

# CHAPTER 24 Electric Potential

## Answers to Understanding the Concepts Questions

1. The volt is defined so that  $1 \text{ N/C} = 1 \text{ V} \cdot \text{m}$ . Cross-multiplying yields  $1 \text{ V} \cdot \text{C} = 1 \text{ N} \cdot \text{m} = 1 \text{ J}$ .
2. The electric field due to the charged plane is  $E = \sigma/2\epsilon_0$ . If we put a negative test charge  $q$  next to the plane then it will experience an electric force of  $F = qE = q\sigma/2\epsilon_0$ . Measure  $F$  (say, by means of finding the resulting acceleration of the charge toward the plane) and we can obtain the value of  $\sigma$ .
3. If charge is placed inside the hollow space in a spherical metal shell, there will be an electric field. The field lines will join the charges to the inner surface of the shell where induced charges appear so as to yield a net charge inside any surface entirely within the spherical shell. To make a constant field in a small region, insert a small uniformly charged plane within the space; near that plane the field is constant.
4. The reason why the electric field near the surface of Earth points downward is because Earth is negatively charged. The metal rod, when in contact with the ground, would also be negatively charged as it forms part of an equipotential body with Earth. Since electric charges tend to congregate in sharp corners, we can expect a higher surface charge density near the end of the rod that is exposed to the air, and the electric field just outside that end is expected to be greater than the ambient value of  $100 \text{ V/m}$ .
5. The question is analogous to asking for the source of the energy which moves a test mass in a gravitational field that arises from a distribution of masses. The test charge -- like the test mass -- has potential energy by virtue of being in the field of the existing charge distribution (mass distribution) and some of this potential energy is converted to kinetic energy in giving the test charge (test mass) some motion. The source of the potential energy is the work that had to be done to assemble the charge distribution (mass distribution) by bringing in the constituent charges (constituent masses) from infinity.
6. If the kinetic energy of the charge changes, then the work done on the unit test charge would no longer be equal to the change in electric potential. For example, if the kinetic energy of the charge increases by  $1 \text{ J}$ , then in addition to changing the potential energy of the test charge, which requires a certain amount of work, one must also expend an additional  $1 \text{ J}$  of work on the charge to cause the increase in its kinetic energy.
7. The net electric force due to the dipole on a point charge is the difference between the force exerted by the positive charge and that exerted by the negative charge. This difference is greater at  $\theta = 0$  or  $\pi$ , so the net force due to the dipole is greater at  $\theta = 0$  or  $\pi$  than at  $\pi/2$ .
8. No. They are at the same (high) potential as the Van de Graaff generator itself and are insulated from the ground. In fact if they were grounded (say, by having their bare feet touch the ground) then there would be a potential difference between their hands, which are touching the generator, and their feet, which are grounded. Such potential difference could be dangerous, as it would drive a current through the body.
9. The surface of a conductor will always be an equipotential in equilibrium. If a charge is placed on an

electric surface, there is a short time during which it is localized. After that short time interval it distributes itself over the surface so that there is no force on any part of the charge, and equilibrium is achieved. The mention of a "short time" indicates that the statement that a conductor forms an equipotential is specifically true for static fields. The question then becomes more a matter of finding the time scale that distinguishes static from nonstatic fields.

10. According to Eq. (24-29), the electric field equals to the negative value of the gradient of the potential function. If we add a constant term to the potential to shift the location of zero potential to any fixed point  $(x_0, y_0, z_0)$ , i.e., change  $V(x, y, z)$  to  $V(x, y, z) - V(x_0, y_0, z_0)$ , whereupon  $V = 0$  at  $(x_0, y_0, z_0)$ , we will not be changing the value of  $E(x, y, z)$ . This is because the gradient of the additional constant term is zero:  $\partial V(x_0, y_0, z_0)/\partial x = \partial V(x_0, y_0, z_0)/\partial y = \partial V(x_0, y_0, z_0)/\partial z = 0$ .
11. The point of the demonstration is to place charge on the person. Once that happens, the individual hairs share the charge and repel each other much like the leaves of an electroscope. If the person is not on an insulated mat, then the charge from the generator will flow through the person as current, with potentially painful or even fatal results.
12. The zero potential can be defined at any point where the charge density is finite. Even if Earth is negatively charged we can still define its potential as zero. Remember, it's the potential *difference* that's physically relevant, not the potential itself.
13. Knowledge of the potential at a point does not allow us to determine the electric field. The simplest way to see this is to observe that potential energy contains an arbitrary constant. In contrast, if we know the potential at two adjacent points, then we know a potential difference, and this has physical meaning. In fact, the difference in the potential can allow us to find the electric field in the direction of the vector that connects the two adjacent points, as can be seen from Eq. (24-9); the points  $b$  and  $a$  are taken near each other.
14. The field lines seem to be denser near the sharp edges of the conductors (i.e., the two ends of the rod and the tip of the tear-drop shaped conductor). This is of course expected; see the discussion next to the figure in the textbook.
15. Yes. What matters is the final configuration of the charged systems, not how it was assembled. This is clear from Eq. (24.18).
16. The electric fields are very large at sharp corners of any charged object, and with large fields breakdown becomes much more likely. Smooth spherical surfaces minimize this possibility by minimizing the presence of points.
17. The net work done by an electrostatic force is always zero as we move a test charge along any enclosed path. By definition, then, the electrostatic force is conservative.
18. Nothing of physical significance would change, since potential energy, and thus electric potential, are not specified to within an additive constant.
19. Yes. For example, consider a pair of charges,  $+q$  and  $-q$ , separated from each other by a distance  $2r$ . If we take  $V = 0$  at infinity, then the potential at the midpoint of the line connecting the two charges is  $V = kq/r + (-kq/r) = 0$ .

20. The potential at a fixed point tells us nothing about the electric field because it is only differences in potentials that give us information about the electric field. (See the discussion of question 13.) If, however, we know the value of the potential in the vicinity of the points where it is zero, we can do better.
21. Yes. For example, take two concentric spherical shells of radii  $r_1$  and  $r_2$ , respectively, and charge them uniformly such that  $q_1/q_2 = -r_1/r_2$ . Then the potential everywhere inside the smaller sphere is  $V = kq_1/r_1 + kq_2/r_2 = 0$ .
22. If we interpret “the easiest way” as the steepest path, then we want to look for a direction in which the lines of constant elevation are the closest from each other. In Fig. 24-10, this is roughly in the “south-east” direction. If you move perpendicularly to the contours then your elevation drops the fastest. This is what happens to a ball if you let it roll off the top of the peak from rest. (And if you follow a certain contour then your elevation does not change, of course.) If we think of this plot as an equipotential plot for a two-dimensional charge distribution, then each contour represents a certain equipotential, and the electric field lines are always perpendicular to the equipotentials.

**Solutions to Problems**

1. With the reference level at infinity, the electrostatic potential energy of the two protons is

$$U = (1/4\pi\epsilon_0)(e^2/r) \\ = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(1.6 \times 10^{-19} \text{ C})^2/(5 \times 10^{-15} \text{ m}) = \boxed{4.6 \times 10^{-14} \text{ J}}.$$

2. With the reference level at infinity, the electrostatic potential energy of the two charges is

$$U = (1/4\pi\epsilon_0)(q_1q_2/r) = - (1/4\pi\epsilon_0)(Ze^2/r) \\ = - (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(92)(1.6 \times 10^{-19} \text{ C})^2/(3 \times 10^{-12} \text{ m}) = \boxed{-7.1 \times 10^{-15} \text{ J}}.$$

3. With the reference level at infinity, the potential energy of the two charges is

$$U = (1/4\pi\epsilon_0)(q_1q_2/r) \\ = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(7.0 \times 10^{-7} \text{ C})(3.0 \times 10^{-6} \text{ C})/(0.20 \text{ m}) = \boxed{9.5 \times 10^{-2} \text{ J}}.$$

4. The raisin will move directly away from the origin to infinity. Because the raisin starts with no kinetic energy, the initial potential energy becomes its final kinetic energy:

$$K_f = U_i = \boxed{9.5 \times 10^{-2} \text{ J}}.$$

5. (a) Because there is no other charge present, no force is required to bring the charge from infinity:

$$W = \boxed{0}.$$

- (b) There is now a potential energy of the two charges, with the reference level at infinity. We find the work done by the electric field from

$$W = -\Delta U = (-1/4\pi\epsilon_0)(q_1q_2/r_1) \\ = - (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(3 \times 10^{-6} \text{ C})(5 \times 10^{-6} \text{ C})/(0.10 \text{ m}) = \boxed{-1.35 \text{ J}}.$$

- (c) The work done by the external agent is the negative of the work done by the electric field:

$$W_F = -W = \boxed{+1.35 \text{ J}}.$$

6. We find the work done by an outside agent from the work-energy theorem:

$$W = \Delta K + \Delta U = 0 + U_b - U_a = (1/4\pi\epsilon_0)q_1q_2(1/r_b - 1/r_a) \\ = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(+2.0 \times 10^{-5} \text{ C})(-9.0 \times 10^{-4} \text{ C})[1/(5 \times 10^{-5} \text{ m}) - 1/(5 \times 10^{-4} \text{ m})] = \boxed{-2.9 \times 10^{-5} \text{ J}}.$$

7. The potential energy is a scalar that depends only on the distance. The distances of the third charge from each of the others are

$$r_{a1} = [(30 \text{ cm})^2 + 0 + (50 \text{ cm} - 5 \text{ cm})^2]^{1/2} = 54.1 \text{ cm};$$

$$r_{a2} = [(30 \text{ cm})^2 + 0 + (15 \text{ cm} + 5 \text{ cm})^2]^{1/2} = 62.6 \text{ cm}.$$

The potential energy is

$$U_a = (1/4\pi\epsilon_0)(q_1q_3/r_{a1} + q_2q_3/r_{a2}) = (1/4\pi\epsilon_0)q_3(q_1/r_{a1} + q_2/r_{a2}) \\ = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(0.20 \times 10^{-6} \text{ C})[(3.0 \times 10^{-6} \text{ C})/(0.541 \text{ m}) + (-3.0 \times 10^{-6} \text{ C})/(0.626 \text{ m})] \\ = \boxed{1.36 \times 10^{-3} \text{ J}}.$$

When the charge is placed at (30 cm, 0 cm, 0 cm), the distances become

$$r_{b1} = r_{b2} = [(30 \text{ cm})^2 + 0 + (5 \text{ cm})^2]^{1/2} = 30.4 \text{ cm}.$$

The potential energy is

$$U_b = (1/4\pi\epsilon_0)[(q_1q_3/r_{b1}) + (q_2q_3/r_{b2})] = (1/4\pi\epsilon_0)q_3[(q_1/r_{b1}) + (q_2/r_{b2})] \\ = (1/4\pi\epsilon_0)(q_3/r_{b1})(q_1 + q_2) = \boxed{0}, \text{ because } q_1 = -q_2.$$

8. (a)  $U_a = (1/4\pi\epsilon_0)(q_1q_3/r_{a1} + q_2q_3/r_{a2}) = (1/4\pi\epsilon_0)q_3(q_1/r_{a1} + q_2/r_{a2})$   
 $= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(-0.20 \times 10^{-6} \text{ C})[(3.0 \times 10^{-5} \text{ C})/(0.541 \text{ m}) + (3.0 \times 10^{-5} \text{ C})/(0.626 \text{ m})]$   
 $= \boxed{-1.9 \times 10^{-2} \text{ J}}$   
 $U_b = (1/4\pi\epsilon_0)(q_1q_3/r_{b1} + q_2q_3/r_{b2}) = (1/4\pi\epsilon_0)q_3(q_1/r_{b1} + q_2/r_{b2})$   
 $= (1/4\pi\epsilon_0)(q_3/r_{b1})(q_1 + q_2)$   
 $= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)[(-0.20 \times 10^{-6} \text{ C})/(0.304 \text{ m})][(3.0 \times 10^{-5} \text{ C}) + (3.0 \times 10^{-5} \text{ C})] = \boxed{-3.6 \times 10^{-2} \text{ J}}$   
 (b) Changing the sign of the third charge will change the sign of the potential energy:  
 $U_a = \boxed{+1.9 \times 10^{-2} \text{ J}}, U_b = \boxed{+3.6 \times 10^{-2} \text{ J}}$

9. The distances between the two charges are  
 $r_i = [(12 \text{ cm} - 12 \text{ cm})^2 + (60 \text{ cm} - 25 \text{ cm})^2 + (-50 \text{ cm} - 0)^2]^{1/2} = 61.0 \text{ cm};$   
 $r_f = [(12 \text{ cm} - 12 \text{ cm})^2 + (50 \text{ cm} - 25 \text{ cm})^2 + (25 \text{ cm} - 0)^2]^{1/2} = 35.4 \text{ cm}.$

The work done by an external agent to move the second charge is

$$W = \Delta U = (1/4\pi\epsilon_0)q_1q_2(1/r_f - 1/r_i)$$

$$= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(1.5 \times 10^{-6} \text{ C})(-3 \times 10^{-6} \text{ C}) [1/(0.351 \text{ m}) - 1/(0.610 \text{ m})] = \boxed{-4.8 \times 10^{-2} \text{ J}}$$

10. A stable radius requires a minimum in the potential energy. With the reference level at infinity, the potential energy of two like charges is positive and increases as  $r$  decreases. Thus there will be no minimum and no stable orbit.

11. The electric potential for two charges is

$$V = (1/4\pi\epsilon_0)(q_1/r_1 + q_2/r_2).$$

Because both charges are negative, the only place where  $V$  can be 0 is  $\boxed{r = \infty}$ .

12. The potential is a scalar that depends only on the distance. The potential for two charges is

$$V = (1/4\pi\epsilon_0)(q_1/r_1 + q_2/r_2).$$

If the potential is 0 at a point  $x$ , we have

$$0 = (1/4\pi\epsilon_0)[(3 \times 10^{-6} \text{ C})/|(x - 14 \text{ cm})| + (-4 \times 10^{-6} \text{ C})/|(x - 15 \text{ cm})|],$$

which gives  $3|x - 15 \text{ cm}| = 4|x - 14 \text{ cm}|$ . No position between the two charges gives  $V = 0$ .

For a point outside the two charges, we have

$$3(x_2 - 15 \text{ cm}) = 4(x_2 - 14 \text{ cm}), \text{ which gives } x = \boxed{19 \text{ cm}}.$$

13. We find the value of the charge from

$$V = (1/4\pi\epsilon_0)q/r$$

$$0.12 \text{ V} = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)q/(2.5 \times 10^{-3} \text{ m}), \text{ which gives } q = \boxed{3.3 \times 10^{-14} \text{ C}}.$$

14. Because the total energy of the proton is conserved, we have

$$\Delta K + \Delta U = 0;$$

$$\frac{1}{2}m(v_B^2 - v_A^2) + q(V_B - V_A) = 0;$$

$$V_B - V_A = -\frac{1}{2}(m/q)(v_B^2 - v_A^2)$$

$$= -\frac{1}{2}[(1.67 \times 10^{-27} \text{ kg})/(1.6 \times 10^{-19} \text{ C})][(8 \times 10^5 \text{ m/s})^2 - (5 \times 10^4 \text{ m/s})^2] = \boxed{+3.3 \text{ kV}}.$$

15. We find the work done by an external agent from the work-energy theorem:

$$W = \Delta K + \Delta U = 0 + q(V_b - V_a)$$

$$= (3.0 \times 10^{-7} \text{ C})[+17 \text{ kV} - (+3.0 \text{ kV})](10^3 \text{ V/kV}) = \boxed{+4.2 \times 10^{-3} \text{ J}}.$$

16. We find the work done by an external agent from the work-energy theorem:

$$W = \Delta K + \Delta U = 0 + q(V_b - V_a) = (3 \times 10^{-8} \text{ C})[+27 \text{ kV} - (+16 \text{ kV})](10^3 \text{ V/kV}) = \boxed{+3.3 \times 10^{-4} \text{ J}}.$$

17. We find the potential energy of the system of charges by adding the work required to bring the three charges in from infinity successively:

$$W_1 = q_1 V_0 = 0;$$

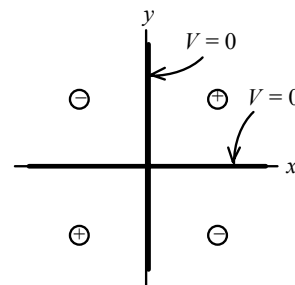
$$W_2 = q_2 V_1 = q_2(1/4\pi\epsilon_0)q_1/r_{12} = (1/4\pi\epsilon_0)q_1q_2/r_{12};$$

$$W_3 = q_3 V_2 = q_3(1/4\pi\epsilon_0)(q_1/r_{13} + q_2/r_{23}) = (1/4\pi\epsilon_0)(q_1q_3/r_{13} + q_2q_3/r_{23}).$$

The total potential energy is

$$\begin{aligned} U &= W_1 + W_2 + W_3 = (1/4\pi\epsilon_0)(q_1q_2/r_{12} + q_1q_3/r_{13} + q_2q_3/r_{23}) \\ &= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)[(2 \text{ mC})(0.5 \text{ mC})/(1 \text{ m}) + (2 \text{ mC})(-1.5 \text{ mC})/(0.5 \text{ m}) + \\ &\quad (0.5 \text{ C})(-1.5 \text{ C})/(1.5 \text{ m})](10^{-3} \text{ C/mC})^2 = \boxed{-5.0 \times 10^4 \text{ J}}. \end{aligned}$$

18. For each positive-negative pair of equal charges,  $V = 0$  at points that are equidistant from the charges. From the placement of the charges, we see that the points in the  $xz$ -plane and in the  $yz$ -plane will have  $V = 0$ .



19. (a) The diameter of a circle subtends an angle of  $90^\circ$  at any point on the circle. Thus the distance from the negative charge to the point is

$$r_2 = [(2R)^2 - r_1^2]^{1/2} = [(50 \text{ cm})^2 - (30 \text{ cm})^2]^{1/2} = 40 \text{ cm}.$$

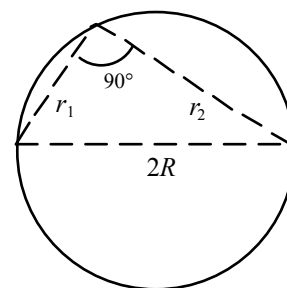
The potential at the point is

$$\begin{aligned} V &= (1/4\pi\epsilon_0)(q_1/r_1 + q_2/r_2) \\ &= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)[(24 \times 10^{-8} \text{ C})/(0.30 \text{ m}) + (-10 \times 10^{-8} \text{ C})/(0.40 \text{ m})] \\ &= \boxed{+5.0 \times 10^3 \text{ V}}. \end{aligned}$$

- (b) The work required is

$$W = q \Delta V = (-0.2 \times 10^{-6} \text{ C})(5.0 \times 10^3 \text{ V} - 0) = \boxed{-1.0 \times 10^{-3} \text{ J}}.$$

The negative value indicates that the negative charge wants to “fall” to the higher potential.



20. The origin is equidistant from the three charges. If the side of the triangle is  $L$ , the distance from the center to a corner is

$$r = (L/2)/\cos 30^\circ = (3 \text{ cm})/(2 \cos 30^\circ) = 1.73 \text{ cm}.$$

The potential is

$$V = (1/4\pi\epsilon_0)(q_1/r_1 + q_2/r_2 + q_3/r_3).$$

Because the charges and distances are the same, we have

$$V = (1/4\pi\epsilon_0)[3(q_1/r_1)] = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(3)(0.5 \times 10^{-6} \text{ C})/(1.73 \times 10^{-2} \text{ m}) = \boxed{7.8 \times 10^5 \text{ V}}.$$

21. If we let  $q = 10^{-6} \text{ C}$ , the charges are

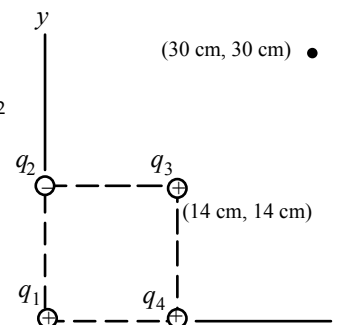
$$q_1 = 2q, q_2 = -3q, q_3 = 5q, q_4 = 3q.$$

The distances from each charge to the point are

$$r_1 = [(30 \text{ cm})^2 + (30 \text{ cm})^2]^{1/2} = 42.4 \text{ cm};$$

$$r_2 = r_4 = [(30 \text{ cm})^2 + (16 \text{ cm})^2]^{1/2} = 34.0 \text{ cm};$$

$$r_3 = [(16 \text{ cm})^2 + (16 \text{ cm})^2]^{1/2} = 22.6 \text{ cm}.$$



The potential at the point is

$$\begin{aligned}
 V &= (1/4\pi\epsilon_0)(q_1/r_1 + q_2/r_2 + q_3/r_3 + q_4/r_4) \\
 &= (1/4\pi\epsilon_0)q[2/r_1 + (-3/r_2) + 5/r_3 + 3/r_4] \\
 &= (1/4\pi\epsilon_0)q(2/r_1 + 5/r_3) \\
 &= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(10^{-6} \text{ C})[2/(0.424 \text{ m}) + 5/(0.226 \text{ m})] = \boxed{+2.4 \times 10^5 \text{ V}}.
 \end{aligned}$$

22. We find the potential energy of the system of charges by adding the work required to bring the three charges in from infinity successively:

$$W_1 = q_1 V_0 = 0;$$

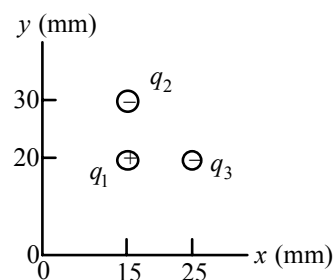
$$W_2 = q_2 V_1 = q_2(1/4\pi\epsilon_0)q_1/r_{12} = (1/4\pi\epsilon_0)q_1 q_2/r_{12};$$

$$W_3 = q_3 V_2 = q_3(1/4\pi\epsilon_0)(q_1/r_{13} + q_2/r_{23}) = (1/4\pi\epsilon_0)(q_1 q_3/r_{13} + q_2 q_3/r_{23}).$$

The total potential energy is

$$\begin{aligned}
 U &= W_1 + W_2 + W_3 = (1/4\pi\epsilon_0)[q_1 q_2/r_{12} + q_1 q_3/r_{13} + q_2 q_3/r_{23}] \\
 &= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)[(5 \mu\text{C})(-3 \mu\text{C})/(10 \times 10^{-3} \text{ m}) + \\
 &\quad (3 \mu\text{C})(-2 \mu\text{C})/(10 \times 10^{-3} \text{ m}) + (-3 \mu\text{C})(-2 \mu\text{C})/(10\sqrt{2} \times 10^{-3} \\
 &\quad \text{m})](10^{-6} \text{ C}/\mu\text{C})^2 \\
 &= \boxed{-18.7 \text{ J}}.
 \end{aligned}$$

The order in which the charges are brought in does not matter.



23. The distances from each charge to the origin are

$$r_1 = [(15 \text{ mm})^2 + (20 \text{ mm})^2]^{1/2} = 25.0 \text{ mm};$$

$$r_2 = [(15 \text{ mm})^2 + (30 \text{ mm})^2]^{1/2} = 33.5 \text{ mm};$$

$$r_3 = [(25 \text{ mm})^2 + (20 \text{ mm})^2]^{1/2} = 32.0 \text{ mm}.$$

The potential at the point is

$$\begin{aligned}
 V &= (1/4\pi\epsilon_0)(q_1/r_1 + q_2/r_2 + q_3/r_3) \\
 &= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(10^{-6} \text{ C})[5/(0.025 \text{ m}) + (-3)/(0.0335 \text{ m}) + \\
 &\quad (-2)/(0.032 \text{ m})] \\
 &= \boxed{+4.3 \times 10^5 \text{ V}}.
 \end{aligned}$$

24. The positive charge must be released from the positive plate. We take the negative plate to be the reference level of potential. We find the speed that the pellet has at the negative plate from conservation of energy:

$$\Delta K = -\Delta U;$$

$$\frac{1}{2}mv_f^2 - 0 = -q(0 - V) = qV;$$

$$\frac{1}{2}(2 \times 10^{-6} \text{ kg})v_f^2 = (3 \times 10^{-7} \text{ C})(600 \text{ V}), \text{ which gives}$$

$$v_f = \boxed{13 \text{ m/s}}.$$

25. (a) Let  $q_1 = +12 \mu\text{C}$ ,  $y_1 = +5.0 \text{ cm}$ ,  $q_2 = -20 \mu\text{C}$ , and  $y_2 = -9.0 \text{ cm}$ . The distance  $r_1$  between  $q_1$  and the point  $(x, 0) = (12.0 \text{ cm}, 0)$  is  $r_1 = (x^2 + y_1^2)^{1/2} = [(12.0 \text{ cm})^2 + (5.0 \text{ cm})^2]^{1/2} = 13 \text{ cm}$ , while that between  $q_2$  and the same point is  $r_2 = (x^2 + y_2^2)^{1/2} = [(12.0 \text{ cm})^2 + (-9.0 \text{ cm})^2]^{1/2} = 15 \text{ cm}$ . The potential at that point due to the two charges is then

$$\begin{aligned}
 V(x, 0) &= (1/4\pi\epsilon_0)q_1/r_1 + (1/4\pi\epsilon_0)q_2/r_2 \\
 &= (9.0 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)[(+12 \times 10^{-6} \text{ C})/(0.13 \text{ m}) + (-20 \times 10^{-6} \text{ C})/(0.15 \text{ m})] \\
 &= \boxed{-3.7 \times 10^5 \text{ V}}.
 \end{aligned}$$

- (b) The point  $(0, 0)$  is a distance  $y_1 = 5.0 \text{ cm}$  from  $q_1$  and  $|y_2| = 9.0 \text{ cm}$  from  $q_2$ . Thus

$$\begin{aligned}
 V(0, 0) &= (1/4\pi\epsilon_0)q_1/y_1 + (1/4\pi\epsilon_0)q_2/|y_2| \\
 &= (9.0 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)[(+12 \times 10^{-6} \text{ C})/(0.050 \text{ m}) + (-20 \times 10^{-6} \text{ C})/(0.090 \text{ m})] \\
 &= \boxed{+1.6 \times 10^5 \text{ V}} \\
 \vec{E}(0, 0) &= [(1/4\pi\epsilon_0)q_1/y_1^2](-\hat{j}) + [(1/4\pi\epsilon_0)q_2/y_2^2]\hat{j} \\
 &= (9.0 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)[- (+12 \times 10^{-6} \text{ C})/(0.050 \text{ m})^2 + (-20 \times 10^{-6} \text{ C})/(0.090 \text{ m})^2]\hat{j} \\
 &= \boxed{-(6.5 \times 10^7 \text{ V/m})\hat{j}}
 \end{aligned}$$

26. Put the origin of the  $x$ -axis at  $q_1 = 2.5 \mu\text{C}$ , and the other charge,  $q_2 = 7.5 \mu\text{C}$ , is located at  $x = L = 0.80 \text{ m}$  on the axis. Consider a point with coordinate  $x$  ( $0 < x < L$ ). The electric potential at that point is the sum of those due to the two charges:

$$\begin{aligned}
 V(x) &= (1/4\pi\epsilon_0)q_1/x + (1/4\pi\epsilon_0)q_2/(L - x) \\
 &= (9.0 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)[(2.5 \times 10^{-6} \text{ C})/x + (7.5 \times 10^{-6} \text{ C})/(0.80 \text{ m} - x)] \\
 &= \boxed{[2.25 \times 10^4/x + 6.75 \times 10^4/(0.80 - x)] \text{ V, where } x \text{ is in meters.}}
 \end{aligned}$$

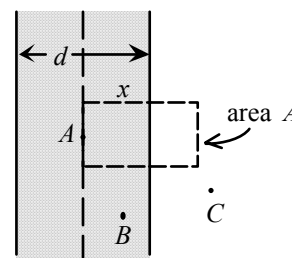
$$\begin{aligned}
 \text{Set } E(x) &= -dV/dx = -(d/dx)[2.25 \times 10^4/x + 6.75 \times 10^4/(0.80 - x)] \\
 &= 2.25 \times 10^4/x^2 - 6.75 \times 10^4/(0.80 - x)^2 = 0
 \end{aligned}$$

to obtain the position where  $E = 0$ :  $x = \boxed{0.29 \text{ m}}$ .

If the test charge  $q$  is positive, then as it moves from the equilibrium toward either  $q_1$  or  $q_2$  closely enough, it will be pushed back. So the equilibrium is stable.

If the test charge is negative then the equilibrium is unstable.

You can verify that by taking  $d^2V/dx^2$  at the equilibrium position. It turns out that  $d^2V/dx^2 > 0$  at the equilibrium position, so the electrostatic energy  $U = qV$  is a minimum for  $q > 0$ , indicating stable equilibrium; and  $U = qV$  is a maximum for  $q < 0$ , indicating unstable equilibrium.



27. (a) From the symmetry of the charge distribution, we see that the electric field is perpendicular to the slab and away from the centerline.

This means that

$$E_A = \boxed{0}.$$

To find the field at  $B$ , we construct a cylinder of height  $x$  with its axis perpendicular to the slab as a Gaussian surface. One end of area  $A$  is placed on the centerline, where the field is 0. Because the field is parallel to the sides of the cylinder, there is flux only through the outer end, so we have

$$\oint \vec{E} \cdot d\vec{A} = EA = Q_{\text{enclosed}}/\epsilon_0.$$

If the end is at point  $B$ , the enclosed charge is  $\rho Ax$  and we have

$$\begin{aligned}
 E_B &= \rho x/\epsilon_0 = (10^{-5} \text{ C/m}^3)x/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \\
 &= \boxed{(1.13 \times 10^6 \text{ N/C} \cdot \text{m})x, \quad x < 1 \text{ cm}.}
 \end{aligned}$$

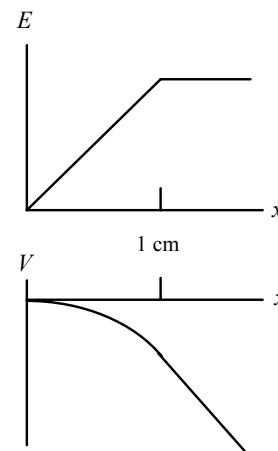
If the end is at point  $C$ , the enclosed charge is  $\rho Ad/2$  and we have

$$\begin{aligned}
 E_C &= \rho d/2\epsilon_0 = (10^{-5} \text{ C/m}^3)(2 \times 10^{-2} \text{ m})/2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \\
 &= \boxed{1.13 \times 10^4 \text{ N/C}, \quad x > 1 \text{ cm}.}
 \end{aligned}$$

As expected, the field outside the slab is uniform.

- (b) We find the potential from the field by integrating over a path perpendicular to the slab:

$$\begin{aligned}
 V_B - V_A &= -\int_0^x \vec{E}_{\text{inside}} \cdot d\vec{s} = 0 - (1.13 \times 10^6 \text{ N/C} \cdot \text{m}) \int_0^x x' dx' \\
 &= -(5.65 \times 10^5 \text{ V/m}^2)x^2, \quad x < 1 \text{ cm}.
 \end{aligned}$$



(c)

The potential at the edge of the slab is



$$V_{\text{edge}} = - (5.65 \times 10^5 \text{ V/m}^2)(0.01 \text{ m})^2 = - 56.5 \text{ V}.$$

For the potential at point C we have

$$\begin{aligned} V_C &= V_{\text{edge}} - \int_{\text{edge}}^x \vec{E}_{\text{outside}} \cdot d\vec{s} = - (56.5 \text{ V}) - (1.13 \times 10^4 \text{ N/C}) \int_{0.01 \text{ m}}^x dx' \\ &= - 56.5 \text{ V} - (1.13 \times 10^4 \text{ V/m})(x - 0.01 \text{ m}), \quad x > 1 \text{ cm} \\ &= + 56.5 \text{ V} - (1.13 \times 10^4 \text{ V/m})x, \quad x > 1 \text{ cm}. \end{aligned}$$

28. We need to find the potential energy stored in the charge  $Q$  distributed uniformly over the spherical shell. We do this by successively bringing a differential charge in from infinity. The total potential energy is the sum (integral) of the differential work done. As we bring in a differential charge, the charge  $q$  already on the shell appears to be a point charge, so the work required to bring in the next differential charge is

$$dW = (1/4\pi\epsilon_0)(q/r) dq.$$

The potential energy is the total work required:

$$U_1 = W_1 = \frac{1}{4\pi\epsilon_0} \int_0^Q \frac{q}{R} dq = \frac{1}{2} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R} \right).$$

For a shell with half the radius, we have

$$U_2 = \frac{1}{2} Q^2 / 4\pi\epsilon_0 (R/2) = 2U_1.$$

The work required to move the charges is

$$W = \Delta U = 2U_1 - U_1 = U_1 = \boxed{\frac{1}{2} Q^2 / 4\pi\epsilon_0 R}.$$

29. We consider the sphere to consist of an infinite number of spherical shells with thickness  $dr$  and charge  $dq = \rho 4\pi r^2 dr$ , where the density of charge is

$$\rho = 3Q/4\pi R^3.$$

We choose the potential reference level at infinity.

At a point outside the sphere, all of the shells, and thus the sphere, are equivalent to point charges:

$$V_{\text{outside}} = \boxed{Q/4\pi\epsilon_0 r, \text{ when } r > R}.$$

At a point inside the sphere,  $r < R$ , there are two contributions to the potential.

All of the shells with radius less than  $r$  are equivalent to point charges:

$$V_1 = q/4\pi\epsilon_0 r = (\rho 4\pi r^3/3)/4\pi\epsilon_0 r = \rho r^2/3\epsilon_0.$$

For a shell with radius greater than  $r$ , the potential anywhere inside is constant and equal to the potential on the shell:

$$dV = dq/4\pi\epsilon_0 r = \rho 4\pi r^2 dr/4\pi\epsilon_0 r = \rho r dr/\epsilon_0.$$

We find the potential contribution from all of the shells with  $r < r' < R$  by integrating:

$$V_2 = \frac{\rho}{\epsilon_0} \int_r^R r' dr' = \frac{\rho}{\epsilon_0} \left( \frac{r'^2}{2} \right) \Big|_r^R = \frac{\rho}{2\epsilon_0} (R^2 - r^2).$$

The total potential is

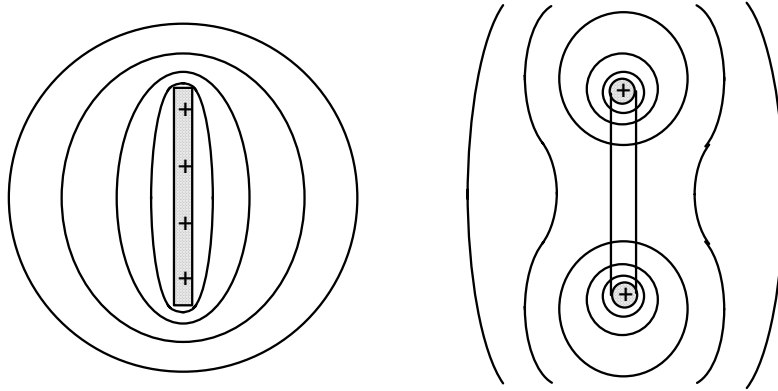
$$\begin{aligned} V_{\text{inside}} &= V_1 + V_2 = (\rho r^2/3\epsilon_0) + [\rho(R^2 - r^2)/2\epsilon_0] \\ &= (\rho/\epsilon_0)[(R^2/2) - (r^2/6)] = (3Q/4\pi\epsilon_0 R^3)[(R^2/2) - (r^2/6)] \\ &= \boxed{(Q/8\pi\epsilon_0 R)[3 - (r/R)^2], \text{ when } r < R}. \end{aligned}$$

If we compare the values at  $r = R$ , we get

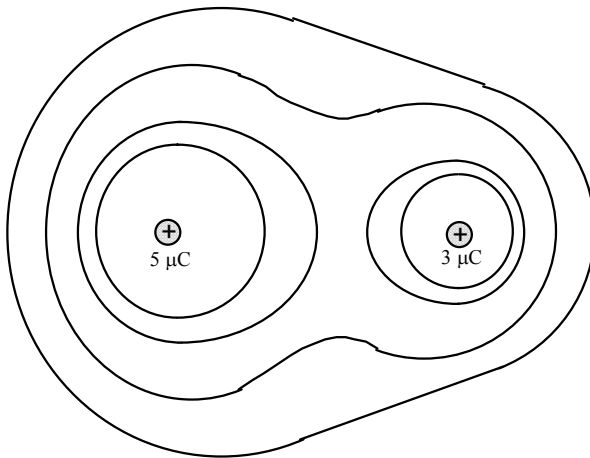
$$V_{\text{outside}} = Q/4\pi\epsilon_0 R \quad \text{and} \quad V_{\text{inside}} = (Q/8\pi\epsilon_0 R)(3 - 1) = Q/4\pi\epsilon_0 R = V_{\text{outside}}.$$

30. (a)

(b)

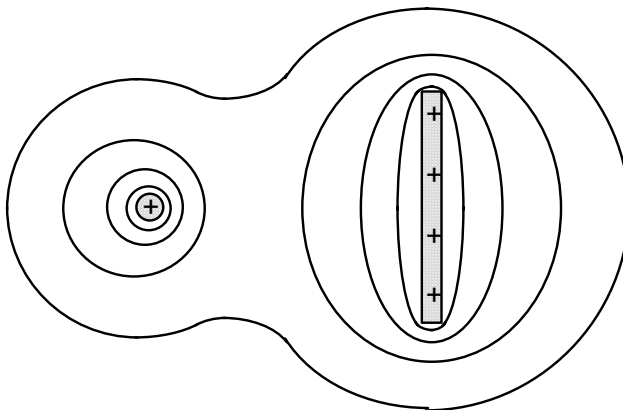


31.

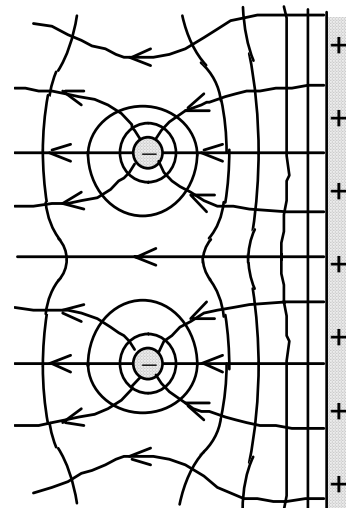


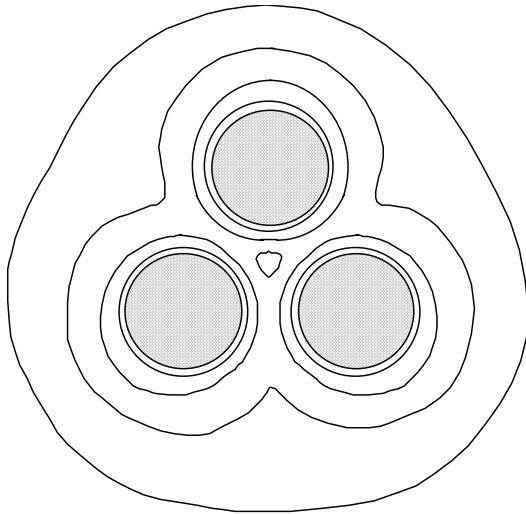
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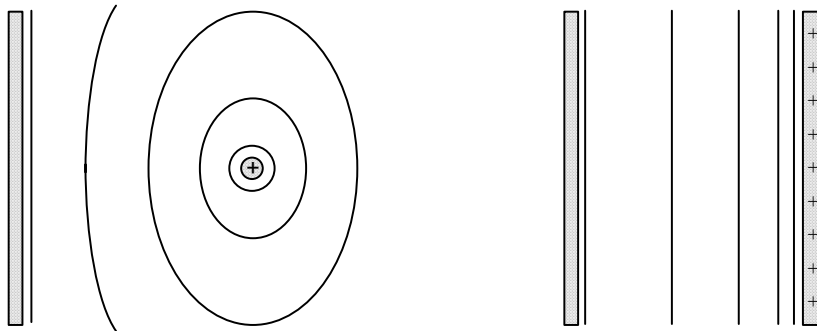


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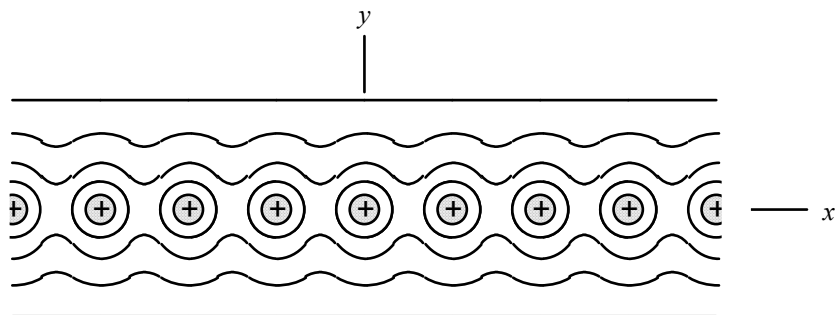




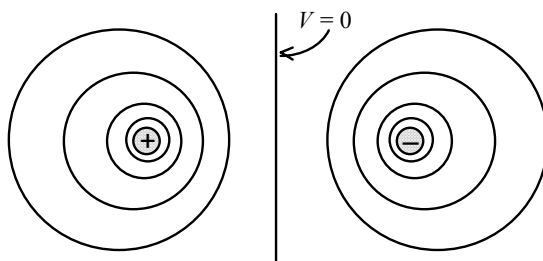
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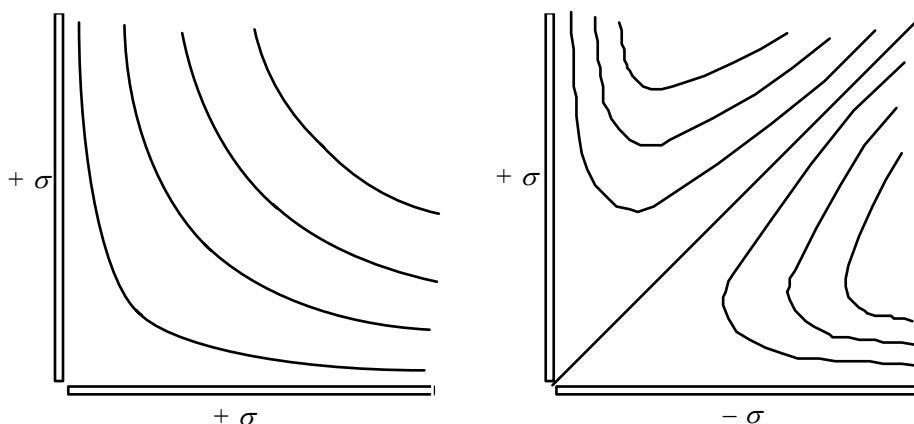
36.



37.



38.



39. From the spatial dependence of the electric potential,  $V(x, y, z) = Q/4\pi\epsilon_0 x$ , we find the components of the electric field from the partial derivatives of  $V$ :

$$E_x = -\partial V/\partial x = Q/4\pi\epsilon_0 x^2;$$

$$E_y = -\partial V/\partial y = 0;$$

$$E_z = -\partial V/\partial z = 0.$$

We can write the electric field:  $\vec{E} = \boxed{(Q/4\pi\epsilon_0 x^2) \hat{i}}$ .

40. From the spatial dependence of the electric potential,  $V(x, y, z) = Ax^2y^2 + Byz^2 + C$ , we find the components of the electric field from the partial derivatives of  $V$ :

$$E_x = -\partial V/\partial x = -2Axy^2;$$

$$E_y = -\partial V/\partial y = -2Ax^2y - Bz^2;$$

$$E_z = -\partial V/\partial z = -2Byz.$$

We can write the electric field:  $\vec{E} = \boxed{-(2Axy^2)\hat{i} - (2Ax^2y + Bz^2)\hat{j} - (2Byz)\hat{k}}$ .

41. From the spatial dependence of the electric potential,  $V(x) = a_0 + a_1x$ , the electric field will have only an  $x$ -component, which we find from the partial derivative of  $V$ :

$$E_x = -\partial V/\partial x = -a_1 = -(-6.68 \text{ V/m}) = +6.68 \text{ V/m}.$$

We can write the electric field:  $\vec{E} = \boxed{(6.68 \text{ V/m}) \hat{i}}$ .

42. Because the electric field is along the  $x$ -axis, we find the the field from

$$\begin{aligned} \vec{E} &= -(\partial V/\partial x) \hat{i} \\ &= -(\partial/\partial x)[Q/4\pi\epsilon_0(R^2 + x^2)^{1/2}] \hat{i} = (-Q/4\pi\epsilon_0)(-1/2)[2x/(R^2 + x^2)^{3/2}] \hat{i}; \\ \vec{E} &= \boxed{Qx/[4\pi\epsilon_0(R^2 + x^2)^{3/2}] \hat{i}}. \end{aligned}$$

43. With the dipole pointing in the  $x$ -direction, the potential is

$$V = (p \cos \theta)/4\pi\epsilon_0 r^2 = px/4\pi\epsilon_0 r^3.$$

We find the components of the electric field from the partial derivatives of  $V$ . For the  $x$ -component, we have

$$E_x = -\partial V/\partial x = -(p/4\pi\epsilon_0 r^3) - [(3px/4\pi\epsilon_0 r^4)(\partial r/\partial x)].$$

From  $r^2 = x^2 + y^2 + z^2$ , we have

$$2r(\partial r/\partial x) = 2x, \text{ or } \partial r/\partial x = x/r, \text{ so we get}$$

$$E_x = -(p/4\pi\epsilon_0 r^3) + (3px^2/4\pi\epsilon_0 r^5).$$

Similarly, we have

$$E_y = -\partial V/\partial y = -[-(3px/4\pi\epsilon_0 r^4)(\partial r/\partial y)] \\ = +3pxy/4\pi\epsilon_0 r^5;$$

$$E_z = -\partial V/\partial z = -[-(3px/4\pi\epsilon_0 r^4)(\partial r/\partial z)] \\ = +3pxz/4\pi\epsilon_0 r^5.$$

Along the bisector (the  $y$ -axis),  $x = 0$ , so we have

$$\vec{E} = \boxed{-(p/4\pi\epsilon_0 r^3) \hat{i}}.$$

Note that the symmetry along the  $y$ -axis shows us that the field there has only an  $x$ -component.

44. From the symmetry of the charge distribution, we know that the electric field is radial. From the spatial dependence of the electric potential,  $V(r) = (Q/2\pi\epsilon_0)[A(r/R) + B(r/R)^2 + C]$ , we find the electric field from

$$E_r = -\partial V/\partial r = \boxed{-(Q/2\pi\epsilon_0)[(A/R) + (2Br/R^2)] \text{ radial}}.$$

If the potential is zero at the surface, we have

$$V(R) = 0 = (Q/2\pi\epsilon_0)(A + B + C), \text{ which gives } \boxed{C = -A - B}.$$

45. From the symmetry of the charge distribution, we know that the electric field is radial, so we find the electric field from

$$E_r = -\partial V/\partial r.$$

For  $r < R$ , we have

$$V_{r < R} = (Q/4\pi\epsilon_0 R)[-2 + 3(r/R)^2]; \\ E_{r < R} = -(Q/4\pi\epsilon_0 R)(+6r/R^2) = -(Q/4\pi\epsilon_0)(6r/R^3), \text{ or} \\ \vec{E}_{r < R} = \boxed{(Q/4\pi\epsilon_0)(6r/R^3) \hat{r}}.$$

For  $r > R$ , we have

$$V_{r > R} = Q/4\pi\epsilon_0 r; \\ E_{r > R} = -(-Q/4\pi\epsilon_0 r^2), \text{ or} \\ \vec{E}_{r > R} = \boxed{(Q/4\pi\epsilon_0) \hat{r}}.$$

46. If we choose a sphere of radius  $r < R$  as a Gaussian surface, we have

$$\oint \vec{E} \cdot d\vec{A} = E4\pi r^2 = Q_{\text{enclosed}}/\epsilon_0, \text{ or} \\ Q_{\text{enclosed}} = -\epsilon_0(Q/4\pi\epsilon_0)(6r/R^3)4\pi r^2 = -6Qr^3/R^3.$$

We set up the integral to find the enclosed charge, by using a spherical shell of radius  $r'$  and thickness  $dr'$  for the differential element. We also write the right-hand side as an integral:

$$\int_0^r \rho 4\pi r'^2 dr' = \left(-18Q/R^3\right) \int_0^r r'^2 dr'.$$

Comparing the two integrands, we see that  $\boxed{\text{for } r < R, \rho = -4.5Q/\pi R^3, \text{ a constant}}.$

If the Gaussian surface is just inside  $r = R$ , the total enclosed charge is  $-6Q$ .

If we choose a sphere with a radius  $r$  just slightly greater than  $R$  as a Gaussian surface, we have

$$\oint \vec{E} \cdot d\vec{A} = E4\pi R^2 = Q_{\text{enclosed}}/\epsilon_0, \text{ or} \\ Q_{\text{enclosed}} = \epsilon_0(Q/4\pi\epsilon_0 R^2)4\pi R^2 = Q.$$

Because there is a charge of  $-6Q$  inside the sphere, there must be a charge of  $+7Q$  on the surface of the sphere to give a net charge of  $Q$ :

$$\boxed{r = R, q = +7Q}.$$

If we choose a sphere with a radius  $r > R$  as a Gaussian surface, we have

$$\oint \vec{E} \cdot d\vec{A} = E4\pi r^2 = Q_{\text{enclosed}}/\epsilon_0, \text{ or}$$

$$Q_{\text{enclosed}} = \epsilon_0 (Q/4\pi\epsilon_0 r^2) 4\pi r^2 = Q.$$

Because this is the net charge on the sphere, we have  $r > R, \rho = 0$ .

47. From the spatial dependence of the electric potential,

$$V(x, y) = (Q/4\pi\epsilon_0 L)\{\tan^{-1}[y/(x - a_0)] - 2 \tan^{-1}(y/x) + \tan^{-1}[y/(x + a_0)]\},$$

we find the components of the electric field from the partial derivatives of  $V$ :

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = \frac{-Q}{4\pi\epsilon_0 L} \left\{ \left[ \frac{1}{1 + \left(\frac{y}{x-a_0}\right)^2} \right] \left[ \frac{-y}{(x-a_0)^2} \right] - 2 \left[ \frac{1}{1 + \left(\frac{y}{x}\right)^2} \right] \left( \frac{-y}{x^2} \right) + \left[ \frac{1}{1 + \left(\frac{y}{x+a_0}\right)^2} \right] \left[ \frac{-y}{(x+a_0)^2} \right] \right\} \\ &= \frac{-Q}{4\pi\epsilon_0 L} \left\{ -\left[ \frac{y}{(x-a_0)^2 + y^2} \right] + \left( \frac{2y}{x^2 + y^2} \right) - \left[ \frac{y}{(x+a_0)^2 + y^2} \right] \right\}. \end{aligned}$$

We expand the denominators and use the approximation,  $1/(1 \pm z) \approx 1 - z$ , when  $z \ll 1$ :

$$\begin{aligned} E_x &= \frac{-Q}{4\pi\epsilon_0 L} \left[ -\left( \frac{y}{x^2 + y^2 - 2a_0x + a_0^2} \right) + \left( \frac{2y}{x^2 + y^2} \right) - \left( \frac{y}{x^2 + y^2 + 2a_0x + a_0^2} \right) \right] \\ &= \frac{-Qy}{4\pi\epsilon_0 L(x^2 + y^2)} \left\{ \left[ \frac{1}{1 - (2a_0x - a_0^2)/(x^2 + y^2)} \right] - 2 + \left[ \frac{1}{1 + (2a_0x - a_0^2)/(x^2 + y^2)} \right] \right\} \\ &\approx \frac{-Qy}{4\pi\epsilon_0 L(x^2 + y^2)} \left( 1 + \frac{2a_0x - a_0^2}{x^2 + y^2} - 2 + 1 - \frac{2a_0x - a_0^2}{x^2 + y^2} \right) = \frac{-2Qya_0^2}{4\pi\epsilon_0 L(x^2 + y^2)^2}. \end{aligned}$$

We use the same approximation for the  $y$ -component:

$$\begin{aligned} E_y &= -\frac{\partial V}{\partial y} = \frac{-Q}{4\pi\epsilon_0 L} \left\{ \left( \frac{1}{1 + [y/(x - a_0)]^2} \right) \left( \frac{1}{x - a_0} \right) - 2 \left[ \frac{1}{1 + (y/x)^2} \right] \left( \frac{1}{x} \right) + \left( \frac{1}{1 + [y/(x + a_0)]^2} \right) \left( \frac{1}{x + a_0} \right) \right\} \\ &= \frac{-Q}{4\pi\epsilon_0 L} \left\{ \left[ \frac{x - a_0}{(x - a_0)^2 + y^2} \right] - \left( \frac{2x}{x^2 + y^2} \right) + \left[ \frac{x + a_0}{(x + a_0)^2 + y^2} \right] \right\} \\ &= \frac{-Q}{4\pi\epsilon_0 L} \left[ \left( \frac{x - a_0}{x^2 + y^2 - 2a_0x + a_0^2} \right) - \left( \frac{2x}{x^2 + y^2} \right) + \left( \frac{x + a_0}{x^2 + y^2 + 2a_0x + a_0^2} \right) \right] \\ &= \frac{-Qx}{4\pi\epsilon_0 L(x^2 + y^2)} \left\{ \left[ \frac{1 - (a_0/x)}{1 - (2a_0x - a_0^2)/(x^2 + y^2)} \right] - 2 + \left[ \frac{1 + (a_0/x)}{1 + (2a_0x - a_0^2)/(x^2 + y^2)} \right] \right\} \\ &\approx \frac{-Qx}{4\pi\epsilon_0 L(x^2 + y^2)} \left[ \left( 1 - \frac{a_0}{x} \right) \left( 1 + \frac{2a_0x - a_0^2}{x^2 + y^2} \right) - 2 + \left( 1 + \frac{a_0}{x} \right) \left( 1 - \frac{2a_0x - a_0^2}{x^2 + y^2} \right) \right] = \frac{-6Qxa_0^2}{4\pi\epsilon_0 L(x^2 + y^2)^2}. \end{aligned}$$

The total electric field is

$$\begin{aligned}\vec{E} &= \frac{2Qa_0^2}{4\pi\epsilon_0 L(x^2 + y^2)^2}(-y\hat{i} - 3x\hat{j}) = \frac{2Qa_0^2}{4\pi\epsilon_0 Lr^4}(-r\sin\theta\hat{i} - 3r\cos\theta\hat{j}) \\ &= \frac{2Qa_0^2}{4\pi\epsilon_0 Lr^3}(-\sin\theta\hat{i} - 3\cos\theta\hat{j}).\end{aligned}$$

48. For two large, parallel plates, we have

$$E = -\Delta V/d;$$

$$7 \times 10^3 \text{ V/m} = (200 \text{ V})/d, \text{ which gives } d = 0.029 \text{ m} = \boxed{2.9 \text{ cm}}.$$

49. We find the charge from the expression for the potential on the axis of the ring:

$$V = (1/4\pi\epsilon_0)Q/(R^2 + x^2)^{1/2};$$

$$5 \text{ V} = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)Q/[(0.10 \text{ m})^2 + (0.15 \text{ m})^2]^{1/2}, \text{ which gives } Q = \boxed{1.0 \times 10^{-10} \text{ C}}.$$

50. (a) Choosing  $y$  up as positive, we find the potential from the field by integrating:

$$V = -\int \vec{E} \cdot d\vec{s} = -(-E) \int dy = \boxed{Ey + (\text{a constant})}.$$

- (b) The most convenient reference point is  $V = 0$  at surface.

- (c) The electric potential energy is  $U_e = \boxed{qEh}$ .

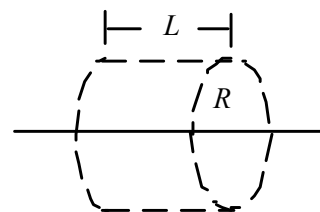
The gravitational potential energy is  $U_g = \boxed{mgh}$ , so they have the same form.

- (d) For the electric force to balance the force of gravity, we need a negative charge with magnitude

given by

$$qE = mg;$$

$$q(100 \text{ N/C}) = (50 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{4.9 \text{ C of negative charge}}.$$



51. From symmetry, we know that the electric field will be radially away from the line charge, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length  $L$  and radius  $R$ , centered on the line. On the ends of this surface, the electric field is not constant, but  $\vec{E}$  and  $d\vec{A}$  are perpendicular, so we have  $\vec{E} \cdot d\vec{A} = 0$ . On the curved side, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ . For Gauss' law, we have

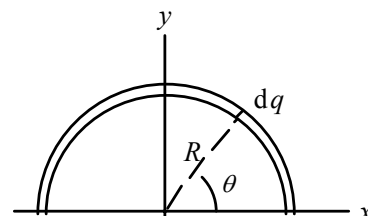
$$\oint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0;$$

$$E2\pi RL = \lambda L/\epsilon_0, \text{ which gives}$$

$$E = 2\lambda/4\pi\epsilon_0 R \text{ radial.}$$

We find the potential by integrating along a radial line:

$$V = -\int \vec{E} \cdot d\vec{R} = -(2\lambda/4\pi\epsilon_0) \int dR/R = \boxed{-(2\lambda/4\pi\epsilon_0) \ln(R) + (\text{a constant})}.$$



52. The potential at the origin from a differential element of the charge is

$$dV = (1/4\pi\epsilon_0)(dq/R).$$

To find the potential at the origin, we add (integrate) the contributions from all elements:



$$\begin{aligned} V &= \int (1/4\pi\epsilon_0)(dq/R) \\ &= (1/4\pi\epsilon_0 R) \int dq = q/4\pi\epsilon_0 R = \lambda\pi R/4\pi\epsilon_0 R \\ &= \boxed{\lambda/4\epsilon_0}. \end{aligned}$$

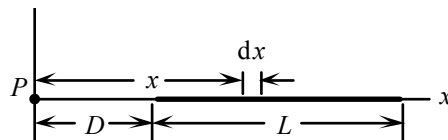
53. We choose the point  $P$  as the origin. The potential at  $P$  from a differential element of the rod, which has a charge

$$dq = (q/L) dx, \text{ is}$$

$$dV = (1/4\pi\epsilon_0)(dq/x).$$

To find the potential at  $P$ , we add (integrate) the contributions from all elements:

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0} \int_D^{D+L} \left(\frac{q}{L}\right) \frac{dx}{x} = \frac{q}{4\pi\epsilon_0 L} \ln \left(\frac{D+L}{D}\right) \\ &= (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2 \times 10^{-6} \text{ C})}{0.2 \text{ m}} \ln \left(\frac{0.1 \text{ m} + 0.2 \text{ m}}{0.1 \text{ m}}\right) = 9.9 \times 10^4 \text{ V}. \end{aligned}$$



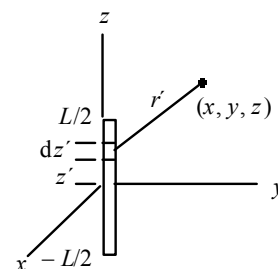
54. We choose a differential element of the rod at position  $z'$ , length  $dz'$ , and charge  $\lambda dz'$ . From the diagram, we see that

$$r^2 = x^2 + y^2 + z^2 \text{ and } r'^2 = x^2 + y^2 + (z - z')^2.$$

The potential from the differential element at the point  $(x, y, z)$  is

$$dV = (1/4\pi\epsilon_0) dq/r' = (\lambda/4\pi\epsilon_0) dz'/r'.$$

The potential from the rod is



$$\begin{aligned} V &= \frac{\lambda}{4\pi\epsilon_0} \int_{z'=-L/2}^{z'=L/2} \frac{dz'}{r'} = \frac{\lambda}{4\pi\epsilon_0} \int_{z'=-L/2}^{z'=L/2} \frac{dz'}{\sqrt{x^2 + y^2 + (z - z')^2}} \\ &= \frac{\lambda}{4\pi\epsilon_0} \ln \left[ z - z' + \sqrt{x^2 + y^2 + (z - z')^2} \right] \Big|_{z'=-L/2}^{z'=L/2} = \frac{\lambda}{4\pi\epsilon_0} \ln \left\{ \frac{z - (L/2) + \sqrt{x^2 + y^2 + [z - (L/2)]^2}}{z + (L/2) + \sqrt{x^2 + y^2 + [z + (L/2)]^2}} \right\} \\ &= \frac{\lambda}{4\pi\epsilon_0} \ln \left\{ \frac{z + (L/2) + \sqrt{x^2 + y^2 + [z + (L/2)]^2}}{z - (L/2) + \sqrt{x^2 + y^2 + [z - (L/2)]^2}} \right\}. \end{aligned}$$

To find the potential when  $r \gg L$ , we use the approximations  $(1 \pm u)^{1/2} \approx 1 \pm (u/2)$ ,  $1/(1 \pm u) \approx 1 - u$ , and  $\ln(1 + u) \approx u$ , when  $u \ll 1$ :

$$\begin{aligned} V &= \frac{\lambda}{4\pi\epsilon_0} \ln \left\{ \frac{z + (L/2) + \sqrt{x^2 + y^2 + [z + (L/2)]^2}}{z - (L/2) + \sqrt{x^2 + y^2 + [z - (L/2)]^2}} \right\} \\ &= \frac{\lambda}{4\pi\epsilon_0} \ln \left[ \frac{z + (L/2) + \sqrt{r^2 + zL + (L/2)^2}}{z - (L/2) + \sqrt{r^2 - zL + (L/2)^2}} \right] \approx \frac{\lambda}{4\pi\epsilon_0} \ln \left[ \frac{z + (L/2) + r\sqrt{1 + (zL/r^2)}}{z - (L/2) + r\sqrt{1 - (zL/r^2)}} \right] \\ &\approx \frac{\lambda}{4\pi\epsilon_0} \ln \left\{ \frac{z + (L/2) + r[1 + (zL/2r^2)]}{z - (L/2) + r[1 - (zL/2r^2)]} \right\} = \frac{\lambda}{4\pi\epsilon_0} \ln \left\{ \frac{(z+r)[1 + (L/2r)]}{(z+r)[1 - (L/2r)]} \right\} \\ &\approx \frac{\lambda}{4\pi\epsilon_0} \ln \left[ \frac{1 + (L/2r)}{1 - (L/2r)} \right] = \frac{2\lambda}{4\pi\epsilon_0} \ln \left[ 1 + (L/2r) \right] \approx \frac{2\lambda}{4\pi\epsilon_0} \frac{L}{2r} = \frac{\lambda L}{4\pi\epsilon_0 r} \\ &= \frac{Q}{4\pi\epsilon_0 r}, \text{ where } Q = \lambda L \text{ and } r \gg L. \end{aligned}$$

55. (a) The potential for the two charges is the sum:

$$V = \frac{1}{4\pi\epsilon_0} \left( \frac{3q_0}{x-x_0} \right) + \frac{1}{4\pi\epsilon_0} \left( \frac{-q_0}{x+\frac{x_0}{2}} \right) = \frac{q_0}{4\pi\epsilon_0} \left[ \left( \frac{3}{x-x_0} \right) - \left( \frac{1}{x+\frac{x_0}{2}} \right) \right]$$

- (b) When  $x \gg x_0$ , we use the approximation  $1/(1 \pm u) \approx 1 - u + u^2 - u^3 + \dots$ , when  $u \ll 1$ :

$$\begin{aligned} V &= \frac{q_0}{4\pi\epsilon_0 x} \left[ \left( \frac{3}{1 - \frac{x_0}{x}} \right) - \left( \frac{1}{1 + \frac{x_0}{2x}} \right) \right] \\ &= \frac{q_0}{4\pi\epsilon_0} \left\{ \frac{3}{x} \left[ 1 + \frac{x_0}{x} + \left( \frac{x_0}{x} \right)^2 + \left( \frac{x_0}{x} \right)^3 + \dots \right] - \frac{1}{x} \left[ 1 - \frac{x_0}{2x} + \left( \frac{x_0}{2x} \right)^2 - \left( \frac{x_0}{2x} \right)^3 + \dots \right] \right\} \\ &= \frac{q_0}{4\pi\epsilon_0} \left[ \frac{2}{x} + \frac{7}{2} \frac{x_0}{x^2} + \frac{11}{4} \frac{x_0^2}{x^3} + \frac{25}{8} \frac{x_0^3}{x^4} + \dots \right] \end{aligned}$$

- (c) At large values of  $x$ , the contribution of each term to the total decreases.

The first term from part (b) is the potential of a point charge, with  $q_{\text{net}} = 2q_0$ .

The second term is the potential of a dipole, with  $p = 7q_0 x_0 / 2$ .

- (d) For the point charge plus the dipole to be within 1% of the exact answer, we have

$$\frac{2}{x} + \frac{7}{2} \frac{x_0}{x^2} = 0.99 \left[ \frac{3}{x-x_0} - \frac{1}{x+\frac{x_0}{2}} \right] = \frac{0.99}{x} \left[ \frac{3}{1 - (x_0/x)} - \frac{2}{2 + (x_0/x)} \right]$$

If we let  $x_0/x = y$ , we have

$$2 + \frac{7}{2}y = 0.99 \left[ \frac{3(2+y) - 2(1-y)}{(1-y)(2+y)} \right] = 0.99 \left( \frac{4+5y}{2+y-y^2} \right)$$

A numerical solution gives  $y = -0.0825, +0.0875$ . The values of  $x$  are  $-12.1x_0, +11.4x_0$ .

Thus if  $|x| > 12.1x_0$ , the point charge plus the dipole will be within 1% of the exact answer.

56. The minimum work brings the charge to the point with no kinetic energy. We use the expression for the potential on the axis of a disk:

$$\begin{aligned} W_{\infty \rightarrow a} &= q(V_a - V_{\infty}) = q(Q/2\pi\epsilon_0 R^2)[(R^2 + x^2)^{1/2} - x] \\ &= [(3.2 \times 10^{-7} \text{ C})(6.0 \times 10^{-8} \text{ C})(2)(9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)/ \\ &\quad (0.028 \text{ m})^2] \{ [(0.088 \text{ m})^2 + (0.028 \text{ m})^2]^{1/2} - 0.088 \text{ m} \} \\ &= [1.8 \times 10^{-3} \text{ J}] \end{aligned}$$

57. There is no change in the kinetic energy. We use the expression for the potential on the axis of a ring:

$$\begin{aligned} W_{a \rightarrow b} &= q(V_b - V_a) = q(Q/4\pi\epsilon_0)[(R^2 + x_b^2)^{-1/2} - (R^2 + x_a^2)^{-1/2}] \\ &= (-8.5 \times 10^{-8} \text{ C})(3.5 \times 10^{-7} \text{ C})(9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2) \times \\ &\quad \{ [(0.24 \text{ m})^2 + (0.85 \text{ m})^2]^{-1/2} - [(0.24 \text{ m})^2 + (0.28 \text{ m})^2]^{-1/2} \} \\ &= [+4.2 \times 10^{-4} \text{ J}] \end{aligned}$$

58. At the surface of a sphere, we have

$V = Q/4\pi\epsilon_0 R$  and  $E = Q/4\pi\epsilon_0 R^2$ , which gives

$$V = ER = (2.8 \times 10^6 \text{ V/m})(0.03 \text{ m}) = 8.4 \times 10^4 \text{ V} = [84 \text{ kV}]$$



59. After the connection, the two spheres must have the same potential:

$$V = (1/4\pi\epsilon_0)(q_1'/r_1) = (1/4\pi\epsilon_0)(q_2'/r_2), \text{ or } q_1' = (r_1/r_2)q_2'.$$

Because charge is conserved we have

$$q_1 + q_2 = q_1' + q_2'.$$

When we combine these two equations, we get

$$q_2' = (q_1 + q_2)[r_2/(r_1 + r_2)].$$

The amount of charge that moves between the two spheres is

$$\Delta q_2 = q_2' - q_2 = \boxed{(q_1 r_2 - q_2 r_1)/(r_1 + r_2)}.$$

60. The electric field that causes air to ionize is  $E = 2.8 \times 10^6 \text{ V/m}$ . Thus

$$\Delta V = E \Delta x = (2.8 \times 10^6 \text{ V/m})(0.002 \text{ m}) = 5.6 \times 10^3 \text{ V} \approx \boxed{6 \text{ kV}}.$$

61. The potential of the dome of radius  $R$  carrying a charge  $Q$  is

$$V = Q/4\pi\epsilon_0 R. \text{ Solve for } Q:$$

$$Q = 4\pi\epsilon_0 R V = (0.61 \text{ m})(5.5 \times 10^6 \text{ V})/(9.0 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2) = \boxed{3.7 \times 10^{-4} \text{ C}}.$$

The energy gained by a proton (charge  $e$ ) after being accelerated by this potential is

$$eV = e(5.5 \text{ MV}) = \boxed{5.5 \text{ MeV}}, \text{ or } (5.5 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = \boxed{8.8 \times 10^{-13} \text{ J}}.$$

To find the resulting speed  $v$  of the proton, let

$$eV = \frac{1}{2}mv^2, \text{ or } v = (2eV/m)^{1/2} = [2(8.8 \times 10^{-13} \text{ J})/(1.67 \times 10^{-27} \text{ kg})]^{1/2} = \boxed{3.2 \times 10^7 \text{ m/s}}.$$

62. We call the radius of the initial drops  $R_1$  and the radius of the combined drop  $R_2$ . Because the volume of the mercury does not change, we have

$$\frac{4}{3}\pi R_2^3 = 2\left(\frac{4}{3}\pi R_1^3\right), \text{ which gives } R_2/R_1 = (2)^{1/3}.$$

We use the potential for a spherical charge:

$$V_1 = q/4\pi\epsilon_0 R_1, \text{ and } V_2 = 2q/4\pi\epsilon_0 R_2, \text{ so we have}$$

$$V_2/V_1 = 2R_1/R_2 = 2(2)^{-1/3}; \quad V_2 = 2(2)^{-1/3}(70 \text{ V}) = \boxed{1.1 \times 10^2 \text{ V}}.$$

63. We label the larger sphere 1 and the smaller sphere 2. The wire connecting the spheres means that the potentials of the spheres are the same:

$$Q_1/4\pi\epsilon_0 R_1 = Q_2/4\pi\epsilon_0 R_2, \text{ which gives } Q_1 = (R_1/R_2)Q_2 = 3Q_2.$$

We combine this with the conservation of charge:

$$Q_1 + Q_2 = Q, \text{ to get } Q_2 = \boxed{Q/4} \text{ and } Q_1 = \boxed{3Q/4}.$$

64. (a) Inside the inner shell, there is no charge, so we have

$$E = \boxed{0, r < R}.$$

Between the two shells, the electric field is that of the inner shell:

$$E = \boxed{q/4\pi\epsilon_0 r^2 \text{ radial}, R < r < 1.5R}.$$

Outside the two shells, the two shells look like a point charge with  $Q = q - 3q = -2q$ :

$$E = -2q/4\pi\epsilon_0 r^2 \text{ radial}, 1.5R < r.$$

- (b) We add the potentials from the two shells at each location. Because the potential inside a spherical shell is constant and equal to the potential on the surface, we have

$$V_R = (q/4\pi\epsilon_0 R) + (-3q/4\pi\epsilon_0 1.5R) = -q/4\pi\epsilon_0 R$$

$$V_{1.5R} = (q/4\pi\epsilon_0 1.5R) + (-3q/4\pi\epsilon_0 1.5R) = -4q/4\pi\epsilon_0 3R.$$

The potential difference is

$$V_{1.5R} - V_R = \boxed{-q/12\pi\epsilon_0 R}.$$

- (c) When the two shells are connected, the potential difference between the two shells must be 0. The system can be considered as one conductor, which can have no charge inside.

All of the charge moves to the outer shell, with  $q_{\text{net}} = -2q$ .

65. The number of raindrops per unit volume is  $n = 1.2 \times 10^{10}/\text{m}^3$ . Each raindrop carries a charge  $q = 16 \times 10^{-19} \text{ C}$ , so the total charge  $Q$  on a spherical piece of cloud of volume  $V$  is

$Q = nqV = nq(\frac{4}{3}\pi R^3)$ , where  $R$  is the radius of the cloud. The electric field  $E$  outside the cloud is then  $E = Q/4\pi\epsilon_0 R^2 = nq(\frac{4}{3}\pi R^3)/4\pi\epsilon_0 R^2 = nqR/3\epsilon_0$ . Equate this to  $E_{\text{max}} = 3.2 \times 10^6 \text{ V/m}$  to obtain the radius at which electrical breakdown occurs:

$$\begin{aligned} R &= 3\epsilon_0 E_{\text{max}}/nq \\ &= (3(8.85 \times 10^{-12} \text{ N} \cdot \text{m}^2/\text{C}^2)(3.2 \times 10^6 \text{ V/m})/[(1.2 \times 10^{10}/\text{m}^3)(16 \times 10^{-19} \text{ C})]) \\ &= 4.4 \times 10^3 \text{ m} = \boxed{4.4 \text{ km}}. \end{aligned}$$

66. Let the charge on the sphere of radius  $R_1 = 0.05 \text{ m}$  be  $Q_1$  and that on the other one be  $Q_2$ . Then  $Q_1 + Q_2 = Q = 40 \mu\text{C}$ .

Also, Since the spheres are connected by a metal wire they must be at the same potential:

$$\begin{aligned} V_1 &= V_2; \\ Q_1/4\pi\epsilon_0 R_1 &= Q_2/4\pi\epsilon_0 R_2. \text{ Thus} \\ Q_1 &= Q/(1 + R_2/R_1) = 40 \mu\text{C}/(1 + 0.05 \text{ m}/0.08 \text{ m}) = \boxed{25 \mu\text{C}} \text{ and} \\ Q_2 &= Q - Q_1 = 40 \mu\text{C} - 25 \mu\text{C} = \boxed{15 \mu\text{C}}. \end{aligned}$$

67. (a) The wire connecting the spheres means that the potentials of the spheres are the same:

$$\begin{aligned} q_1/4\pi\epsilon_0 R_1 &= q_2/4\pi\epsilon_0 R_2; \\ q_1 &= (R_1/R_2)q_2 = [(20 \text{ mm})/(100 \text{ mm})]q_2, \text{ which gives } q_2 = 5q_1. \end{aligned}$$

The Coulomb force is

$$\begin{aligned} F &= (1/4\pi\epsilon_0)(q_1 q_2/r^2); \\ 3.5 \text{ N} &= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(5q_1^2)/(0.25 \text{ m})^2, \text{ which gives} \\ q_1 &= 2.2 \times 10^{-6} \text{ C} = \boxed{2.2 \mu\text{C}} \text{ and } q_2 = 5q_1 = \boxed{11 \mu\text{C}}. \end{aligned}$$

- (b) The electric fields at the surfaces of the spheres are

$$\begin{aligned} E_1 &= (1/4\pi\epsilon_0)(q_1/R_1^2) = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(2.2 \times 10^{-6} \text{ C})/(0.020 \text{ m})^2 = \boxed{4.95 \times 10^7 \text{ V/m, radial}}. \\ E_2 &= (1/4\pi\epsilon_0)(q_2/R_2^2) = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(11 \times 10^{-6} \text{ C})/(0.100 \text{ m})^2 = \boxed{9.90 \times 10^6 \text{ V/m, radial}}. \end{aligned}$$

68. (a) At the surface of the balloon, we have

$$V_1 = (1/4\pi\epsilon_0)(q/R_1) = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(1.5 \times 10^{-5} \text{ C})/(4.30 \text{ m}) = \boxed{3.1 \times 10^4 \text{ V}}.$$

- (b) Because the charge is conserved, we have

$$V_2 = (1/4\pi\epsilon_0)(q/R_2) = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(1.5 \times 10^{-5} \text{ C})/(3.10 \text{ m}) = \boxed{4.4 \times 10^4 \text{ V}}.$$

- (c) The energy increases because the outside pressure compresses the balloon and therefore does positive work. This increases the charge density on the surface, which means that the positive charges are forced closer.

69. (a) The increase in kinetic energy comes from the decrease in potential energy, which means the proton must go from high to low potential:

$$\begin{aligned} \Delta K &= K - 0 = -\Delta U = -q\Delta V; \\ K &= -(+1 \text{ e})(-5.5 \times 10^6 \text{ V}) = \boxed{+5.5 \times 10^6 \text{ eV}}. \end{aligned}$$

To convert units, we have

$$K = (+5.5 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = \boxed{8.8 \times 10^{-13} \text{ J}}.$$

- (b) We find the final speed from

$$\begin{aligned} K &= \frac{1}{2}mv^2; \\ 8.8 \times 10^{-13} \text{ J} &= \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})v^2, \text{ which gives} \\ v &= \boxed{3.3 \times 10^7 \text{ m/s}}. \end{aligned}$$

70. The maximum potential is reached when the charge on the sphere creates an electric field large enough to break down the air. At the surface of a sphere, we have

$$E = (1/4\pi\epsilon_0)(Q/R^2) \quad \text{and} \quad V = (1/4\pi\epsilon_0)(Q/R), \quad \text{or}$$

$$V = ER = (2.8 \times 10^6 \text{ V/m})(0.41 \text{ m}) = \boxed{1.1 \times 10^6 \text{ V}}.$$

We find the charge on the sphere from

$$V = (1/4\pi\epsilon_0)(Q/R);$$

$$1.1 \times 10^6 \text{ V} = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)Q/(0.41 \text{ m}), \text{ which gives } Q = \boxed{5.2 \times 10^{-5} \text{ C}}.$$

71. (a) The maximum potential is reached when the charge on the sphere creates an electric field large enough to break down the air. At the surface of a sphere, we have

$$E = (1/4\pi\epsilon_0)(Q/R^2) \quad \text{and} \quad V = (1/4\pi\epsilon_0)(Q/R), \quad \text{or}$$

$$V = ER = (3 \times 10^6 \text{ V/m})(1.3 \text{ m}) = \boxed{3.9 \times 10^6 \text{ V}}.$$

- (b) The increase in kinetic energy comes from the decrease in potential energy, which means the proton must go from high to low potential:

$$\Delta K = K - 0 = -\Delta U = -q \Delta V;$$

$$K = - (+1 \text{ e})(-3.9 \times 10^6 \text{ V}) = +3.9 \times 10^6 \text{ eV} = \boxed{3.9 \text{ MeV} \quad (6.2 \times 10^{-13} \text{ J})}.$$

- (c) We find the charge on the sphere from

$$V = (1/4\pi\epsilon_0)(Q/R);$$

$$3.9 \times 10^6 \text{ V} = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)Q/(1.3 \text{ m}), \text{ which gives } Q = \boxed{5.6 \times 10^{-4} \text{ C}}.$$

72. For parallel plates, we have

$$\Delta V = Ed;$$

$$5 \times 10^3 \text{ V} = (3 \times 10^6 \text{ V/m})d, \text{ which gives } d = 1.7 \times 10^{-3} \text{ m} = \boxed{1.7 \text{ mm}}.$$

73. The potential at a point on the  $x$ -axis between the disks is

$$\begin{aligned} V &= \frac{Q}{2\pi\epsilon_0 R^2} \left[ \sqrt{R^2 + (a+x)^2} - (a+x) + \sqrt{R^2 + (a-x)^2} - (a-x) \right] \\ &= \frac{Q}{2\pi\epsilon_0 R^2} \left[ \sqrt{R^2 + (a+x)^2} + \sqrt{R^2 + (a-x)^2} - 2a \right]. \end{aligned}$$

74. The potential of the two rings at a point on the  $x$ -axis is

$$V = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{\sqrt{R^2 + (x-a)^2}} - \frac{1}{\sqrt{R^2 + (x+a)^2}} \right].$$

75. When  $x \gg a$  and  $x \gg R$ , we write the solution for the two rings as

$$\begin{aligned} V &= \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{x\sqrt{\left(\frac{R}{x}\right)^2 + \left(1 - \frac{a}{x}\right)^2}} - \frac{1}{x\sqrt{\left(\frac{R}{x}\right)^2 + \left(1 + \frac{a}{x}\right)^2}} \right] \\ &= \frac{Q}{4\pi\epsilon_0 x} \left( \frac{1}{\sqrt{1 - \frac{2a}{x} + \frac{a^2 + R^2}{x^2}}} - \frac{1}{\sqrt{1 + \frac{2a}{x} + \frac{a^2 + R^2}{x^2}}} \right). \end{aligned}$$

Because  $x \gg a$  and  $x \gg R$ , we use the approximation  $(1+u)^{-1/2} \approx 1 - \frac{1}{2}u$  to get

$$V \approx \frac{Q}{4\pi\epsilon_0 x} \left[ 1 + \frac{a}{x} - \frac{1}{2} \left( \frac{a^2 + R^2}{x^2} \right) - 1 + \frac{a}{x} + \frac{1}{2} \left( \frac{a^2 + R^2}{x^2} \right) \right] \approx \frac{Q}{4\pi\epsilon_0 x} \left( \frac{2a}{x} \right) \approx \frac{Qa}{2\pi\epsilon_0 x^2}.$$

We see that the potential is that of a dipole; from far away the disks approximate two point charges separated by  $2a$ .

76. If the second charge is stationary, the total energy is potential, which we find by considering one charge to be at the potential created by the other charge:

$$U = (-q)(1/4\pi\epsilon_0)(Q/r) = \boxed{-qQ/4\pi\epsilon_0 r}$$

For the circular motion, the Coulomb force must provide the centripetal acceleration:

$$F = qQ/4\pi\epsilon_0 r^2 = mv^2/r, \text{ which gives } mv^2 = qQ/4\pi\epsilon_0 r.$$

The total energy is

$$E = K + U \\ = \frac{1}{2}mv^2 + (-qQ/4\pi\epsilon_0 r) = \left(\frac{1}{2} - 1\right)(qQ/4\pi\epsilon_0 r) = \boxed{-qQ/8\pi\epsilon_0 r}.$$

Because the force is a central force, the angular momentum must be conserved; thus the angular velocity is constant.

77. From Table 24-1, with  $V = 0$  at  $r = \infty$ , we have the potential inside a nonconducting sphere:

$$V = (Q/8\pi\epsilon_0 R)[3 - (r^2/R^2)].$$

The potential energy of a charge  $-q$  is

$$U = -qV = \boxed{-(qQ/8\pi\epsilon_0 R)(3 - r^2/R^2)}.$$

The variable part of the potential energy has the form of the elastic potential energy of a spring:

$$U = \frac{1}{2}kr^2,$$

so the motion can be an oscillation, like the mass on a spring.

Comparing the coefficients, we have

$$k = \boxed{qQ/4\pi\epsilon_0 R^3}.$$

78. With all electrons at infinity, which is the reference level, no work is required to place the first electron at  $x = -6 \mu\text{m}$ :

$$U_1 = W_1 = \boxed{0, \text{ first electron at } x = -6 \mu\text{m}}.$$

To place the second electron at  $x = +6 \mu\text{m}$ , we have

$$U_2 = W_2 = qV_a = (-e)(1/4\pi\epsilon_0)(-e/r_a) = (1/4\pi\epsilon_0)(e^2/r) \\ = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(1.6 \times 10^{-19} \text{ C})^2/(6 \times 10^{-6} \text{ m}) \\ = \boxed{3.9 \times 10^{-23} \text{ J, second electron at } x = +6 \mu\text{m}}.$$

To place the third electron at  $x = 0 \text{ nm}$ , we have

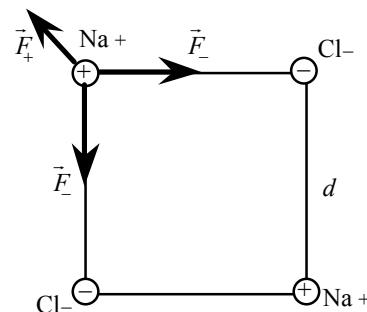
$$U_3 = W_3 = qV_b = (-e)(1/4\pi\epsilon_0)[(-e/r_b) + (-e/r_b)] = (1/4\pi\epsilon_0)(2e^2/r_b) \\ = (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)2(1.6 \times 10^{-19} \text{ C})^2/(6 \times 10^{-6} \text{ m}) \\ = \boxed{5.9 \times 10^{-23} \text{ J, third electron at } x = 0}.$$

The order in which the electrons are moved will affect the individual terms but not the total energy of  $9.8 \times 10^{-23} \text{ J}$  ( $6.0 \times 10^{-4} \text{ eV}$ ).

79. If we consider the sodium ion in the upper left corner, we see from symmetry that the net force must be along the diagonal:

$$F_{\text{net}} = 2F_- \cos 45^\circ - F_+ = (e^2/4\pi\epsilon_0)\{(2 \cos 45^\circ/d^2) - [1/(d\sqrt{2})^2]\} \\ = (e^2/4\pi\epsilon_0 d^2)(\sqrt{2} - 1) \\ = [(1.6 \times 10^{-19} \text{ C})^2(9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)/(2.5 \times 10^{-19} \text{ m})^2](\sqrt{2} - 1) \\ = \boxed{3.4 \times 10^{-9} \text{ N toward the other Na}^+}.$$

We find the work required from the potential energy change:





$$\begin{aligned}
 W = \Delta U &= e(V_{\infty} - V_{\text{corner}}) = (e/4\pi\epsilon_0)\{0 - [-(2e/d) + (e/d\sqrt{2})]\} \\
 &= (e^2/4\pi\epsilon_0 d)[2 - (1/\sqrt{2})] \\
 &= [(1.6 \times 10^{-19} \text{ C})^2 (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2) / (2.5 \times 10^{-19} \text{ m})][2 - (1/\sqrt{2})] \\
 &= \boxed{1.2 \times 10^{-18} \text{ J} \quad (7.4 \text{ eV})}.
 \end{aligned}$$

80. Because the point is not far away from the dipole, we find the potential from the sum of the potentials of two point charges:

$$\begin{aligned}
 V &= \frac{1}{4\pi\epsilon_0} \left\{ \frac{q}{[x^2 + (y-a)^2]^{1/2}} + \frac{-q}{[x^2 + (y+a)^2]^{1/2}} \right\} \\
 &= \frac{q}{4\pi\epsilon_0} \left\{ \frac{1}{[x^2 + (y-a)^2]^{1/2}} - \frac{1}{[x^2 + (y+a)^2]^{1/2}} \right\}.
 \end{aligned}$$

We find the components of the electric field from the partial derivatives of  $V$ :

$$\begin{aligned}
 E_x &= -\frac{\partial V}{\partial x} = \frac{-q}{4\pi\epsilon_0} \left\{ \frac{-x}{[x^2 + (y-a)^2]^{3/2}} - \frac{-x}{[x^2 + (y+a)^2]^{3/2}} \right\} \\
 &= \frac{q}{4\pi\epsilon_0} \left\{ \frac{x}{[x^2 + (y-a)^2]^{3/2}} - \frac{x}{[x^2 + (y+a)^2]^{3/2}} \right\}; \\
 E_y &= -\frac{\partial V}{\partial y} = \frac{-q}{4\pi\epsilon_0} \left\{ \frac{-(y-a)}{[x^2 + (y-a)^2]^{3/2}} - \frac{-(y+a)}{[x^2 + (y+a)^2]^{3/2}} \right\} \\
 &= \frac{q}{4\pi\epsilon_0} \left\{ \frac{y-a}{[x^2 + (y-a)^2]^{3/2}} - \frac{y+a}{[x^2 + (y+a)^2]^{3/2}} \right\}.
 \end{aligned}$$

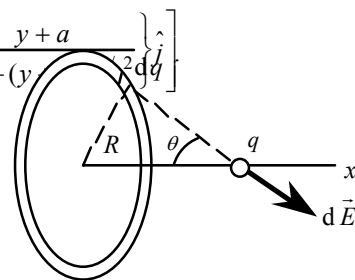
The electric field at the point  $(x, y)$  is

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \left[ \left\{ \frac{x}{[x^2 + (y-a)^2]^{3/2}} - \frac{x}{[x^2 + (y+a)^2]^{3/2}} \right\} \hat{i} + \left\{ \frac{y-a}{[x^2 + (y-a)^2]^{3/2}} - \frac{y+a}{[x^2 + (y+a)^2]^{3/2}} \right\} \hat{j} \right]$$

At the point  $P$ ,  $r = 3a$   $45^\circ$  from the  $y$ -axis, and  $x = y = 3a \cos 45^\circ = 2.12a$ .

Thus

$$V_P = \boxed{0.152q/4\pi\epsilon_0 a}, \text{ and } \vec{E}_P = \boxed{(q/4\pi\epsilon_0 a^2)(0.114\hat{i} + 0.023\hat{j})}.$$



81. From Example 24-9, we know the potential on the axis of a ring is

$$V = Q/4\pi\epsilon_0(R^2 + x^2)^{1/2}.$$

From symmetry, the electric field is along the  $x$ -axis, which we find from

$$\begin{aligned}
 \vec{E} &= -(\partial V/\partial x)\hat{i} \\
 &= -(\partial/\partial x)[Q/4\pi\epsilon_0(R^2 + x^2)^{1/2}]\hat{i} \\
 &= (-Q/4\pi\epsilon_0)(-!)(2x)/(R^2 + x^2)^{3/2}\hat{i} = \boxed{[(Qx/4\pi\epsilon_0(R^2 + x^2)^{3/2})]\hat{i}}.
 \end{aligned}$$

To find the field from direct integration, we use the diagram. Choosing a differential element of the ring, we see that the symmetry of the charge distribution means that we need to integrate the  $x$ -component:

$$E = \int dE_x = \int (1/4\pi\epsilon_0)[dq/(R^2 + x^2)] \cos \theta.$$

To perform the integration, we must reduce the integrand to one variable.

If we compare the two ways, we see that the direct integration method requires the selection of differential elements and the use of symmetry to handle the vector components, plus the actual integration. Because potential is a scalar, finding the field by differentiating  $V$  is generally easier.

82. For the positron, the increase in kinetic energy comes from the decrease in potential energy:

$$K_f - K_i = - (U_f - U_i) = - e(V_f - V_i);$$

$$K_f - 0 = - e(1/4\pi\epsilon_0)(0 - e/r_i);$$

$$K_f = (e^2/4\pi\epsilon_0)(1/r_0) = (1.60 \times 10^{-19} \text{ C})^2(9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)/(6.5 \times 10^{-10} \text{ m}) = \boxed{3.5 \times 10^{-19} \text{ J} (2.2 \text{ eV})}.$$

83. We find the change in potential energy from

$$\Delta U = - e \Delta V = - e(Ze/4\pi\epsilon_0)(1/r_2 - 1/r_1) = - (Ze^2/4\pi\epsilon_0)(1/r_2 - 1/r_1)$$

$$= - (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(2)(1.60 \times 10^{-19} \text{ C})^2(1/2 - 1/3)/(10^{-10} \text{ m}) = \boxed{- 7.68 \times 10^{-19} \text{ J}}.$$

For the electron orbiting the nucleus, the attractive Coulomb force provides the centripetal acceleration:

$$Ze^2/4\pi\epsilon_0 r^2 = mv^2/r, \text{ which gives } mv^2 = Ze^2/4\pi\epsilon_0 r.$$

The change in kinetic energy is

$$\Delta K = \Delta(mv^2) = (Ze^2/4\pi\epsilon_0)(1/r_2 - 1/r_1)$$

$$= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(2)(1.60 \times 10^{-19} \text{ C})^2 (1/2 - 1/3)/(10^{-10} \text{ m}) = \boxed{+ 3.84 \times 10^{-19} \text{ J}}.$$

The change in total energy is

$$\Delta E = \Delta K + \Delta U = + 3.84 \times 10^{-19} \text{ J} + (- 7.68 \times 10^{-19} \text{ J}) = \boxed{- 3.84 \times 10^{-19} \text{ J}}.$$

We see that the energy decreases as the electron gets closer to the nucleus; the energy is carried off by light emitted by the electron.

84. In the diagram shown to the right, the distances between each charge, labeled 1 through 4, to the point P of interest, are given by

$$r_1 = r_3 = (R^2 + a^2)^{1/2} = (R^2 + L^2)^{1/2},$$

$$r_2 = R - a = R - L/\sqrt{2}, \text{ and}$$

$$r_4 = R + a = R + L/\sqrt{2}.$$

The potential at point P is then

$$V = (1/4\pi\epsilon_0)(Q_1/r_1 + Q_2/r_2 + Q_3/r_3 + Q_4/r_4)$$

$$= (1/4\pi\epsilon_0)[Q/(R^2 + L^2)^{1/2} -$$

$$Q/(R - L/\sqrt{2}) + Q/(R^2 + L^2)^{1/2}$$

$$- \frac{Q}{(R + L/\sqrt{2})}]$$

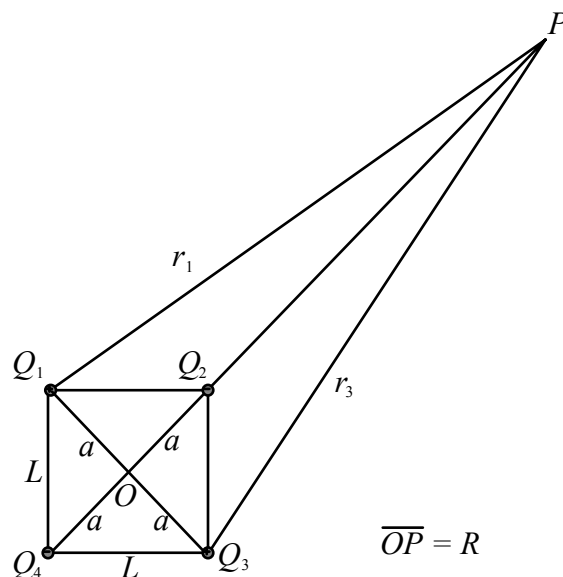
$$= (2Q/4\pi\epsilon_0 R)[(1 + L^2/2R^2)^{-1/2} - (1 - L^2/2R^2)^{-1/2}]$$

$$\approx (2Q/4\pi\epsilon_0 R)[(1 - L^2/4R^2) - (1 + L^2/2R^2)]$$

$$= \boxed{- 3QL^2/(8\pi\epsilon_0 R^3)}.$$

Note that here we made use of the approximation  $(1 + x)^n \approx nx$ , for  $x = L^2/2R^2 \ll 1$ . Also, we assumed that point P is aligned with the two negative charges ( $Q_2 = Q_4 = -Q$ ). Otherwise  $V$  will differ by a negative sign.

85. Assume that  $r_1 > r_2$ . Place the origin of the coordinate system at the center of both shells. For  $r > r_2$  the electric field is identical to that of a point charge,  $Q = q_1 + q_2$ , at the origin. So



$$V = (1/4\pi\epsilon_0)Q/r = \boxed{(1/4\pi\epsilon_0)(q_1 + q_2)/r \quad (r_2 < r)}.$$

Between  $r_1$  and  $r_2$ , the  $E$ -field produced by  $q_1$  is zero, so the potential due to  $q_1$  remains the same as its value at  $r_1$ , i.e.,  $(1/4\pi\epsilon_0)q_1/r_1$ . For  $q_2$ , the  $E$ -field is still equivalent to that of a point charge  $q_2$  at the origin, so the contribution to  $V$  due to  $q_2$  is  $(1/4\pi\epsilon_0)q_2/r$ . Add both contributions up to obtain

$$V = \boxed{(1/4\pi\epsilon_0)(q_2/r_2 + q_1/r) \quad (r_1 < r < r_2)}.$$

Once  $r < r_1$ , there is no electric field, so the potential no longer changes once it reaches its value at  $r_1$ . Thus

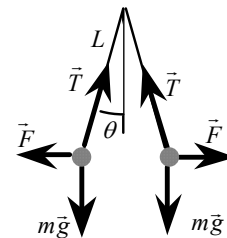
$$V = \boxed{(1/4\pi\epsilon_0)(q_1/r_1 + q_2/r_2) \quad (r < r_1)}.$$

86. Because the work done by the electric field of the dipole is independent of the path, we have

$$W_{a \rightarrow b} = q_0(V_b - V_a).$$

The initial and final points are not far from the dipole, so we find the potentials for two point charges:

$$\begin{aligned} W_{a \rightarrow b} &= q_0[(1/4\pi\epsilon_0)(q/r_{b2} - q/r_{b1}) - (1/4\pi\epsilon_0)(q/r_{a2} - q/r_{a1})] \\ &= (1/4\pi\epsilon_0)(q_0q)(1/r_{b2} - 1/r_{b1} - 1/r_{a2} + 1/r_{a1}) \\ &= (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2)(3.0 \times 10^{-6} \text{ C})(5.0 \times 10^{-6} \text{ C})[1/(0.2 \text{ m}) - 1/(0.6 \text{ m}) - 1/(0.8 \text{ m}) + 1/(0.4 \text{ m})] \\ &= \boxed{+0.62 \text{ J}}. \end{aligned}$$



87. (a) From the force diagram, we apply  $\sum \vec{F} = 0$ :

$$\text{horizontal: } T \sin \theta = F = kq^2/r^2;$$

$$\text{vertical: } T \cos \theta = mg.$$

If we divide the two equations, we get

$$\tan \theta = F/mg = kq^2/r^2 mg = kq^2/(2L \sin \theta)^2 mg$$

$$\tan 30^\circ = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.0 \times 10^{-6} \text{ C})^2/[2(0.80 \text{ m}) \sin 30^\circ]^2 m(9.8 \text{ m/s}^2),$$

$$\text{which gives } m = \boxed{9.9 \times 10^{-3} \text{ kg}}.$$

- (b) With the electric potential reference level at infinity and the gravitational potential reference level at

$\theta = 0^\circ$ , we have

$$\begin{aligned} U &= qV + mgy = (1/4\pi\epsilon_0)(q^2/2L \sin \theta) + mg(L - L \cos \theta) \\ &= [(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2 \times 10^{-6} \text{ C})^2/2(0.80 \text{ m})(\sin \theta)] + (9.9 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)(0.80 \text{ m})(1 - \cos \theta) \\ &= \boxed{0.023/\sin \theta + 0.078(1 - \cos \theta)}. \end{aligned}$$

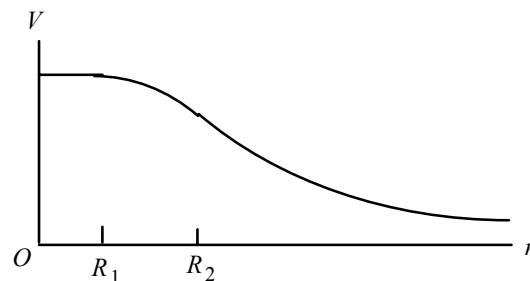
88. If we neglect end effects, the electric field (the potential gradient) of each plane is uniform. We find the work required to move the second plane from

$$W = q_2 \Delta V = q_2 (-E_1 \Delta x)$$

$$= -\sigma_2 L^2 (\sigma_1 / 2\epsilon_0) [(a - (x_2 - x_1))] = \boxed{(\sigma_1 \sigma_2 L^2 / 2\epsilon_0)(x_2 - x_1 - a)}.$$

89. We use the analogy to the charged spherical shell. When we are outside a charged cylindrical shell of radius  $r'$ , the potential is that of a line charge:  $V = -(\lambda/2\pi\epsilon_0) \ln(r/a)$  with  $V = 0$  at  $r = a$ . When we are inside a charged cylindrical shell of radius  $r'$ , the potential is the potential on the surface:  $V = -(\lambda/2\pi\epsilon_0) \ln(r'/a)$ . For a point inside the cylinder,  $r < R$ , the potential has two contributions: the sum (integral) of the shells inside  $r$  and the sum (integral) of the shells outside  $r$ . For a shell of radius  $r'$ , the linear charge density is  $d\lambda = \rho 2\pi r' dr'$ . We find the potential from

$$\begin{aligned}
 V &= \int_0^r -\frac{\rho 2\pi r' dr'}{2\pi\epsilon_0} \ln\left(\frac{r}{a}\right) + \int_r^R -\frac{\rho 2\pi r' dr'}{2\pi\epsilon_0} \ln\left(\frac{r'}{a}\right) \\
 &= -\frac{\rho}{\epsilon_0} \ln\left(\frac{r}{a}\right) \int_0^r r' dr' - \frac{\rho}{\epsilon_0} \int_r^R r' \ln\left(\frac{r'}{a}\right) dr' \\
 &= -\frac{\rho}{\epsilon_0} \ln\left(\frac{r}{a}\right) \frac{r^2}{2} - \frac{\rho}{\epsilon_0} \left\{ \frac{r'^2}{2} \left[ \ln\left(\frac{r'}{a}\right) - \frac{1}{2} \right] \right\} \bigg|_r^R \\
 &= -\frac{\rho r^2}{2\epsilon_0} \ln\left(\frac{r}{a}\right) - \frac{\rho R^2}{2\epsilon_0} \left[ \ln\left(\frac{R}{a}\right) - \frac{1}{2} \right] + \frac{\rho r^2}{2\epsilon_0} \left[ \ln\left(\frac{r}{a}\right) - \frac{1}{2} \right] \\
 &= -\frac{\rho r^2}{4\epsilon_0} - \frac{\rho R^2}{2\epsilon_0} \left[ \ln\left(\frac{R}{a}\right) - \frac{1}{2} \right], \quad r < R.
 \end{aligned}$$



For a point outside the cylinder,  $r > R$ , all of the cylindrical shells appear to be line charges:

$$\begin{aligned}
 V &= \int_0^R -\frac{\rho 2\pi r' dr'}{2\pi\epsilon_0} \ln\left(\frac{r}{a}\right) = -\frac{\rho}{\epsilon_0} \ln\left(\frac{r}{a}\right) \int_0^R r' dr' \\
 &= -\frac{\rho R^2}{2\epsilon_0} \ln\left(\frac{r}{a}\right), \quad r > R.
 \end{aligned}$$

90. The charge density of the dielectric shell is

$$\begin{aligned}
 \rho &= Q / [\pi(R_2^3 - R_1^3)] \\
 &= (5 \times 10^{-6} \text{ C}) / [\pi[(0.45 \text{ m})^3 - (0.16 \text{ m})^3]] \\
 &= 1.4 \times 10^{-5} \text{ C/m}^3.
 \end{aligned}$$

In the region where  $r < R_1 = 16 \text{ cm}$ , there is no electric field,

so the potential is constant. Inside the dielectric shell,

$R_1 < r < R_2 = 45 \text{ cm}$ , we see from Table 24-1 that the

potential is proportional to  $-r^2$ . Outside the dielectric shell,  $r > R_2$ , the shell is equivalent to a point charge at

the center, so the potential is proportional to  $1/r$ .

We find the potential at  $r = 0$  by adding (integrating)

the potentials of the differential spherical shells between  $R_1$  and  $R_2$ :

$$\begin{aligned}
 V &= k \int (\rho 4\pi r^2 dr) / r = 2\pi k \rho (R_2^2 - R_1^2) \\
 &= 2\pi (9 \times 10^9 \text{ C}^2/\text{N} \cdot \text{m}^2) (1.4 \times 10^{-5} \text{ C/m}^3) [(0.45 \text{ m})^2 - (0.16 \text{ m})^2] \\
 &= \boxed{1.4 \times 10^5 \text{ V}, \quad r = 0}.
 \end{aligned}$$

Because the potential is constant from  $r = 0$  to  $r = R_1$ , the potential at the inner radius is

$$\boxed{1.4 \times 10^5 \text{ V}, \quad r = R_1}.$$

Because the potential outside the shell is the same as that of a point charge, we find the potential at  $r = R_2$  from

$$\begin{aligned}
 V &= (1/4\pi\epsilon_0)(Q/R_2) \\
 &= (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C})(5 \times 10^{-6} \text{ C}) / (0.45 \text{ m}) = \boxed{1.0 \times 10^5 \text{ V}, \quad r = R_2}.
 \end{aligned}$$

91. To find the total potential energy of the sphere, we consider it to be made up of differential shells and add (integrate) the work required to bring each shell in from infinity.

If a sphere of radius  $r < R$  has been formed, the potential at the surface is

$$V = (1/4\pi\epsilon_0)(q/r) = (1/4\pi\epsilon_0)(\rho) \pi r^3 / r = \rho r^2 / 3\epsilon_0.$$

The work to bring the charge of the next shell,  $dq = \rho 4\pi r^2 dr$ , in from infinity is

$$dW = dq V = (\rho 4\pi r^2 dr)(\rho r^2 / 3\epsilon_0).$$

The total work and thus the total potential energy stored is

$$W = \int_0^R \frac{\rho^2 4\pi r^4}{3\epsilon_0} dr = \frac{4\pi\rho^2}{3\epsilon_0} \int_0^R r^4 dr = \frac{4\pi\rho^2 R^5}{15\epsilon_0}.$$