

# CHAPTER 23 Gauss' Law

## Answers to Understanding the Concepts Questions

1. The value of temperature at each point forms a scalar field. Since there is no directionality associated with temperature, flux, which measures an abstract sort of flow across a surface, cannot be associated with a temperature field. However, given a temperature field, one can calculate at each point a vector, termed the *temperature gradient*, representing the change in temperature. This vector field has the components  $(\partial T/\partial x, \partial T/\partial y, \partial T/\partial z)$ , where you will recall that the symbol  $\partial$  refers to partial differentiation. Because this field has directionality, it is possible to define a flux for it.
2. The opening is presumably very small in comparison with the size of the sphere. So the "open" sphere can be thought of as a closed one, only with a small patch removed.
3. According to Gauss' law, the net charge enclosed by the surface is zero. This does not, however, mean that the electric field over the surface is always zero. The simplest counter example would be a uniform field produced by some charge distribution outside the surface. The flux of a uniform field over any enclosed surface is always zero, yet the field itself is not. Also, the surface can enclose an equal amount of positive and negative charges, producing a non-zero field but a zero net flux over the surface.
4. Suppose we consider a charge-free region, and there is a break (discontinuity) in a field line. It is then possible to construct a Gaussian surface that envelops the tip of the break in the field line. There will be a net flux across that surface, but on the other hand, there is no charge in the region. Thus a break in an electric field line in a charge-free region violates Gauss' law. The only way to satisfy Gauss' law is to insist that when a field line ends, it ends on a charge.
5. Consider, for example, a spherical Gaussian surface of radius  $r$  centered at the location of a point charge  $q$ . The electric flux through this surface is  $\Phi = EA = E(4\pi r^2)$ . If  $E = q/r^2$  then  $\Phi = (q/r^2)(4\pi r^2) = 4\pi q$ , which depends on the radius  $r$  of the Gaussian surface, rather than just the charge  $q$  enclosed -- this is contradictory to Gauss' law.
6. If the point charge is located at the center of a certain face (face A) of the cube, then by symmetry the electric flux through each of the four faces that are perpendicular to face A is identical. However, the flux through the remaining face that is directly opposite to face A is different. Therefore, in this case symmetry alone does not provide a simple answer to the flux through each face.
7. The reconciliation follows by considering a pill-box Gaussian surface on the first plate, with its flat ends, of area  $A$ , extending just outside of the plate itself. The charge density is not changed, so that the  $Q/\epsilon_0$  part of Gauss' law is unchanged. The flux through the end surfaces, however, is changed. On the side of the plate away from the second, negatively charged, plate, the flux through the end of the pill-box is  $(\sigma/2\epsilon_0)A$  from the positive plate, and  $-(\sigma/2\epsilon_0)A$  from the negative plate. These cancel. The flux through the end surface of the pill-box between the plates is  $(\sigma/2\epsilon_0)A$  from the positive plate and a like amount from the negative plate. These add to a total of  $(\sigma/\epsilon_0)A$ . There is no conflict.

8. No. It only means that the net charge enclosed by the surface is zero, since the flux is proportional to the net charge enclosed. As an example, consider a spherical shell of radius  $r$  that is uniformly charged to a total charge  $q$ . At the center of the sphere is a point charge  $-q$ . If we draw a spherical Gaussian surface of radius  $R > r$ , concentric with the charged shell, then  $E = 0$  everywhere on the gaussian surface -- and yet there are charged enclosed by it (although no net charge).
9. The electric field produced by a uniformly charged spherical shell is zero anywhere inside the shell. The force it exerts on a charge there is therefore zero.
10. Gauss' law for fluid flow involves the fluid flux, given by  $\Phi = \int \vec{v} \cdot d\vec{A}$ . This flux describes the rate at which the fluid crosses the surface. For a closed surface there will be a net outflow of fluid only if there is a source of fluid somewhere within the enclosed volume. Thus Gauss' law will read  $\Phi = S$ , where  $S$  is the rate at which fluid is "created" inside the surface by a source, in  $\text{m}^3/\text{s}$ . If there are sources (faucets) in the region, then  $S$  is positive; if there are sinks (drains) in the region, then  $S$  is negative. Evaporation acts as a sink; that is, a negative contribution to the flux. Looking at the net flux, it is impossible to separate evaporation from any other type of sink. In the case of electricity, the analog of evaporation would be the disappearance of electric charge. There are deep principles that argue against that, and therefore one would not expect  $S$  to change with time unless charges actually cross the boundary of the enclosed surface.
11. Yes. The charges would be deposited over the exterior surface of the aluminum shell, in such a way that the electric field inside the shell remains zero.
12. Like the Coulomb force, the gravitational force is also a central force with inverse-square dependency on distance, so Gauss' law applies to it as well. If we compare the Coulomb force,  $F_E = (1/4\pi\epsilon_0) q_1 q_2 / r^2$ , with the gravitational force,  $F_g = G m_1 m_2 / r^2$ , we find that in writing down Gauss' law for gravitational field we need to make the following substitution:  $q$  to  $m$ ,  $\vec{E}$  ( $= \vec{F}_E / q$ ) to  $\vec{g}$  ( $= \vec{F}_g / m$ ), and  $1/4\pi\epsilon_0$  to  $G$  (or  $1/\epsilon_0$  to  $4\pi G$ ). Also, the gravitational flux is negative due to the attractive nature of the force. Thus  $\Phi_E = \oint \vec{E} \cdot d\vec{A} = q/\epsilon_0$  becomes  $\Phi_g = \oint \vec{g} \cdot d\vec{A} = -4\pi G m$ .
13. Symmetry does allow us to state that the electric field is parallel to the vector normal to the surface of the torus. Thus Gauss' law gives us a value for the integral  $\int E dA$ . Because of the curvature of the surface,  $E$  is not the same on the inner part of the torus as on the outer part, and therefore the integral cannot be converted to the form  $E \int dA = EA$ .
14. Assuming that the two point charges are fixed so that they cannot annihilate each other, the resulting electric field is zero inside the conductor. Outside the conductor, the field lines extend from the charges, and always intersect the surface of the conductor perpendicularly.
15. With Gauss' law we can show that there is no charge in the region of uniform electric field. Take a Gaussian surface in the shape of a can with the two ends perpendicular to the constant field direction. The net flux through the surface is zero, and so the net charge inside the region is zero. The surface can be anywhere within the large region, so that there is no net charge anywhere. If you are worried that this does not rule out equal positive and negative charges inside the region, just make the can smaller. No matter how small the can's volume, there is no net charge.
16. The flux also triples. This is because  $\vec{E}$  is now  $3\vec{E}$  (as it is proportional to the charge that produces it).

17. For a charged line of finite length, the electric field is not uniform over the Gaussian cylinder in question. For example, the value of the  $E$ -field close to one end of the line is not quite the same as that near the center of the line. Therefore we can no longer evaluate the electric flux as  $\oint \vec{E} \cdot d\vec{A} = EA$ .
18. The flux is proportional to the charge enclosed, so as  $q$  doubles the flux also doubles, to  $8.0 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}$ . Doubling the side length of the cube is irrelevant to the answer, since the flux depends only on the charge enclosed, not on the size and shape of the Gaussian surface.
19. Let's use Gauss' law together with a Gaussian surface in the form of a tiny pill-box whose flat ends are perpendicular to the  $z$ -axis. Since the  $z$ -component of  $\vec{E}$  vanishes and the other components are independent of  $z$ , the net flux through this Gaussian surface is independent of  $z$ , and the net charge can only depend on  $x$  and  $y$ . Thus the charge density must also be independent of  $z$ .
20. Imagine a Gaussian surface that enclosed a certain volume  $V$  of the region. The charge enclosed by the surface is  $q = \rho V$ . By measuring the electric field on the surface we can find the flux  $\Phi$  through the surface. But according to Gauss' law  $\Phi = q/\epsilon_0 = \rho V/\epsilon_0$ , so  $\rho = \epsilon_0 \Phi/V$ .
21. No. If the charge distribution is not uniform then the  $E$ -field would not exhibit the symmetry that allowed us to write  $\oint \vec{E} \cdot d\vec{A} = EA$ . The resulting  $E$ -field is not the same. Consider, for example, a highly asymmetrical case where all the charges on the shell is concentrated at one point on the shell. The resulting electric field is that of a point charge located on the shell, and that is certainly very different from the result of a uniformly charged shell.

## Solutions to Problems

- The electric field of the plate is perpendicular to the plate with magnitude  $E = \sigma/2\epsilon_0$ .
  - Because the circle is parallel to the plate, the area vector is perpendicular to the plate. The flux through the circle is
 
$$\Phi = \iint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \vec{A} = E\pi R^2 = \boxed{\sigma\pi R^2/2\epsilon_0}.$$
  - The angle between the field vector and the area vector is  $30^\circ$ . The flux through the circle is
 
$$\Phi = \iint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \vec{A} = (\sigma\pi R^2/2\epsilon_0) \cos 30^\circ = \boxed{0.866 \sigma\pi R^2/2\epsilon_0}.$$

- The angle between the field vector and the area vector is  $48^\circ$ . The flux through the square is
 
$$\Phi = \iint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \vec{A} = EA \cos 48^\circ = (1325 \text{ N/C})(0.27 \text{ m})^2 \cos 48^\circ = \boxed{65 \text{ N} \cdot \text{m}^2/\text{C}}.$$

- On the ends of the cylinder the electric field is not constant, but it is always perpendicular to the area vector of the surface. On the sides of the cylinder the electric field is constant and parallel to the area vector. The flux through the cylinder is

$$\begin{aligned} \Phi &= \iiint \vec{E} \cdot d\vec{A} = \iint_{\text{end}} \vec{E} \cdot d\vec{A} + \iint_{\text{end}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} \\ &= 0 + 0 + EA_{\text{side}} = \frac{\lambda}{2\pi\epsilon_0 R} (2\pi Rh) = \frac{\lambda h}{\epsilon_0}. \end{aligned}$$

We see that the result is independent of  $R$ , so we get the same flux through a cylinder of radius  $2R$ .

- The electric field and the area vector are parallel. Because the electric field varies over the surface, we find the flux by integrating:

$$\begin{aligned} \Phi &= \iint_{\text{square}} \vec{E} \cdot d\vec{A} = \iint_{\text{end}} (5xz \hat{k}) \cdot (dx dy \hat{k}) = 5z \int_{-1}^2 dy \int_{-1}^2 x dx \\ &= 5z (y) \Big|_{-1}^2 \left( \frac{x^2}{2} \right) \Big|_{-1}^2 = 5(3)[2 - (-1)] \left[ \frac{(2)^2}{2} - \frac{(-1)^2}{2} \right] \\ &= 68 \text{ N} \cdot \text{m}^2/\text{C}. \end{aligned}$$

- Because each side of the cube has an area vector parallel to one of the coordinate axes, the scalar product for the side involves only one component of the electric field. The total flux through the cube is

$$\begin{aligned} \Phi &= \iiint \vec{E} \cdot d\vec{A} = \iint_{x=0} \vec{E} \cdot d\vec{A} + \iint_{x=1} \vec{E} \cdot d\vec{A} + \iint_{y=0} \vec{E} \cdot d\vec{A} + \iint_{y=1} \vec{E} \cdot d\vec{A} + \iint_{z=0} \vec{E} \cdot d\vec{A} + \iint_{z=1} \vec{E} \cdot d\vec{A} \\ &= \iint_{x=0} (5x) dy dz + \iint_{x=1} (5x) dy dz + \iint_{y=0} (-3y) dx dz + \iint_{y=1} (-3y) dx dz + \\ &\quad \iint_{z=0} (4z) dx dy + \iint_{z=1} (4z) dx dy \\ &= 0 + (5)(1)(1)(1) + 0 + (-3)(1)(1)(1) + 0 + (4)(1)(1)(1) = +6 \text{ N} \cdot \text{m}^2/\text{C}. \end{aligned}$$

- The area vector is perpendicular to the plate and thus parallel to the electric field. The flux through the loop is

$$\Phi = \iint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \vec{A} = EA = (150 \text{ N/C})(4 \times 10^{-4} \text{ m}^2) = \boxed{6 \times 10^{-2} \text{ N} \cdot \text{m}^2/\text{C}}.$$

- (b) The angle between the field vector and the area vector is  $30^\circ$ . The flux through the circle is

$$\Phi = \iint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \vec{A} = EA \cos 25^\circ = (150 \text{ N/C})(4 \times 10^{-4} \text{ m}^2) \cos 25^\circ = \boxed{5.4 \times 10^{-2} \text{ N} \cdot \text{m}^2/\text{C}}.$$

- (c) The angle between the field vector and the area vector is  $330^\circ$ . The flux through the circle is

$$\Phi = \iint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \vec{A} = EA \cos 335^\circ = (150 \text{ N/C})(4 \times 10^{-4} \text{ m}^2) \cos 335^\circ = \boxed{5.4 \times 10^{-2} \text{ N} \cdot \text{m}^2/\text{C}}.$$

There is no change.

7. The electric field and the area vector are parallel. Because the electric field varies over the surface, we find the flux by integrating. We choose a circular ring of radius  $r$  and thickness  $dr$  as the differential element:

$$\begin{aligned}\Phi &= \iint \vec{E} \cdot d\vec{A} = \int_0^R E_0 \left(1 - \frac{r}{R}\right) (2\pi r dr) \\ &= 2\pi E_0 \int_0^R \left(r - \frac{r^2}{R}\right) dr = 2\pi E_0 \left(\frac{r^2}{2} - \frac{r^3}{3R}\right) \Big|_0^R = \frac{\pi E_0 R^2}{3}.\end{aligned}$$

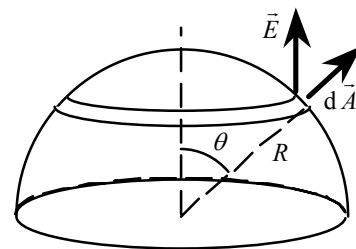
8. Because the angle between the electric field and the area varies over the surface of the hemisphere, we find the flux by integration. We choose a strip at an angle  $\theta$  with a thickness  $R d\theta$ , as shown in the diagram. The area of this strip is

$$dA = (2\pi R \sin \theta) R d\theta = 2\pi R^2 \sin \theta d\theta.$$

From the diagram, we see that  $\theta$  is the angle between  $\vec{E}$  and  $d\vec{A}$ , so we have

$$\begin{aligned}\Phi &= \iint \vec{E} \cdot d\vec{A} = \int_0^{\pi/2} E (\cos \theta) 2\pi R^2 \sin \theta d\theta \\ &= E 2\pi R^2 \left(\frac{\sin^2 \theta}{2}\right) \Big|_0^{\pi/2} = E \pi R^2.\end{aligned}$$

This is the flux of a constant field through the area of a circle of radius  $R$ .

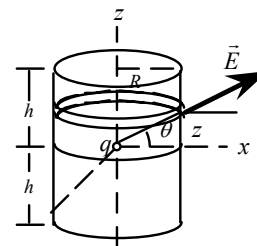


9. Because the angle between the electric field and the area varies over the surface of the hemisphere, we find the flux by integration. We choose a band at an elevation  $z$ , which corresponds to an angle  $\theta$  such that  $z = R \tan \theta$ . The band has thickness  $dz = R \sec^2 \theta d\theta$  so the area of this band is

$$dA = 2\pi R dz = 2\pi R^2 \sec^2 \theta d\theta.$$

From the symmetry we see that the flux will be the same for the upper and lower halves of the surface, so we double the result of the integration over the top half. The angle  $\theta$  ranges from 0 to  $\theta_0$ , with  $\sin \theta_0 = h/R$ . From the diagram, we see that  $\theta$  is the angle between  $\vec{E}$  and  $d\vec{A}$  for all elements of the band, so we have

$$\begin{aligned}\Phi &= \iint \vec{E} \cdot d\vec{A} = 2 \int_0^{\theta_0} \frac{1}{4\pi\epsilon_0} \frac{q}{(R/\cos \theta)^2} 2\pi R^2 \sec^2 \theta (\cos \theta) d\theta \\ &= \frac{q}{\epsilon_0} \int_0^{\theta_0} \cos \theta d\theta = \frac{q}{\epsilon_0} (\sin \theta) \Big|_0^{\theta_0} = \frac{q}{\epsilon_0} \frac{1}{\sqrt{1 + (R^2/h^2)}}.\end{aligned}$$



10. If the charge is placed a very small distance above the center, the radial electric field through the hemisphere is constant in magnitude and always perpendicular to the surface ( $\vec{E}$  and  $d\vec{A}$  parallel). The flux through the hemisphere is

$$\Phi_{\text{hemisphere}} = \iint \vec{E} \cdot d\vec{A} = EA = (q/4\pi\epsilon_0 R^2)(4\pi R^2) = q/2\epsilon_0.$$

The direct calculation of the flux through the planar circle is more difficult; however we can use the symmetry of the electric field of the point charge. The flux above the horizontal plane must be equal to the flux below the horizontal plane:

$$\Phi_{\text{circle}} = \Phi_{\text{hemisphere}}.$$

Thus, the total flux is

$$\Phi = \Phi_{\text{hemisphere}} + \Phi_{\text{circle}} = (q/2\epsilon_0) + (q/2\epsilon_0) = \boxed{q/\epsilon_0}.$$

11. Because each side of the parallelepiped has an area vector parallel to one of the coordinate axes, the scalar product for each side involves only the component of the electric field in the direction of the axis. Because the electric field is a function of  $x$ ,  $y$ , and  $z$ , the magnitude of its components will be different on opposite sides of the parallelepiped.

For example, along the  $x$ -axis, we have

$$\vec{E}(x + dx, y, z) = \vec{E}(x, y, z) + [\partial \vec{E}(x, y, z)/\partial x] dx,$$

or the equivalent three component equations.

The area vector always points out of the surface. We find the differential flux through the two sides perpendicular to the  $x$ -axis, with area  $dy dz$ , from

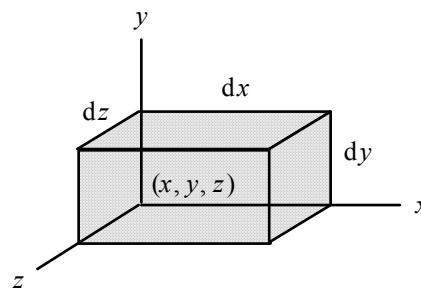
$$\begin{aligned} \Phi_1 &= \vec{E} \cdot d\vec{A} \\ &= (E_x \hat{i} + E_y \hat{j} + E_z \hat{k}) \cdot dy dz (-\hat{i}) + \{(E_x \hat{i} + E_y \hat{j} + E_z \hat{k}) + [(\partial E_x/\partial x) \hat{i} + (\partial E_y/\partial x) \hat{j} + (\partial E_z/\partial x) \hat{k}]\} \cdot dy dz \hat{i} \\ &= -E_x dy dz + E_x dy dz + (\partial E_x/\partial x) dx dy dz = (\partial E_x/\partial x) dx dy dz. \end{aligned}$$

If we apply a similar analysis to the other pairs of sides, we have

$$\Phi_2 = (\partial E_y/\partial y) dy dx dz \quad \text{and} \quad \Phi_3 = (\partial E_z/\partial z) dz dx dy.$$

The total flux through the surface is

$$\begin{aligned} \Phi &= \Phi_1 + \Phi_2 + \Phi_3 \\ &= (\partial E_x/\partial x) dx dy dz + (\partial E_y/\partial y) dy dx dz + (\partial E_z/\partial z) dz dx dy \\ &= [(\partial E_x/\partial x) + (\partial E_y/\partial y) + (\partial E_z/\partial z)] dx dy dz. \end{aligned}$$



12. The flux is directly dependent on the enclosed charge:

$$\Phi = Q/\epsilon_0, \quad \text{or} \quad Q = \epsilon_0 \Phi = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(-5.7 \times 10^{-5} \text{ N} \cdot \text{m}^2/\text{C}) = \boxed{-5.0 \times 10^{-16} \text{ C}}.$$

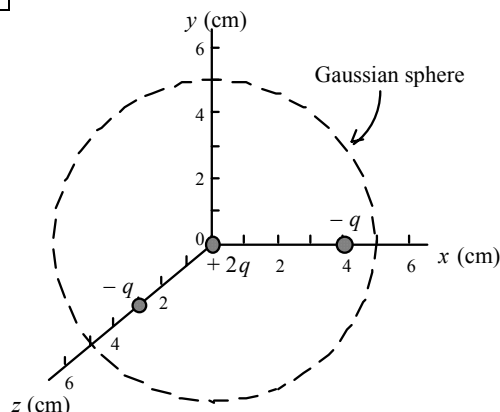
13. (a) We use the spherical surface within the charged surface as a Gaussian surface. Because there is no enclosed charge the total electric flux through the surface is zero.
- (b) We use the spherical surface outside the charged surface as a Gaussian surface. Because all of the charge is enclosed, the total electric flux through the surface is

$$\begin{aligned} \Phi &= \oint \vec{E} \cdot d\vec{A} = Q/\epsilon_0 \\ &= (10^{-3} \text{ C})/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = \boxed{1.13 \times 10^8 \text{ N} \cdot \text{m}^2/\text{C}}. \end{aligned}$$

14. Because all of the charge is enclosed, the total electric flux through the surface is

$$\begin{aligned} \Phi &= \oint \vec{E} \cdot d\vec{A} = Q/\epsilon_0 \\ &= (120 \times 10^{-9} \text{ C})/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = \boxed{1.36 \times 10^4} \\ &\quad \boxed{\text{N} \cdot \text{m}^2/\text{C}}. \end{aligned}$$

15. (a) For the Gaussian sphere we have
- $$\Phi = \oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}}/\epsilon_0 = (-q + 2q - q)/\epsilon_0 = 0.$$
- The net electric flux through the surface is zero.
- (b) Some electric field lines from the positive charge to the negative charges will pierce the sphere; however, every line that comes out through the sphere at some point will enter the sphere at some other point.



16. Because the charge is placed at the midpoint, we know from symmetry that the flux is the same through the two ends. From Gauss' law, we have

$$\Phi = Q/\epsilon_0 = \Phi_{\text{ends}} + \Phi_{\text{curved}};$$

$$(1.2 \times 10^4 \text{ C})/\epsilon_0 = 2(4.5 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}) + \Phi_{\text{curved}}, \text{ which gives } \Phi_{\text{curved}} = \boxed{4.6 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}}.$$

17. (a) The total electric flux through the surface depends only on the enclosed charge:

$$\Phi = \oint \vec{E} \cdot d\vec{A} = Q/\epsilon_0;$$

$$-4 \times 10^2 \text{ N} \cdot \text{m}^2 = Q/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2), \text{ which gives } Q = \boxed{-3.54 \times 10^{-9} \text{ C}}.$$

- (b) The total electric flux through the closed surface does not depend on the shape of the surface.

$$\text{The enclosed charge is } Q = \boxed{-3.54 \times 10^{-9} \text{ C}}.$$

- (c) Similar to (b), the enclosed charge is  $Q = \boxed{-3.54 \times 10^{-9} \text{ C}}.$

18. The total electric flux through the surface depends only on the enclosed charge:

$$\Phi = \oint \vec{E} \cdot d\vec{A} = Q/\epsilon_0 = (420 \times 10^{-6} \text{ C})/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 4.7 \times 10^7 \text{ N} \cdot \text{m}^2/\text{C}.$$

Because the charge is at the center of the cube, we know from symmetry that each of the six sides has the same flux through it:

$$\Phi_{\text{side}} = (1/6)\Phi_{\text{total}} = (1/6)(4.7 \times 10^7 \text{ N} \cdot \text{m}^2/\text{C}) = \boxed{7.9 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}}.$$

19. Because the charge at the origin is at the center of the cube, we know from symmetry that it will produce a flux out of each side that is 1/6 of the total flux it produces:

$$\Phi_1 = (1/6)\Phi_{\text{charge}} = (1/6)(Q/\epsilon_0)$$

$$= (1/6)(5 \times 10^{-8} \text{ C})/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 9.4 \times 10^2 \text{ N} \cdot \text{m}^2/\text{C}.$$

Because the uniform field is parallel to the  $x$ -axis, it produces no flux through the sides parallel to the  $x$ -axis. Through the sides parallel to the  $yz$ -plane, the uniform field produces a flux

$$\Phi_2 = EA = (3000 \text{ N/C})(0.20 \text{ m})^2 = 1.2 \times 10^2 \text{ N} \cdot \text{m}^2/\text{C}.$$

Because this flux enters the cube from the  $+x$ -axis and leaves the cube toward the  $-x$ -axis, we have

$$\begin{array}{l} 9.4 \times 10^2 \text{ N} \cdot \text{m}^2/\text{C} \text{ out of the sides parallel to the } xy\text{- or } yz\text{-planes,} \\ 10.6 \times 10^2 \text{ N} \cdot \text{m}^2/\text{C} \text{ out of the side perpendicular to the } -x\text{-axis,} \\ 8.2 \times 10^2 \text{ N} \cdot \text{m}^2/\text{C} \text{ out of the side perpendicular to the } +x\text{-axis.} \end{array}$$

20. The gravitational field is

$$\vec{g} = \frac{\vec{F}}{m} = -\frac{GM}{r^2} \hat{r}.$$

If we compare this to the electric field,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r},$$

we see that  $E \rightarrow \vec{g}$ ,  $Q \rightarrow -M$ , and  $\epsilon_0 \rightarrow 1/4\pi G$ . If we make these substitutions in Gauss' law for the electric field, we have

$$\Phi = \oint \vec{E} \cdot d\vec{A} = q/\epsilon_0 \rightarrow \Phi = \oint \vec{g} \cdot d\vec{A} = -M/(1/4\pi G) = -4\pi GM,$$

which is Gauss' law for the gravitational field, where  $M$  is the enclosed mass.

To find the gravitational field within a sphere of uniform mass density, we must select a Gaussian surface. From symmetry, we know that the field is radial, since all directions must be equivalent. If we choose a spherical surface, the field will be constant and parallel to the area vector, so we have

$$\oint \vec{g} \cdot d\vec{A} = g \oint dA = -4\pi GM_{\text{enclosed}};$$

$$g4\pi r^2 = -4\pi G\rho(\pi r^3), \text{ which gives } g = -\pi G\rho r, \text{ or } g = \boxed{-GMr/R^3 \text{ toward the center, } r < R}.$$

At a point outside the sphere, we know from symmetry that the field is radial, since all directions must



be equivalent. Over a spherical surface, the field will be constant and parallel to the area vector, so

$$\oint \vec{g} \cdot d\vec{A} = g \oint dA = -4\pi GM_{\text{enclosed}};$$

$$g4\pi r^2 = -4\pi GM, \text{ which gives } g = \boxed{-GM/r^2 \text{ toward the center, } r > R}.$$

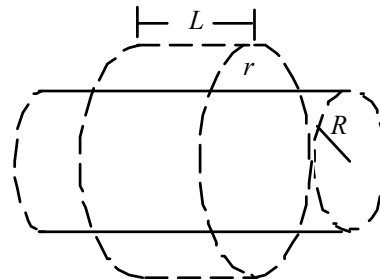
21. From symmetry, we know that, outside the charge, the electric field will be radially away from the axis of the cylinder, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length  $L$  and radius  $r$ , centered on the axis. On the ends of this surface, the electric field is not constant but  $\vec{E}$  and  $d\vec{A}$  are perpendicular, so we have  $\vec{E} \cdot d\vec{A} = 0$ . On the curved side, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ . For Gauss' law, we have

$$\oiint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0;$$

$$E2\pi rL = r\pi R^2L/\epsilon_0, \text{ which gives}$$

$$E = \boxed{\rho R^2/2\epsilon_0 r}.$$

Note that the result is independent of  $L$ , as it must be.

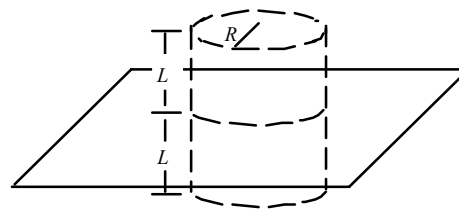


22. For a Gaussian surface, we choose a cylinder with radius  $R$  and length  $2L$ , with its axis perpendicular to the plate. Because the charge density of the plate is uniform, we know that the electric field must be perpendicular to the plate and may depend only on the distance from the plate. By arranging the surface so that the ends are equidistant from the plate, we know that the flux through the ends must be the same, while the flux through the side must be zero. When we apply Gauss' law, we have

$$\oiint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 2EA_{\text{end}} + 0 = Q/\epsilon_0;$$

$$2E\pi R^2 = \sigma\pi R^2/\epsilon_0, \text{ which gives}$$

$$E = \boxed{\sigma/2\epsilon_0}.$$



23. For the long rod we have

$$E_{\text{rod}} = (1/2\pi\epsilon_0)\lambda/r = 2(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.5 \times 10^{-8} \text{ C/m})/(10^{-2} \text{ m}) = 1.17 \times 10^4 \text{ N/C}.$$

For the point charge we have

$$E_{\text{point charge}} = (1/4\pi\epsilon_0)q/r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.5 \times 10^{-8} \text{ C})/(10^{-2} \text{ m}) = 5.85 \times 10^5 \text{ N/C}.$$

The ratio is

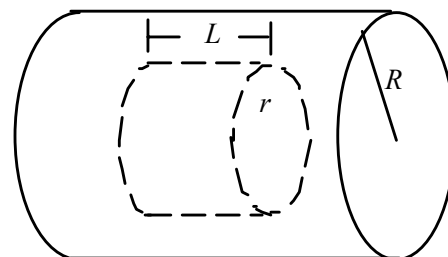
$$\boxed{E_{\text{rod}}/E_{\text{point charge}} = 0.02 = 2\%}.$$

24. From symmetry, we know that the electric field will be radially away from the axis of the cylinder, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length  $L$  and radius  $r$ , centered on the axis. On the ends of this surface, the electric field is not constant but  $\vec{E}$  and  $d\vec{A}$  are perpendicular, so we have  $\vec{E} \cdot d\vec{A} = 0$ . On the curved side, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ . For Gauss' law, we have

$$\oiint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0;$$

$$E2\pi rL = \rho\pi r^2L/\epsilon_0, \text{ which gives}$$

$$E = \boxed{\rho r/2\epsilon_0}.$$



25. We choose a Gaussian surface with a top surface just above the ground and a bottom surface below the ground, each of area 40 acres, and the sides perpendicular to the ground. There is no flux through the sides, because  $\vec{E} \cdot d\vec{A} = 0$ . There is no flux through the bottom, because  $\vec{E} = 0$ . When we apply Gauss' law to this surface, we get

$$\begin{aligned}\oiint \vec{E} \cdot d\vec{A} &= \iint_{\text{top}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} + \iint_{\text{bottom}} \vec{E} \cdot d\vec{A} \\ &= -EA_{\text{top}} + 0 + 0 = Q/\epsilon_0, \text{ which gives} \\ Q &= -\epsilon_0 EA \\ &= -(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(110 \text{ N/C})(60 \text{ acres})(4 \times 10^3 \text{ m}^2/\text{acre}) = \boxed{-2.3 \times 10^{-4} \text{ C}}.\end{aligned}$$

26. We assume that the smaller cylinder is positive. From the symmetry of the charge distribution, we know that the electric field must be radial, away from the axis of the cylinders, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length  $L$  and radius  $r$ , centered on the axis. On the ends of this surface, the electric field is not constant but  $\vec{E}$  and  $d\vec{A}$  are perpendicular, so we have  $\vec{E} \cdot d\vec{A} = 0$ . On the curved side, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ .

For the region inside the smaller cylinder,  $r < r_1$ , we apply Gauss' law:

$$\begin{aligned}\oiint \vec{E} \cdot d\vec{A} &= \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0; \\ E2\pi rL &= 0, \text{ because } q = 0 \text{ inside the Gaussian surface, which gives} \\ E &= \boxed{0 \text{ for } r < r_1}.\end{aligned}$$

For the region between the cylinders,  $r_1 < r < r_2$ , we apply Gauss' law:

$$\begin{aligned}\oiint \vec{E} \cdot d\vec{A} &= \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0; \\ E2\pi rL &= \lambda L/\epsilon_0, \text{ because only the smaller cylinder is inside the} \\ \text{Gaussian surface, which gives} \\ E &= \boxed{\lambda/2\pi\epsilon_0 r \text{ radially out for } r_1 < r < r_2}.\end{aligned}$$

For the region outside the larger cylinder,  $r_2 < r$ , we apply Gauss' law:

$$\begin{aligned}\oiint \vec{E} \cdot d\vec{A} &= \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0; \\ E2\pi rL &= \lambda L/\epsilon_0, \text{ because only the smaller cylinder is inside the} \\ \text{Gaussian surface, which gives} \\ E &= \boxed{0 \text{ for } r_2 < r}.\end{aligned}$$

27. From the symmetry of the charge distribution, we know that the electric field must be radial, away from the center of the balloon, with a magnitude independent of the direction. For a Gaussian surface we choose a sphere centered on the balloon with radius  $r = 50$  cm. On this surface, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ . When we apply Gauss' law, we get

$$\begin{aligned}\oint \vec{E} \cdot d\vec{A} &= E4\pi r^2 = Q/\epsilon_0, \text{ which gives} \\ E &= (1/4\pi\epsilon_0)(Q/r^2) \\ &= (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5 \times 10^{-7} \text{ C})/(0.50 \text{ m})^2 \\ &= 1.8 \times 10^4 \text{ N/C}; \\ \vec{E} &= \boxed{(1.8 \times 10^4 \text{ N/C}) \hat{r}}.\end{aligned}$$

If the balloon shrinks, the enclosed charge does not change, so the electric field will be the same:

$$\vec{E} = \boxed{(1.8 \times 10^4 \text{ N/C}) \hat{r}}.$$

28. We assume that the wire is positive. From the symmetry of the charge distribution, we know that the

electric field must be radial, away from the axis, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length  $L$  and radius  $r$ , centered on the axis. On the ends of this surface, the electric field is not constant but  $\vec{E}$  and  $d\vec{A}$  are perpendicular, so we have  $\vec{E} \cdot d\vec{A} = 0$ . On the curved side, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ . For the region between the wire and the cylinder,  $r_1 < r < r_2$ , we apply Gauss' law:

$$\oiint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0;$$

$E2\pi rL = \lambda L/\epsilon_0$ , because only the wire is inside the Gaussian surface, which gives

$$E = \lambda/2\pi\epsilon_0 r$$

$$= (8.5 \times 10^{-9} \text{ C/cm})(10^2 \text{ cm/m})/2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)r = \boxed{(1.6 \times 10^4/r) \text{ N/C radially out.}}$$

If the radius of the Gaussian surface is reduced to the surface of the wire,  $\lambda$  does not change, so we have

$$E_{\text{wire}} = (1.6 \times 10^4)/(5.0 \times 10^{-5} \text{ m}) = \boxed{3.1 \times 10^8 \text{ N/C.}}$$

If the radius of the Gaussian surface increases to the inner surface of the cylinder,  $\lambda$  does not change, so

$$E_{\text{cylinder}} = (1.6 \times 10^4)/(3 \times 10^{-2} \text{ m}) = \boxed{5.1 \times 10^5 \text{ N/C}}, \text{ much less than that in the wire.}$$

29. From the symmetry of the charge distribution, we know that the electric field must be radial, away from the axis of the cylinder, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length  $L$  and radius  $r$ , centered on the axis. On the ends of this surface, the electric field is not constant but  $\vec{E}$  and  $d\vec{A}$  are perpendicular, so we have  $\vec{E} \cdot d\vec{A} = 0$ . On the curved side, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ . For the region where  $r < r_1$ , we apply Gauss' law:

$$\oiint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0;$$

$E2\pi rL = 0$ , because there is no charge inside the Gaussian surface, which gives

$$E = \boxed{0 \text{ for } r < r_1}.$$

For the region where  $r_1 < r < r_2$ , we apply Gauss' law:

$$\oiint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0;$$

$E2\pi rL = \rho(\pi r^2 - \pi r_1^2)L/\epsilon_0$ , which gives

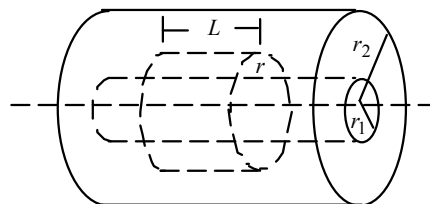
$$\vec{E} = \boxed{[\rho(r^2 - r_1^2)/2\epsilon_0 r]\hat{r} \text{ for } r_1 < r < r_2}.$$

For the region where  $r_2 < r$ , we apply Gauss' law:

$$\oiint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\epsilon_0;$$

$E2\pi rL = \rho(\pi r_2^2 - \pi r_1^2)L/\epsilon_0$ , which gives

$$\vec{E} = \boxed{[\rho(r_2^2 - r_1^2)/2\epsilon_0 r]\hat{r} \text{ for } r_2 < r}.$$



30. The electric field just outside a charged surface is

$$E = \sigma/\epsilon_0, \text{ where } \sigma \text{ is the charge per unit area.}$$

Because the charge density is uniform, for the total charge we have

$$\begin{aligned} q &= \sigma A = \epsilon_0 E(2\pi R h + 2\pi R^2) = \epsilon_0 E 2\pi R(h + R) \\ &= (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.0 \times 10^6 \text{ N/C})2\pi(0.40 \times 10^{-2} \text{ m})[(7.0 \text{ cm} + 0.40 \text{ cm})(10^{-2} \text{ m/cm})] \\ &= \boxed{3.3 \times 10^{-8} \text{ C.}} \end{aligned}$$

31. From the symmetry of the charge distribution, we know that the electric field must be radial, with a magnitude independent of the direction. For a Gaussian surface we choose a sphere of radius  $r$ . On this surface, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ . The charge density is
- $$\rho = Q / [\pi(R_2^3 - R_1^3)].$$

For the region where  $r < R_1$ , we apply Gauss' law:

$$\oint \vec{E} \cdot d\vec{A} = EA = Q / \epsilon_0;$$

$E4\pi r^2 = 0$ , because there is no charge inside the Gaussian surface, which

gives

$$E = 0 \text{ for } r < R_1.$$

For the region where  $R_1 < r < R_2$ , we apply Gauss' law:

$$\oint \vec{E} \cdot d\vec{A} = EA = Q / \epsilon_0;$$

$E4\pi r^2 = \rho \pi(r^3 - R_1^3) / \epsilon_0$ , which gives

$$E = \rho \pi(r^3 - R_1^3) / (4\pi\epsilon_0 r^2) = Q(r^3 - R_1^3) / 4\pi\epsilon_0(R_2^3 - R_1^3)r^2;$$

$$\vec{E} = [Q(r^3 - R_1^3) / 4\pi\epsilon_0(R_2^3 - R_1^3)r^2] \hat{r} \text{ for } R_1 < r < R_2.$$

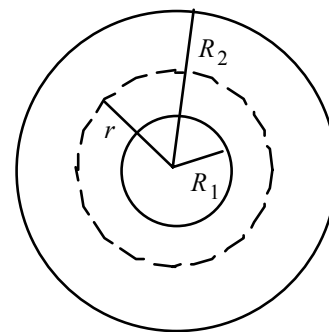
For the region where  $R_2 < r$ , we apply Gauss' law:

$$\oint \vec{E} \cdot d\vec{A} = EA = Q / \epsilon_0;$$

$E4\pi r^2 = Q / \epsilon_0$ , which gives

$$E = (Q / 4\pi\epsilon_0 r^2);$$

$$\vec{E} = (Q / 4\pi\epsilon_0 r^2) \hat{r} \text{ for } R_2 < r.$$



32. (a) From Gauss' law  $\Phi = \oint \vec{E} \cdot d\vec{A} = EA = E(4\pi r^2) = Q / \epsilon_0$ , so the electric field a distance  $r$  from the center of the shell is

$$E = Q / 4\pi\epsilon_0 r^2,$$

where  $Q$  is the net charge enclosed in the spherical region of radius  $r$ . If we choose  $r$  to be just slightly greater than  $R$ , so that the Gaussian surface is inside the metal shell, then  $E = 0$ , and hence  $Q = 0$ . But  $Q$  is the sum of  $q$  and the charge  $q'$  on the inner surface of the shell; i.e.,

$Q = q + q' = 0$ . Thus  $q' = -q$  on the inner surface of the shell.

- (b) Since the shell as a whole is charge-neutral, the charge on its outer surface must be  $+q$ .

- (c) For  $r < d < R$  the charge enclosed by the Gaussian surface is  $Q = q$ , so

$$E = q / 4\pi\epsilon_0 r^2, \text{ radially outward if } q > 0 \text{ and inward if } q < 0.$$

33. (a) The electric field due to a single infinite plane of charge density  $\sigma$  is  $\sigma / 2\epsilon_0$ , as we learned from the textbook. If we set up an  $x$ -axis pointing perpendicularly from the first sheet (with charge density  $\sigma_1$ ) to the second sheet (with charge density  $\sigma_2$ ), where the first sheet is located at  $x = 0$  and the second one at  $x = a$ , then by superposition

$$\begin{aligned} \vec{E} &= \frac{-(\sigma_1 / 2\epsilon_0 + \sigma_2 / 2\epsilon_0)}{\hat{i}} & (x < 0) \\ \vec{E} &= \frac{(\sigma_1 / 2\epsilon_0 - \sigma_2 / 2\epsilon_0)}{\hat{i}} & (0 < x < a) \\ \vec{E} &= \frac{(\sigma_1 / 2\epsilon_0 + \sigma_2 / 2\epsilon_0)}{\hat{i}} & (x > a) \end{aligned}$$

- (b) Since the metal plate is uncharged, when placed in between the two charged plates the densities of the induced charges on both of its surfaces must have the same magnitude and opposite signs. The net electric field due to this pair of surfaces is zero outside the metal, so  $\vec{E}$  remains the same as part

(a) above, except for the interior of the metal, where the field of the induced charge is not zero and cancels with that of the two charged plates, resulting in a zero net field.

34. The positive sheet produces an electric field directed away from the sheet with a magnitude

$$E_+ = \sigma_+/2\epsilon_0 = (5 \times 10^{-6} \text{ C/m}^2)/2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 2.8 \times 10^5 \text{ N/C}.$$

The negative sheet produces an electric field directed toward the sheet with a magnitude

$$E_- = \sigma_-/2\epsilon_0 = (3 \times 10^{-6} \text{ C/m}^2)/2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 1.7 \times 10^5 \text{ N/C}.$$

Between the sheets, the two fields are in the same direction, so we have

$$\begin{aligned} E_{\text{between}} &= E_+ + E_- = 2.8 \times 10^5 \text{ N/C} + 1.7 \times 10^5 \text{ N/C} \\ &= \boxed{4.5 \times 10^5 \text{ N/C toward the negative sheet}}. \end{aligned}$$

Outside the sheets, the two fields are in opposite directions, so we have

$$E_{\text{outside}} = E_+ - E_- = 2.8 \times 10^5 \text{ N/C} - 1.7 \times 10^5 \text{ N/C} = \boxed{1.1 \times 10^5 \text{ N/C away from the sheets}}.$$

35. The positive sheet produces an electric field directed away from the sheet with a magnitude

$$\begin{aligned} E_+ &= \sigma_+/2\epsilon_0 \\ &= (5 \times 10^{-6} \text{ C/m}^2)/2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 2.8 \times 10^5 \text{ N/C}. \end{aligned}$$

The negative sheet produces an electric field directed toward the sheet with a magnitude

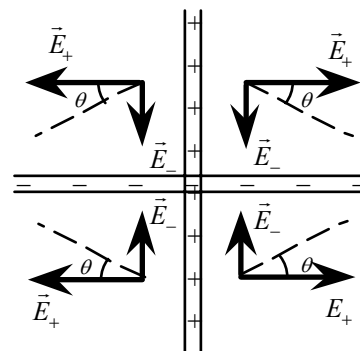
$$\begin{aligned} E_- &= \sigma_-/2\epsilon_0 \\ &= (3 \times 10^{-6} \text{ C/m}^2)/2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 1.7 \times 10^5 \text{ N/C}. \end{aligned}$$

If we consider the 1st quadrant, because the fields are perpendicular, we have

$$\begin{aligned} E &= (E_+^2 + E_-^2)^{1/2} \\ &= [(2.8 \times 10^5 \text{ N/C})^2 + (1.7 \times 10^5 \text{ N/C})^2]^{1/2} = \boxed{3.3 \times 10^5 \text{ N/C}} \quad \text{and} \\ \tan \theta &= E_-/E_+ = (1.7 \times 10^5 \text{ N/C})/(2.8 \times 10^5 \text{ N/C}) = 0.61, \text{ which gives } \theta = \boxed{31^\circ}. \end{aligned}$$

From the diagram, we see that the fields in the other quadrants are mirror images of the field in the 1st quadrant:

$$\begin{array}{ll} \text{1st quadrant: } E \text{ at } -\theta; & \text{2nd quadrant: } E \text{ at } 180^\circ + \theta; \\ \text{3rd quadrant: } E \text{ at } 180^\circ - \theta; & \text{4th quadrant: } E \text{ at } \theta. \end{array}$$



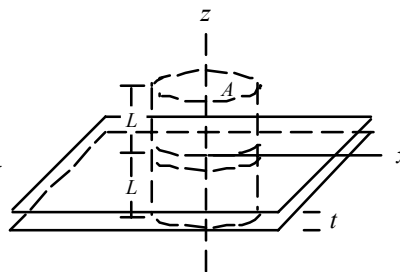
36. Because the slab is infinite, we know from symmetry that the field must be perpendicular to the slab, with a constant magnitude for a constant distance from the slab. If  $r$  is positive, the field will be away from the slab. For a Gaussian surface we choose a cylinder of length  $2L$  and area  $A$ , centered on the axis. On the curved side of this surface, the electric field is not constant but  $\vec{E}$  and  $d\vec{A}$  are perpendicular, so  $\vec{E} \cdot d\vec{A} = 0$ . On the ends, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so  $\vec{E} \cdot d\vec{A} = E dA$ .

To find the field outside the slab, we use the fact that the field will be away from the slab. If we place our Gaussian cylinder so that one end is at  $z = L$  and the other end is at  $z = -L$ , the fields at each end will be directed out of the surface and have the same magnitude. When we apply Gauss' law, we have

$$\begin{aligned} \oiint \vec{E} \cdot d\vec{A} &= \iint_{\text{top}} \vec{E} \cdot d\vec{A} + \iint_{\text{bottom}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} \\ &= EA + EA + 0 = Q/\epsilon_0; \quad \text{which gives} \end{aligned}$$

$$E = \boxed{\rho t/2\epsilon_0 \text{ away from the slab, where } z > t/2 \text{ or } z < -t/2}.$$
 The field is uniform outside the slab.

To find the field inside the slab, we use the same Gaussian surface, with the ends of the cylinder at  $\pm z$ ,  $z < t/2$ . The enclosed charge is only that part of the slab inside the cylinder. Apply Gauss' law:



$$\begin{aligned} \oiint \vec{E} \cdot d\vec{A} &= \iint_{\text{top}} \vec{E} \cdot d\vec{A} + \iint_{\text{bottom}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} \\ &= EA + EA + 0 = Q/\epsilon_0; \text{ which gives} \\ \vec{E} &= \left( \rho/\epsilon_0 \right) \hat{k} \quad \text{where} \quad -t/2 < z < t/2. \end{aligned}$$

37. From the symmetry of the charge distribution, we know that the electric field must be radial, with a magnitude independent of the direction. For a Gaussian surface we choose a sphere of radius  $r$ . On this surface, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have  $\vec{E} \cdot d\vec{A} = E dA$ . The inner charge density is

$$\begin{aligned} \rho_1 &= Q_1/(\pi R_1^3) = 3Q_1/4\pi R_1^3 \\ &= 3(-2 \times 10^{-6} \text{ C})/4\pi(0.03 \text{ m})^3 = -1.77 \times 10^{-2} \text{ C/m}^3. \end{aligned}$$

The outer surface charge density is

$$\sigma_2 = Q_2/4\pi R_2^2 = (5 \times 10^{-6} \text{ C})/4\pi(0.08 \text{ m})^2 = 6.2 \times 10^{-5} \text{ C/m}^2.$$

For the region where  $r < R_1$ , we apply Gauss' law:

$$\begin{aligned} \oint \vec{E} \cdot d\vec{A} &= EA = Q/\epsilon_0; \quad E4\pi r^2 = \rho_1(\pi r^3)/\epsilon_0, \text{ which gives} \\ E &= \rho_1 r/3\epsilon_0 = [(-1.77 \times 10^{-2} \text{ C/m}^3)/3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)]r = (-6.7 \times 10^8) r \text{ N/C with } r \text{ in m;} \\ \vec{E} &= \boxed{(-6.7 \times 10^8 \hat{r}) \text{ N/C with } r \text{ in m}}, \text{ where } r < 3 \text{ cm.} \end{aligned}$$

For the region where  $R_1 < r < R_2$ , we apply Gauss' law:

$$\begin{aligned} \oint \vec{E} \cdot d\vec{A} &= EA = Q/\epsilon_0; \quad E4\pi r^2 = Q_1/\epsilon_0, \text{ which gives} \\ E &= (1/4\pi\epsilon_0)Q_1/r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-2 \times 10^{-6} \text{ C})/r^2 = (-1.8 \times 10^4)/r^2 \text{ N/C with } r \text{ in m;} \\ \vec{E} &= \boxed{[(-1.8 \times 10^4/r^2) \hat{r}] \text{ N/C with } r \text{ in m}}, \text{ where } 3 \text{ cm} < r < 8 \text{ cm.} \end{aligned}$$

For the region where  $R_2 < r$ , we apply Gauss' law:

$$\begin{aligned} \oint \vec{E} \cdot d\vec{A} &= EA = Q/\epsilon_0; \quad E4\pi r^2 = (Q_1 + Q_2)/\epsilon_0, \text{ which gives} \\ E &= (Q_1 + Q_2)/4\pi\epsilon_0 r^2 = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)[(-2 \times 10^{-6} \text{ C}) + (5 \times 10^{-6} \text{ C})]/r^2 \text{ N/C} \\ &= (2.7 \times 10^4)/r^2 \text{ N/C with } r \text{ in m;} \\ \vec{E} &= \boxed{[(2.7 \times 10^4/r^2) \hat{r}] \text{ N/C with } r \text{ in m}}, \text{ where } 8 \text{ cm} < r. \end{aligned}$$

38. The charge within the sphere with  $r = a$  is  $Q_1 = \rho_0(\pi a^3) = \pi a^3 \rho_0$ .

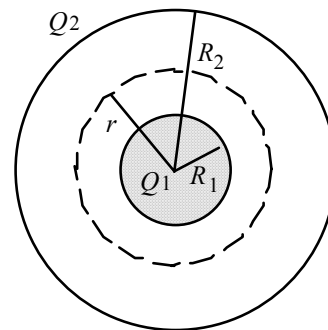
We find the charge within the spherical shell from  $r = a$  to  $r = R$  by choosing a spherical shell of radius  $r$  and thickness  $dr$ , and integrating:

$$\begin{aligned} Q_2 &= \int_a^R \rho_0 4\pi r^2 dr = \int_a^R \rho_0 \left( \frac{r-R}{a-R} \right) 4\pi r^2 dr = \frac{4\pi\rho_0}{(a-R)} \int_a^R (r-R)r^2 dr \\ &= \frac{4\pi\rho_0}{(a-R)} \left[ \frac{1}{4}(R^4 - a^4) - \frac{R}{3}(R^3 - a^3) \right] = \frac{\pi\rho_0}{3} \frac{(4Ra^3 - 3a^4 - R^4)}{(a-R)}. \end{aligned}$$

The flux through each of the spherical surfaces depends only on the enclosed charge, so we have

$$\begin{aligned} \Phi_{r=a} &= Q_{\text{enclosed}}/\epsilon_0 = Q_1/\epsilon_0 = \boxed{\pi a^3 \rho_0/\epsilon_0} \\ \Phi_{r=R} &= Q_{\text{enclosed}}/\epsilon_0 = (Q_1 + Q_2)/\epsilon_0 \\ &= (4\pi a^3 \rho_0/3\epsilon_0) + (\pi\rho_0/3\epsilon_0)[(4Ra^3 - 3a^4 - R^4)/(a-R)] \\ &= (\pi\rho_0/3\epsilon_0)[(4a^4 - 4a^3R + 4Ra^3 - 3a^4 - R^4)/(a-R)] \\ &= (\pi\rho_0/3\epsilon_0)[(a^4 - R^4)/(a-R)] = \boxed{(\pi\rho_0/3\epsilon_0)(a^2 + R^2)(a+R)} \\ \Phi_{r=10R} &= Q_{\text{enclosed}}/\epsilon_0 = (Q_1 + Q_2)/\epsilon_0 = \Phi_{r=R} = \boxed{(\pi\rho_0/3\epsilon_0)(a^2 + R^2)(a+R)}. \end{aligned}$$

From the symmetry of the charge distribution, we know that the electric field must be radial, with a magnitude independent of the direction. On each of the spheres, the field has a constant magnitude and  $\vec{E}$  and  $d\vec{A}$  are parallel, so we have



$$\Phi = \oint \vec{E} \cdot d\vec{A} = EA, \text{ or } E = \Phi/A.$$

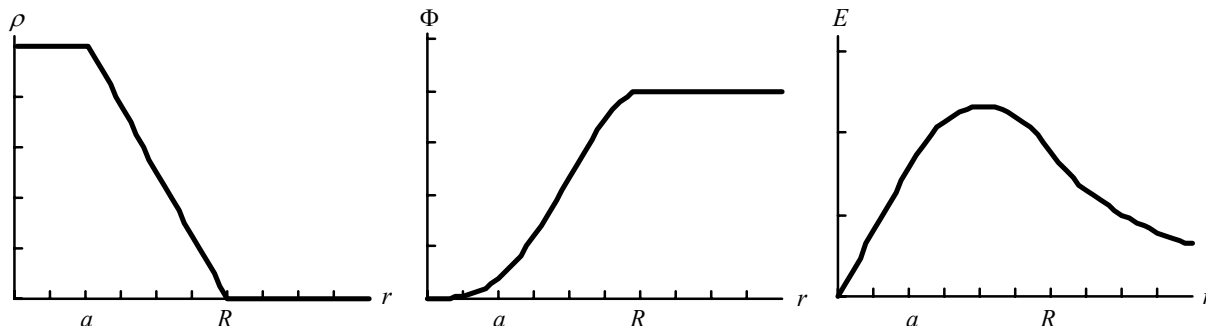
$$E_{r=a} = \Phi_{r=a}/4\pi a^2 = \boxed{\rho_0 a/3\epsilon_0 \text{ radial}}.$$

$$E_{r=R} = \Phi_{r=R}/4\pi R^2 = \boxed{(\rho_0/12\epsilon_0 R^2)(a^2 + R^2)(a + R) \text{ radial}}.$$

$$E_{r=10R} = \Phi_{r=10R}/4\pi(10R)^2 = \boxed{(\rho_0/1200\epsilon_0 R^2)(a^2 + R^2)(a + R) \text{ radial}}.$$

39. If we apply the solution for Problem 38 to the general point  $r$ , we have

	Charge density	Flux	Electric field
$r < a$ :	$\rho_0$	$(4\pi\rho_0/3\epsilon_0)r^3$	$(\rho_0/3\epsilon_0)r$
$a < r < R$ :	$\rho_0(r - R)/(a - R)$	$(\pi\rho_0/3\epsilon_0)(a^4 - 4ar^3 - 3r^4)/(a - r)$	$(\rho_0/12\epsilon_0)(a^4 - 4ar^3 - 3r^4)/(a - r)r^2$
$R < r$ :	0	$(\pi\rho_0/3\epsilon_0)(a^2 + R^2)(a + R)$	$(\rho_0/12\epsilon_0)(a^2 + R^2)(a + R)/r^2$



40. Each plate produces a downward uniform electric field, so the electric field between the plates is

$$E = (\sigma_+ + \sigma_-)/2\epsilon_0 = [(6.5 + 4.8) \times 10^{-6} \text{ C/m}^2]/2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \\ = \boxed{6.4 \times 10^5 \text{ N/C away from the positive plate}}.$$

41. In the region where  $r < R$ , we are inside both spherical shells, so we must have

$$r < R: \vec{E} = \vec{0}.$$

In the region where  $R < r < 2R$ , we are outside the inner shell, so it looks like a point charge; we are inside the outer shell, so it contributes no field:

$$R < r < 2R: \vec{E} = \boxed{(q/4\pi\epsilon_0 r^2) \hat{r}}.$$

In the region where  $2R < r$ , we are outside both shells, so each one looks like a point charge, or a net charge of  $-q$ :

$$2R < r: \vec{E} = \boxed{-(q/4\pi\epsilon_0 r^2) \hat{r}}.$$

42. The electric field between the plates is

$$E = \sigma/\epsilon_0, \text{ so the charge on each plate is}$$

$$Q = \sigma A = \epsilon_0 A E \\ = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.1 \text{ m})^2(3 \times 10^6 \text{ N/C}) = \boxed{2.7 \times 10^{-7} \text{ C}}.$$

43. For a conducting spherical surface, the radial electric field just outside the surface is

$$E = \sigma/\epsilon_0, \text{ so we have}$$

$$\sigma_{\text{max}} = \epsilon_0 E_{\text{max}} = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3 \times 10^6 \text{ N/C}) = \boxed{2.7 \times 10^{-5} \text{ C/m}^2}.$$

44. The flux through a Gaussian surface depends on the enclosed charge.

For the surface at a radius of 50 cm, we have

$$\Phi_2 = (Q_{\text{sphere}} + Q_{\text{shell}})/\epsilon_0.$$

For the surface at a radius of 30 cm, we have

$$\Phi_1 = Q_{\text{sphere}}/\epsilon_0.$$

For the ratio we have



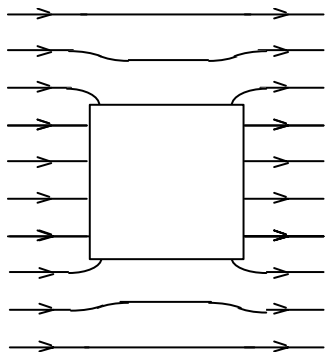
$$\Phi_2/\Phi_1 = (Q_{\text{sphere}} + Q_{\text{shell}})/Q_{\text{sphere}} = 1 + Q_{\text{shell}}/Q_{\text{sphere}};$$

$$(1.6 \times 10^{-7} \text{ N} \cdot \text{m}^2/\text{C})/(0.8 \times 10^{-7} \text{ N} \cdot \text{m}^2/\text{C}) = 1 + Q_{\text{shell}}/Q_{\text{sphere}}, \text{ which gives } \boxed{Q_{\text{sphere}}/Q_{\text{shell}} = 1}.$$

45. For the ratio of charge densities, we have

$$\sigma_{\text{sphere}}/\sigma_{\text{shell}} = (Q_{\text{sphere}}/Q_{\text{shell}})(R_{\text{shell}}/R_{\text{sphere}})^2 = (1)(35 \text{ cm}/25 \text{ cm})^2 = \boxed{1.96}.$$

46.



47. From Gauss' law, we know that the flux through a Gaussian surface depends on the enclosed charge:

$$\Phi = Q_{\text{enclosed}}/\epsilon_0.$$

For the region where  $a < r < b$ , we have  $\Phi_1 = Q/\epsilon_0$ , so the enclosed charge is  $Q$ . This must be the total charge on the inner sphere. Because the sphere is conducting, the charge is located uniformly on the surface, with the charge density

$$\sigma_{\text{inner sphere}} = \boxed{Q/4\pi a^2}.$$

For the region within the shell,  $b < r < R$ , the electric flux is 0, so the net enclosed charge is 0. Because there is a charge  $Q$  on the inner sphere, there must be a charge  $-Q$  on the inner surface of the shell. Because the shell is conducting, the charge is located uniformly on the surface, with the charge density

$$\sigma_{\text{shell, inside}} = \boxed{-Q/4\pi b^2}.$$

For the region outside the shell,  $R < r$ , we have  $\Phi_2 = 2Q/\epsilon_0$ , so the net enclosed charge is  $2Q$ . Because there is a charge  $Q$  on the inner sphere, there must be a charge  $Q$  on the shell. Because there is a charge  $-Q$  on the inner surface, there must be a charge  $+2Q$  on the outer surface, located uniformly on the surface, with the charge density

$$\sigma_{\text{shell, outside}} = +2Q/4\pi R^2 = \boxed{+Q/2\pi R^2}.$$

48. If we choose a Gaussian surface around the earth, we have

$$\oint \vec{E} \cdot d\vec{A} = -EA = Q/\epsilon_0, \text{ so we have}$$

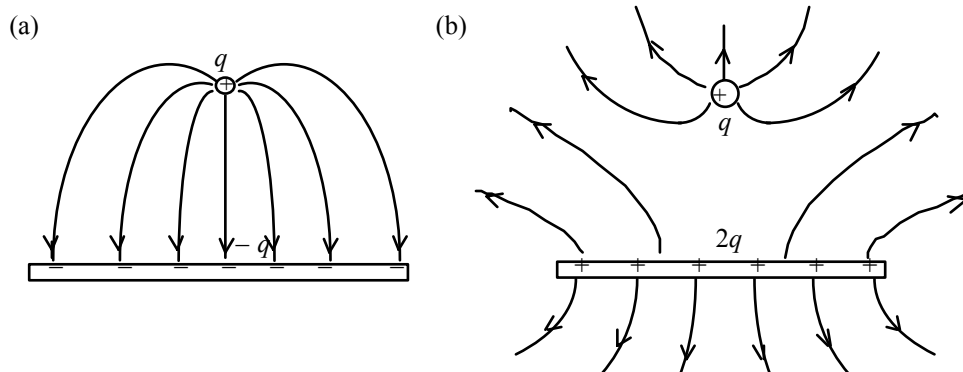
$$Q = -\epsilon_0 4\pi R^2 E$$

$$= -[1/(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)](6.37 \times 10^6 \text{ m})^2(100 \text{ N/C}) = \boxed{-4.5 \times 10^5 \text{ C, on the surface.}}$$

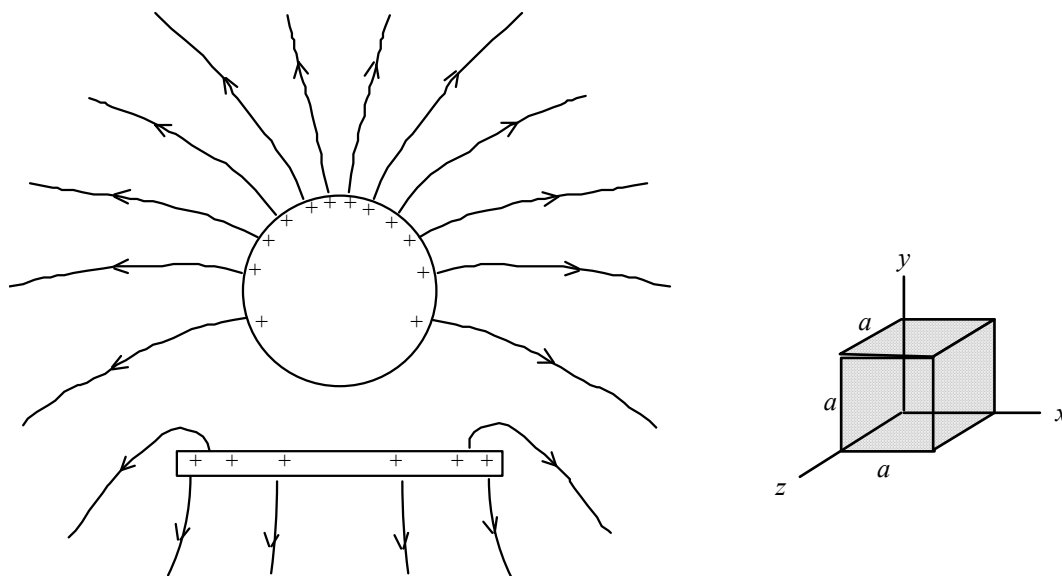
The surface charge density is

$$\sigma = Q/A = \epsilon_0 E = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(-100 \text{ N/C}) = \boxed{-8.9 \times 10^{-10} \text{ C/m}^2}.$$

49.



50.



51. We find the flux through a side from

$$\Phi = \iint \vec{E} \cdot d\vec{A}.$$

For the sides perpendicular to the  $x$ -axis, we have

$$\Phi_{x=0} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (-dA \hat{i}) = -b(0)^2 a^2 = \boxed{0};$$

$$\Phi_{x=a} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (+dA \hat{i}) = b(a)^2 a^2 = \boxed{ba^4}.$$

For the sides parallel to the  $x$ -axis, we have

$$\Phi_{y=0} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (-dA \hat{j}) = \boxed{0};$$

$$\Phi_{y=a} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (+dA \hat{j}) = \boxed{0};$$

$$\Phi_{z=0} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (-dA \hat{k}) = \boxed{0};$$

$$\Phi_{z=a} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (+dA \hat{k}) = \boxed{0}.$$

We use Gauss' law to find the enclosed charge:

$$\Phi = \oint \vec{E} \cdot d\vec{A} = q/\epsilon_0;$$

$$ba^4 = q/\epsilon_0, \text{ which gives } q = \boxed{\epsilon_0 ba^4}.$$

52. From Example 23-7, we know that the electric field inside the sphere is

$$E = (Q/4\pi\epsilon_0)(r/R^3) \text{ radial.}$$

Because the charges have opposite signs, the force on the point charge is toward the center of the sphere, with magnitude

$$F = qQr/4\pi\epsilon_0 R^3,$$

and is a restoring force proportional to the displacement from the center, as in simple harmonic motion, with an effective force constant of

$$k = qQ/4\pi\epsilon_0 R^3.$$

The resulting motion, with  $r = R$  at  $t = 0$ , is

$$r = R \cos(\omega t), \quad \text{with } \omega = (k/m)^{1/2}; \text{ the motion is simple harmonic.}$$

The period of the motion is

$$\tau = 2\pi/\omega = 2\pi(m/k)^{1/2} = 2\pi(4\pi\epsilon_0 m R^3/qQ)^{1/2}.$$

The total energy is the initial potential energy:

$$E = U = \frac{1}{2}kR^2 = \frac{1}{2}(qQ/4\pi\epsilon_0 R^3)R^2 = \frac{qQ}{8\pi\epsilon_0 R}.$$

53. (a) Since no charge is present in the region enclosed by the cap and the flat surface, any electric field lines passing through the flat surface will also pass through the cap. So the electric flux through the flat surface is the same as that through the cap. Since the area of the cap is 0.067, or 6.7%, of that of the sphere, the flux  $\Phi$  through the cap is also 6.7% of  $\Phi_0$ , the flux through the entire sphere:

$$\Phi = 0.067 \Phi_0 = 0.067(Q/\epsilon_0),$$

where we noted that  $\Phi_0 = Q/\epsilon_0$ , due to Gauss' Law.

- (b) Any point on the boundary of the flat surface is also on the Gaussian sphere, so it is a distance  $R$  from the charge  $Q$ . The magnitude of the electric field there is therefore

$$E = Q/4\pi\epsilon_0 R^2.$$

54. (a) The total charge  $Q$  on the sphere satisfies Gauss' law:

$$\Phi = Q/\epsilon_0; \text{ or}$$

$$Q = \Phi\epsilon_0 = (17.1 \text{ N} \cdot \text{m}^2/\text{C}^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \\ = 1.51 \times 10^{-10} \text{ C}.$$

- (b) The electric flux through the upper hemisphere is greater than that through the lower hemisphere, so the charge distribution is non-uniform.

- (c) The sphere is made of insulating material. Otherwise, since it is uniform the charge distribution should also be uniform (as the charges are free to move on the surface of a conductor), and the fluxes through the two hemispheres would be identical.

55. We find the flux through the surface from

$$\Phi = \iint \vec{E} \cdot d\vec{A}.$$

Because  $\vec{E}$  and  $d\vec{A}$  are constant vectors, we have

$$\Phi = \vec{E} \cdot \vec{A} = EA \cos \theta \\ = E(L)(L/\cos \theta)(\cos \theta) = EL^2(\cos \theta/\cos \theta) = EL^2.$$

We choose a Gaussian surface by using the sides of the tube and ends at two different angles. Because there is no charge enclosed, the net flux through the surface is 0. There is no flux through the sides of the tube, so the flux that enters one end must be the same as that which exits the other end, and thus independent of angle.

56. (a) The point  $r = 0.50 \text{ m}$  is outside the inner sphere, so it is equivalent to a point charge. The point is inside the outer sphere, so it makes no contribution to the electric field. The total field is

$$E_a = (Q_{\text{inner}}/4\pi\epsilon_0 r_a^2) + 0 = \sigma_{\text{inner}} 4\pi r_{\text{inner}}^2 / 4\pi\epsilon_0 r_a^2 = \sigma_{\text{inner}} r_{\text{inner}}^2 / \epsilon_0 r_a^2 \\ = (16 \times 10^{-6} \text{ C/m}^2)(0.25 \text{ m})^2 / (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.50 \text{ m})^2$$

$$= \boxed{4.5 \times 10^5 \text{ N/C radial}}$$

- (b) The point  $r = 0.70 \text{ m}$  is outside the inner sphere, so it is equivalent to a point charge. The point is inside the outer sphere, so it makes no contribution to the electric field. The total field is

$$\begin{aligned} E_b &= (Q_{\text{inner}}/4\pi\epsilon_0 r_b^2) + 0 = \sigma_{\text{inner}} 4\pi r_{\text{inner}}^2 / 4\pi\epsilon_0 r_a^2 = \sigma_{\text{inner}} r_{\text{inner}}^2 / \epsilon_0 r_a^2 \\ &= (16 \times 10^{-6} \text{ C/m}^2)(0.25 \text{ m})^2 / (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.70 \text{ m})^2 \\ &= \boxed{2.3 \times 10^5 \text{ N/C radial}} \end{aligned}$$

- (c) Because the outer shell does not contribute to the electric field inside, there will be **no change**.

- (d) The point  $r = 1.0 \text{ m}$  is outside both spheres, so each is equivalent to a point charge.

The total field is

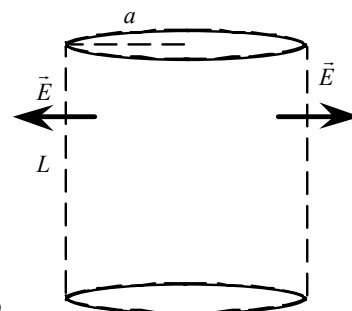
$$\begin{aligned} E_d &= (Q_1 + Q_2)/4\pi\epsilon_0 r_d^2 = (\sigma_{\text{inner}} 4\pi r_{\text{inner}}^2 + \sigma_{\text{outer}} 4\pi r_{\text{outer}}^2) / 4\pi\epsilon_0 r_d^2 \\ &= [(\sigma_{\text{inner}} r_{\text{inner}}^2) + (\sigma_{\text{outer}} r_{\text{outer}}^2)] / \epsilon_0 r_d^2 \\ &= [(16 \times 10^{-6} \text{ C/m}^2)(0.25 \text{ m})^2 + (8 \times 10^{-6} \text{ C/m}^2)(0.75 \text{ m})^2] / [(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.5 \text{ m})^2] \\ &= \boxed{2.8 \times 10^5 \text{ N/C radial}} \end{aligned}$$

57. Because there is no charge enclosed by the tetrahedron, the net flux through all sides is 0:

$$\Phi_{\text{net}} = \Phi_{\text{upper sides}} + \Phi_{\text{bottom}}$$

Thus we find the flux through the three upper sides from

$$\begin{aligned} \Phi_{\text{upper sides}} &= -\Phi_{\text{bottom}} = -(E\hat{k}) \cdot A(-\hat{k}) = +E(L)(L \sin 60^\circ) = \\ &\boxed{0.433EL^2} \end{aligned}$$



58. If we choose a sphere of radius  $r < R$  as a Gaussian surface, we have

$$\oint \vec{E} \cdot d\vec{A} = E4\pi r^2 = Q_{\text{enclosed}}/\epsilon_0, \quad \text{or} \quad Q_{\text{enclosed}} = \epsilon_0 E4\pi r^2.$$

We set up the integral to find the enclosed charge by using a spherical shell of radius  $r'$  and thickness  $dr'$  for the differential element. We also write the right-hand side as an integral:

$$\int_0^r \rho 4\pi r'^2 dr' = \epsilon_0 E4\pi \int_0^r r' dr'.$$

Comparing the two integrands, we see that  $\boxed{\rho \propto 1/r}$ .

As  $r \rightarrow 0$  at the center,  $\boxed{\rho \rightarrow \infty}$ , because the volume of a sphere approaches 0 faster than the area does.

59. From the symmetry of the field we construct a Gaussian surface which is a cylinder of length  $L$  and radius  $a$  with its axis along the axis of the field. Because the field is parallel to the ends of the cylinder, we have

$$\oint \vec{E} \cdot d\vec{A} = E2\pi aL = Q_{\text{enclosed}}/\epsilon_0.$$

If there is a charge distribution  $\rho(r)$  within the cylinder, we have

$$Q_{\text{enclosed}} = \int_0^a 2\pi r L \rho(r) dr. \quad \text{Thus}$$

$$\epsilon_0 E2\pi aL = \int_0^a 2\pi r L \rho(r) dr, \quad \text{or} \quad \epsilon_0 Ea = \int_0^a r \rho(r) dr.$$

We can write the left-hand side as an integral to get

$$\epsilon_0 E \int_0^a dr = \int_0^a r \rho(r) dr.$$

Comparing the two integrands, we see that

$$\rho(r) = \boxed{\epsilon_0 E/r}.$$

Note that this function diverges when  $r \rightarrow 0$ . The required field can be set up only beginning at some distance  $r_0$  from the axis. Within  $r_0$  only the total charge has to correspond to the required field:

$$q/L = \epsilon_0 E 2\pi r_0, \text{ which is finite.}$$

60. (a) For a Gaussian surface within the cylinder just outside the spherical cavity, we have

$$\oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}} / \epsilon_0.$$

Because the field must be 0 inside a conductor, the enclosed charge must be zero. With a charge  $+0.12 \text{ mC}$  at the center, there must be a charge of  $-0.12 \text{ mC}$  on the surface of the cavity.

- (b) There can be no free static charge inside the conductor. If  $-0.12 \text{ mC}$  of the total charge of  $-0.55 \text{ mC}$  on the cylinder resides on the inner surface, the remaining  $-0.43 \text{ mC}$  must be on the outside surface.

61. We can express the linear electric field as  $\vec{E}(x) = bx\hat{i}$ . At  $x = 0.5 \text{ m}$ , we have

$$\vec{E}(0.5 \text{ m}) = (3000 \text{ N/C})\hat{i} = b(0.5 \text{ m})\hat{i}, \text{ which gives } b = 6000 \text{ N/C} \cdot \text{m}.$$

We call the area of the surface oriented in the  $yz$ -plane  $A$ . We choose a Gaussian surface consisting of the boundary of the region parallel to the  $x$ -axis and ends of area  $A$  at  $x = 0$  and  $x = x$ . Because there is no flux through the sides and the field is constant at each end, we have

$$\Phi = \oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}} / \epsilon_0;$$

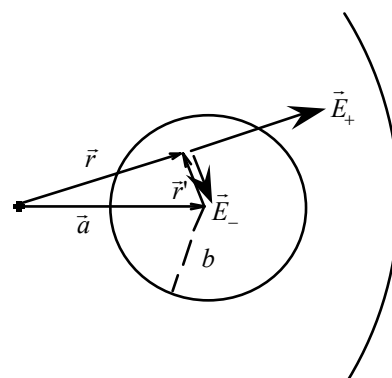
$$E(x)\hat{i} \cdot A\hat{i} + E(0)\hat{i} \cdot A(-\hat{i}) = (1/\epsilon_0) \int \rho A dx, \text{ which gives}$$

$$\int \rho dx = \epsilon_0(bx) = +b\epsilon_0 x.$$

Comparing the two sides, we see that

$$\rho = +b\epsilon_0 = +6000\epsilon_0 \text{ C/m}^3 \text{ (constant)}.$$

Note that for the field to be 0 at  $x = 0$ , there must be external charges at  $x < 0$ .



62. From symmetry, we know that the field inside a uniformly charged sphere must be radial and depends only on the distance from the center. We choose a spherical surface with  $r < R$  for a Gaussian surface. Because  $\vec{E}$  and  $d\vec{A}$  are parallel, we have

$$\oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}} / \epsilon_0;$$

$$E(4\pi r^2) = \rho(\pi r^3) / \epsilon_0, \text{ which gives } E = \rho r / 3\epsilon_0 \text{ radial, or}$$

$$\vec{E} = (\rho / 3\epsilon_0) \vec{r}.$$

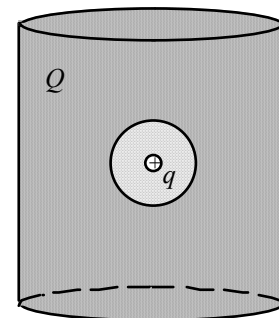
We create the cavity by adding to the original sphere, with density  $\rho$ , a sphere with density  $-\rho$  and radius  $b$ , centered at  $\vec{a}$ . Within the cavity, we are inside both spheres, so their fields are

$$\vec{E}_+ = (\rho / 3\epsilon_0) \vec{r} \quad \text{and} \quad \vec{E}_- = (-\rho / 3\epsilon_0) \vec{r}',$$

where  $\vec{r}'$  is the radius vector for the cavity.

From the diagram, we have  $\vec{r} = \vec{a} + \vec{r}'$ , so the total field is

$$\vec{E} = \vec{E}_+ + \vec{E}_- = (\rho / 3\epsilon_0) \vec{r} + (-\rho / 3\epsilon_0) \vec{r}' = (\rho \vec{r} / 3\epsilon_0) - [\rho(\vec{r} - \vec{a}) / 3\epsilon_0];$$



$$\vec{E} = \left[ (\rho / 3\epsilon_0) \vec{a} \right].$$

Note that the field inside the cavity is uniform.

63. We assume that the positive test charge is at a stable equilibrium point. The electric field there from the other charges is 0. A short distance from the stable equilibrium point, the electric field must be directed toward the point. We choose a small Gaussian surface around the equilibrium point:

$$\oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}} / \epsilon_0.$$

Because the field is directed into the surface, we have

$\oint \vec{E} \cdot d\vec{A} < 0$ , which means that there is a negative charge at the equilibrium point. This is a contradiction, because the only charge there is the positive test charge. Thus the test charge cannot be in stable equilibrium.