is

$$dP/dt = VI_0(-2\alpha k t) / (1 + \alpha k t^2)^2 = -2VI_0\alpha k t / (1 + \alpha k t^2)^2.$$

Because  $dP/dt < 0$ , thermal equilibrium will be reached.

92. The resistance of the wire is  $R = \rho L / A$ , and the current is  $I = JA = nev_d A$ . The power dissipated, which becomes thermal energy, is

$$\begin{aligned} P &= I^2 R = (nev_d A)^2 \rho L / A = (nev_d)^2 A \rho L \\ &= [(8.5 \times 10^{28} \text{ electrons/m}^3)(1.6 \times 10^{-19} \text{ C})(1.2 \times 10^{-5} \text{ m/s})]^2 [\pi(0.5 \times 10^{-3} \text{ m})^2](1.72 \times 10^{-8} \Omega \cdot \text{m})(3 \text{ m}) \\ &= 1.1 \times 10^{-3} \text{ W}. \end{aligned}$$

For the wire to maintain its temperature, thermal energy must be removed at this rate,  $\boxed{1.1 \times 10^{-3} \text{ W}}$ .

93. If a coil has length  $\pi r$ , the number of turns in the coil is

$$N = \pi / 2r, \quad \text{so}$$

$$N_2 / N_1 = r_1 / r_2.$$

Because a turn has a length  $\pi D$ , the length of a wire is

$$L = N\pi D, \quad \text{so}$$

$$L_2 / L_1 = N_2 D_2 / N_1 D_1 = r_1 D_2 / r_2 D_1.$$

The resistance of a wire is

$$R = \rho L / A, \quad \text{so we have}$$

$$\begin{aligned} R_2 / R_1 &= (\rho L_2 / \pi r_2^2) / (\rho L_1 / \pi r_1^2) = (r_1 D_2 / r_2 D_1)(r_1 / r_2)^2 = (D_2 / D_1)(r_1 / r_2)^3 \\ &= [(8 \text{ cm}) / (5 \text{ cm})][(0.6 \text{ mm}) / (0.4 \text{ mm})]^3 = \boxed{5.4}. \end{aligned}$$

94. (a) From the conservation of charge, we know that the current must be constant along the wire.

Because the area is also constant, we have

$$E = \rho j = \rho_0 j e^{-x/L} = (\rho_0 I / A) e^{-x/L} = \boxed{E_0 e^{-x/L}}.$$

- (b) We take the reference level for  $V$  to be  $V = 0$  at  $x = L$ , so  $V = V_0$  at  $x = 0$ . We integrate the relation between the field and the potential,  $E = -dV/dx$ :

$$\int_{V_0}^V dV = - \int_0^x E dx' = -E_0 \int_0^x e^{-x'/L} dx';$$

$$V - V_0 = -E_0 \left( -L e^{-x'/L} \right) \Big|_0^x = +E_0 L (e^{-x/L} - 1) = -E_0 L (1 - e^{-x/L}).$$

We can determine  $E_0$  from our reference level:

$$-V_0 = -E_0 L (1 - e^{-1}), \quad \text{which gives } E_0 L = V_0 / (1 - e^{-1}).$$

The potential is

$$V = V_0 - V_0 (1 - e^{-x/L}) / (1 - e^{-1}) = \boxed{V_0 (e^{-x/L} - e^{-1}) / (1 - e^{-1})}.$$

- (c) We choose a differential segment of the wire at  $x$  with length  $dx$ . We find the resistance by integration:

$$\begin{aligned} R &= \int \frac{\rho dL}{A} = \int_0^L \frac{\rho_0 e^{-x/L} dx}{A} = -\frac{\rho_0 L}{A} e^{-x/L} \Big|_0^L \\ &= -\frac{\rho_0 L}{A} (e^{-1} - 1) = \frac{\rho_0 L}{A} (1 - e^{-1}). \end{aligned}$$

95. If we ignore the change in the dimensions of the wire, the resistance of the wire will be a function of the temperature:

$$R = R_0(1 + \alpha \Delta T) = R_0[1 + \alpha(T - T_0)].$$

Because all of the energy from Joule heating raises the temperature of the wire, we have

$$P = I^2 R = V^2 / R = V^2 / R_0 [1 + \alpha(T - T_0)] = mc(dT/dt), \text{ which gives}$$

$$\boxed{dT/dt = k/[1 + \alpha(T - T_0)], \text{ where } k = V^2/mcR_0.}$$

The solution of this equation will give the temperature as a function of time,  $T(t)$ .

We find the current from

$$I(t) = V/R = \boxed{V/R_0\{1 + \alpha[T(t) - T_0]\}}.$$