

CHAPTER 10 More on Angular Momentum and Torque

Answers to Understanding the Concepts Questions

1. The dimensions of angular momentum are those of mass times speed times distance; that is, $[M(L/T) L] = [M (L/T)^2 T]$. Because the factor $M (L/T)^2$ is the dimension of energy [(mass)(speed)²], the dimensions of angular momentum are indeed those of energy times time, one joule times second is indeed an SI unit for angular momentum.
2. The magnitude of the angular momentum of the comet measured from the center of the Sun is $L = pr$, where r is the shortest distance between the path of the comet and the center of the Sun. Since the comet is headed toward the center of the Sun $r = 0$; so $L = 0$, regardless of the linear momentum p of the comet.
3. If a bicycle is stationary, it has no angular momentum, and a slight imbalance (when the center of mass leaves the vertical line that passes through the contact points between the wheels and the ground) would produce a net torque that causes the bicycle to rotate further to one side. If the bicycle is moving rapidly, it has an angular momentum due to the rotation of the wheels, and the net torque mentioned above would simply cause the direction of the angular momentum to change, and the bicycle would turn instead of falling over. This is analogous to the case of a satellite. If it were to stop moving then Earth's gravitational pull would cause it to fall to the ground; but with sufficient orbital speed that same gravitational force only makes its direction of motion change.
4. Because there is no strong source for a torque on the ball once it starts its flight, the spin maintains its orientation and so does the ball. This suggests the major reasons for putting spin on the ball. First, the effects of air resistance are more predictable if the ball maintains a fixed orientation. In addition, the receiver can more reliably judge the flight of the ball with its fixed orientation, and it is easier to catch.
5. Crouching as low as possible helps lower the center of mass of the system (which consists of the bicycle and the rider), and the lower the center of mass the more stable the system (against toppling over).
6. As the astronauts spin their arms, their bodies spin in an opposite direction in order to maintain their total angular momentum at a constant value (typically zero in this case). When the arms stop spinning, so do their bodies. The fact that after having rotated their bodies the astronauts have changed their orientations in no way contradicts the principle of angular momentum conservation.
7. As a fan spins at a high speed it has a fairly large angular momentum, which points along its axis. If one tries to move it one has to change this angular momentum, which could require a considerable torque.
8. A long and heavy bar has a relatively large rotational inertia. Holding the bar in hand substantially increases the rotational inertia of the system (which consists of the bar and the tightrope walker), so in the event of an unbalanced external torque the resulting angular acceleration of the system would be relatively small, giving the tightrope walker enough time to make necessary adjustments to regain his or her balance.
9. When you use the bent leg technique for lifting, there is little torque on any part of you. In contrast, a bent back must counter the torque you experience from the weight of the lifted object, and it is the muscles of the back and abdomen that must supply the countering torque. The result may be pulled back muscles, or worse, damage to the spinal column.
10. For the sake of clarity, imagine yourself as the astronaut, holding the rotating axle in your hands, with the axle spinning clockwise. The angular momentum of the axle points horizontally away from you, in the positive x -direction: $\vec{L}_i = L\hat{i}$. If you suddenly point the axle upward its angular momentum changes

direction, as it now points upward (z-direction): $\vec{L}_f = L\hat{k}$. The change in the angular momentum of the axles is $\Delta\vec{L} = \vec{L}_f - \vec{L}_i = L(\hat{k} - \hat{i})$. Since you are in outer space there is no external force (or torque) on you, so the total angular momentum of the system (consisting of you and the axle) cannot change. To compensate the change in angular momentum of the axle, your body must start to rotate in such a sense as to generate an angular momentum of $-\Delta\vec{L}$, so as to maintain the total angular momentum of the system. So your axis of spin is along the direction of the vector $\hat{i} - \hat{k}$, which is 45° down from the horizontal.

11. Unless the diver is prepared to use countering revolutions with his or her body (analogous to the technique used by the dropped cat and described in the text), an initial rotation is helpful. The diver will strive to acquire this by using the contact force from the diving board to provide a torque on his or her center of mass. Once there is an initial angular momentum, the diver can use the flexibility of his or her body to change the rate of rotation.
12. As the ball pulls on the rope, the rope in turn pulls the central pole off center with a force tangential to the radius of the pole.
13. Suppose that the ball is initially at rest and the cue strikes it horizontally from the left. The linear impulse on the ball sets it move to the right after the collision. As far as the rotation of the ball is concerned, if the point of impact is above the center of mass of the ball, then the angular impulse delivered by the cue is clockwise, and the ball will start to rotate clockwise after the impact. If the point of impact is below the center of mass of the ball, then the angular impulse delivered by the cue is counterclockwise, causing the ball to rotate counterclockwise after the impact.
14. The hard boiled egg is a solid, and when it is set in rotation internal forces set all the parts in rotation as a rigid body. The large angular momentum is maintained for a long time and the spin resembles that of a top. If the egg has not been cooked, the interior is a liquid, and the shear forces that tend to put the entire egg in rotation when the shell is spun are not entirely effective. The shell may initially spin while the inside does not. The angular momentum is far smaller than it is if the egg is hard-cooked, and the spin rapidly stops. This is a very amusing experiment to try.
15. No. Friction exerted by the ground on your feet provides the torque that keeps your body from spinning in the opposite sense. In the absence of such friction, you would indeed start spinning.
16. Yes. Imagine a uniform bar lying horizontally on a table with the bar's orientation away from you. Then imagine two blows that strike the bar simultaneously, one from the left on the part of the bar closest to you and one from the right on the part of the bar farthest from you. If the blows have equal strength, the net impulse is zero; the center of mass of the bar will not move. But the bar will certainly rotate as the result of a nonzero angular impulse.
17. The magnitude of the angular momentum of a small object about a certain axis is $L = rp$, where p is the magnitude of the linear momentum \vec{p} of that object and r the distance between the vector \vec{p} and the axis. Since the comet moves toward the center of the Sun \vec{p} passes through the axis in our case, i.e., $r = 0$; so $L = 0$.
18. It depends on the type of maneuver being performed. Once airborne the angular momentum of the body about its center of mass cannot change, since the only force that influences the center of mass motion is the weight of the body and it has no lever arm with respect to the center of mass. So, if a particular maneuver requires the body to spin about the center of mass, that would be possible only if the diver has acquired some angular momentum before getting airborne (by pushing against the diving board, for example).
19. We did indeed see that the conservation of the classical angular momentum when there is no net torque is a consequence of Newton's laws, as is the conservation of linear momentum. When quantum physics is involved, the conservation of angular momentum takes on a separate significance.

20. Have the person hold the meter stick horizontally while sitting on the stool. Put the two masses at the center of the meter stick, which coincide with the axis of rotation of the stool. Set the stool into rotation, and use the stopwatch to measure T_i , the time for one revolution. With the stool still rotating, have the person push the two masses to either end of the meter stick. Measure the time T_f for one revolution. Note that the angular momentum of the system consisting of the person (1), the stool (2), and the two masses (3) is essentially conserved. Before the two masses are pushed to the ends of the meter stick $L_i \approx (I_1 + I_2)\omega_i$, and afterwards $L_f \approx (I_1 + I_2 + m_3 d^2)\omega_f$, with d the distance between the center of the meter stick and the end ($d \approx 0.5$ m). From $L_i = L_f$ we get $(I_1 + I_2)\omega_i \approx (I_1 + I_2 + m_3 d^2)\omega_f$, or $(I_1 + I_2)/(I_1 + I_2 + m_3 d^2) \approx \omega_f / \omega_i = T_f / T_i$. Given I_2 , m_3 , d , T_f and T_i , we can then solve for I_1 .
21. The angular momentum associated with the propeller spin is, by application of a right-hand rule, directly away from the pilot. (We reference vector quantities from an origin at the center of the airplane, at the location of the pilot.) In turning to the right, the spin angular momentum is also rotating in that direction; that is, there is a change of angular momentum to the right. To effectuate this change, there must be a torque to the right, and hence a force \vec{F} such that $\vec{r} \times \vec{F}$ is to the right. Since \vec{r} points forward, the force on the nose, where the propeller is located, must be up. Thus the pilot must arrange his ailerons and rudder to introduce a force that will push the nose up, and this will aid the turn.
22. Braking with the rear wheel is a safer choice. If the rider brakes hard with the front wheel from a high speed, the entire bicycle could rotate forward about the contact point between the front wheel and the ground, throwing the rider forward and out of the seat. This could be disastrous.
23. The vertical angular momentum of the system is still conserved. As the student turns the wheel the angular momentum of the wheel is changed, and that must be compensated by the corresponding change in the angular momentum of the student, who must then starts turning, as in the example in the text. The student can only turn half as fast as the previous case, however, as the vertical angular momentum of the wheel has only changed half as much.
24. If we think of the woman and platform as a single isolated system, there are no external forces, so asking for the source of the torque is a bit of a red herring. The system of woman plus platform is one in which the angular momentum is constant. As the woman moves to the center, the rotational inertia of the system decreases, so the platform plus woman must speed up their joint rotation. On the other hand we may think of the platform alone and the woman alone. The only force acting between these two separate systems is the friction at the shoes. If the woman stays at a given radius, then friction from the platform acts centripetally to keep the woman moving in uniform circular motion; by Newton's third law, friction from the woman acts radially outward on the platform and this supplies no torque. However, if the woman moves to the inside of the platform, then her velocity would tend to decrease if the motion continued with the same period, meaning that friction from the platform acts tangentially to slow her down; again by Newton's third law, friction must then act tangentially on the platform, and in a direction that speeds it up. The result is the same as in the description of the woman and platform as a single system.
25. The arrival of the boy increases the rotational inertia of the platform somewhat (as his mass cannot be entirely on the axis of rotation), and if we neglect the effect of friction of the axle then the angular momentum of the boy-platform system is essentially conserved, i.e., $L = I\omega = \text{constant}$. As I increases ω will decrease. However, if the platform is large and massive compared with the size and mass of the boy, then the increase in I would be minimal and ω should not decrease appreciably.
26. We refer to the figure, and think of the origin for the calculation of torques as placed at the center of the flywheel. (a) Without the flywheel, a wave approaching from the side of the boat would lift the side of the boat, so that there is a force that points upward acting at the side of the boat. With the flywheel we want to find the corresponding torque and see how it changes an existing ω . The torque is $\vec{r} \times \vec{F}$, which for \vec{r} pointing from the flywheel center to the side of the boat is oriented to the front of the boat. A vector $\Delta\vec{\omega}$ added to the existing $\vec{\omega}$ reorients $\vec{\omega}$ towards the bow direction, and the boat rotates about a vertical axis to the right with respect to the bow rather than tilting along the long axis of the boat. (b) If a force acts upward at the bow, then \vec{r} runs from the flywheel to the bow, and the torque acts in a direction opposite to the existing $\vec{\omega}$; this has no effect on the motion of the boat.

27. Since the angular momentum is not acquired by the second cylinder, it must be absorbed by Earth.
28. Putting it into a spinning motion provides a prop with some angular momentum (about its center of mass), which basically cannot change once it is airborne (as the only significant external force exerted on it is its weight, which acts through its center of mass and provides no torque about the center of mass.) This helps stabilize its motion.
29. The friction that acts on the ball has two effects: it introduces a torque on the ball and it acts on its center of mass to accelerate the ball. The force is horizontal. We can see whether it is forward or backward by looking at the direction of spin and deducing the effect of the torque. If the spin is forward, then friction must act in the backward direction, and this is sensible. Such a force will accelerate the center of mass in such a way that the horizontal component of the ball's velocity decreases. Let's now assume that the vertical forces acting when the ball hits the floor are elastic, so that the vertical component of the ball reverses but suffers no change in magnitude. The angle the ball's path makes with the horizontal has a tangent that is the ratio of the vertical to the horizontal component of the velocity. If the horizontal component has decreased in magnitude but the vertical component has not, then the angle will be increased on the bounce.
30. Express the rotational kinetic energy K in terms of the spin angular momentum L : $K = \frac{1}{2}I\omega^2 = (L\omega)^2 / 2I = L^2 / 2I$. In the process described $L = \text{constant}$ while I increases. As a result $K = L^2 / 2I$ decreases.

Solutions to Problems

1. (a) From the right-hand rule, the direction of the angular momentum will be down. The magnitude is $L = pd$, where d is the perpendicular distance from the axis to the line of the linear momentum p . Thus

$$\begin{aligned} L &= pd \\ &= (2 \times 10^3 \text{ kg})(200 \text{ km/h})(100 \text{ km})(10^3 \text{ m/km})^2 / (3600 \text{ s/h}) \\ &= \boxed{1.1 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s} \text{ down}}. \end{aligned}$$

- (b) A northeasterly direction is at an angle of 45° with the position line, so
- $$L = (1.1 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s}) \sin 45^\circ = \boxed{7.9 \times 10^9 \text{ kg} \cdot \text{m}^2/\text{s} \text{ down}}.$$

2. The angular momentum is

$$\begin{aligned} \vec{L} &= \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) \\ &= (0.270 \text{ kg})[(0.1\hat{i} - 0.5\hat{j} + 0.2\hat{k}) \text{ m} \times (12\hat{i} - 7\hat{j} - 3\hat{k}) \text{ m/s}] \\ &= (0.270 \text{ kg})(2.9\hat{i} + 2.7\hat{j} + 5.3\hat{k}) \text{ m}^2/\text{s} \\ &= \boxed{(0.78\hat{i} + 0.73\hat{j} + 1.4\hat{k}) \text{ kg} \cdot \text{m}^2/\text{s}}. \end{aligned}$$

3. We assume that the car is traveling east and passing on the north side of you, so the direction of the angular momentum will be down.

- (a) The magnitude is $L = pd$, where d is the perpendicular distance from the axis to the line of the linear momentum p . Thus

$$L = pd = (1000 \text{ kg})(10 \text{ m/s})(5 \text{ m}) = \boxed{5 \times 10^4 \text{ kg} \cdot \text{m}^2/\text{s} \text{ down}}.$$

- (b) Because the perpendicular distance has not changed, the angular momentum is still

$$L = \boxed{5 \times 10^4 \text{ kg} \cdot \text{m}^2/\text{s} \text{ down}}.$$

4. The speed of the orbiting object is given by $v = 2\pi r/T$.

- (a) $L = pd = m(2\pi r/T)r = 2\pi mr^2/T$
- $$= 2\pi(6.0 \times 10^{24} \text{ kg})(1.5 \times 10^{11} \text{ m})^2 / [(1 \text{ yr})(365 \text{ d/yr})(24 \text{ h/d})(3600 \text{ s/h})] = \boxed{2.7 \times 10^{40} \text{ kg} \cdot \text{m}^2/\text{s}}.$$

- (b) $L = 2\pi(7.35 \times 10^{22} \text{ kg})(3.84 \times 10^8 \text{ m})^2 / (27 \text{ d})(24 \text{ h/d})(3600 \text{ s/h}) = \boxed{2.9 \times 10^{34} \text{ kg} \cdot \text{m}^2/\text{s}}.$

- (c) We treat the earth as a rigid sphere, with angular momentum

$$\begin{aligned} L &= I\omega = \frac{2}{5}MR^2\omega \\ &= \frac{2}{5}(6.0 \times 10^{24} \text{ kg})(6.4 \times 10^6 \text{ m})^2[2\pi / (1 \text{ d})(24 \text{ h/d})(3600 \text{ s/h})] \\ &= \boxed{7.1 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}}. \end{aligned}$$

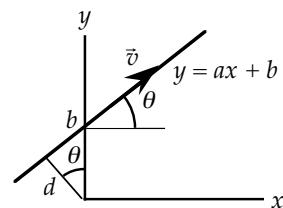
5. We take the direction of the angular momentum to be north along the axis. The angular momentum of rotation about the axis is

$$\begin{aligned} L &= I\omega = mR^2\omega \\ &= (1.8 \text{ kg})(0.18 \text{ m})^2(4.2 \text{ rev/s})(2\pi \text{ rad/rev}) = \boxed{1.5 \text{ kg} \cdot \text{m}^2/\text{s} \text{ north}}. \end{aligned}$$

6. Because the particle is traveling in a straight line, $y = ax + b$, its angular momentum about the origin will be constant. From the diagram the right-hand rule gives its direction as into the page, or $-\hat{k}$. We find the angle θ from $\tan \theta = dy/dx = a$, so $\cos \theta = 1/(1 + a^2)^{1/2}$. The magnitude of the angular momentum is

$$L = pd = mvb \cos \theta = mbv[1/(1 + a^2)^{1/2}]. \text{ Thus}$$

$$\vec{L} = \boxed{-[mbv/\sqrt{1 + a^2}] \hat{k}}.$$



7. For the projectile motion using the coordinate system shown, we find the position and velocity components:

$$x = v_{0x}t = v_x t;$$

$$y = v_{0y}t - \frac{1}{2}gt^2 = 0 - \frac{1}{2}gt^2;$$

$$v_x = \text{constant};$$

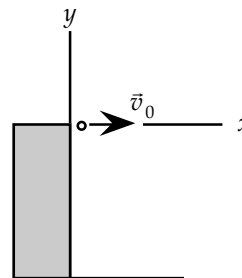
$$v_y = v_{0y} - gt = 0 - gt.$$

When we express these as vectors, we have

$$\vec{r} = v_x t \hat{i} - (\frac{1}{2}gt^2) \hat{j}; \quad \vec{p} = m\vec{v} = mv_x \hat{i} - mgt \hat{j}.$$

The angular momentum is

$$\begin{aligned} \vec{L} &= \vec{r} \times \vec{p} = (v_x t \hat{i} - \frac{1}{2}gt^2 \hat{j}) \times (mv_x \hat{i} - mgt \hat{j}) = -(\frac{1}{2}mgv_x t^2) \hat{k} \\ &= -\frac{1}{2}(0.060 \text{ kg})(9.8 \text{ m/s}^2)(25 \text{ m/s})t^2 \hat{k} \\ &= \boxed{-(7.4t^2 \hat{k}) \text{ kg} \cdot \text{m}^2 / \text{s} \text{ into the page}}. \end{aligned}$$



8. The speed of the axle, the center of mass, is $v = R\omega$. The total angular momentum about a point on the road is the angular momentum of the center of mass and the angular momentum of rotation about the center of mass:

$$\begin{aligned} L &= mvR + I\omega = mR^2\omega + I\omega = (mR^2 + I)\omega \\ &= [(1.5 \text{ kg})(0.38 \text{ m})^2 + (0.28 \text{ kg} \cdot \text{m}^2)](2.5 \text{ rad/s}) = \boxed{1.2 \text{ kg} \cdot \text{m}^2 / \text{s} \text{ along axle}}. \end{aligned}$$

9. From the position, $\vec{r} = \frac{1}{2}at^2 \hat{i} + vt \hat{j} + (\frac{1}{2}bt^2 - wt) \hat{k}$, we find the velocity:

$$\vec{v} = d\vec{r} / dt = at \hat{i} + v \hat{j} + (bt - w) \hat{k}.$$

The angular momentum is

$$\vec{L} = \vec{r} \times \vec{p} = m[\frac{1}{2}at^2 \hat{i} + vt \hat{j} + (\frac{1}{2}bt^2 - wt) \hat{k}] \times [at \hat{i} + v \hat{j} + (bt - w) \hat{k}].$$

When we expand the cross product and simplify, we get

$$\vec{L} = \boxed{\frac{1}{2}(mbvt^2 \hat{i} - mawt^2 \hat{j} - mavt^2 \hat{k})}.$$

10. From the position $\vec{r} = (x_0 + \rho \cos \omega t) \hat{i} + (y_0 + \rho \sin \omega t) \hat{j}$, we find the velocity:

$$\vec{v} = d\vec{r} / dt = (-\rho\omega \sin \omega t) \hat{i} + (\rho\omega \cos \omega t) \hat{j}.$$

The angular momentum is

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = m[(x_0 + \rho \cos \omega t) \hat{i} + (y_0 + \rho \sin \omega t) \hat{j}] \times [(-\rho\omega \sin \omega t) \hat{i} + (\rho\omega \cos \omega t) \hat{j}].$$

This reduces to

$$\begin{aligned} \vec{L} &= m[(x_0 + \rho \cos \omega t)(\rho\omega \cos \omega t) + (y_0 + \rho \sin \omega t)(\rho\omega \sin \omega t)] \hat{k} \\ &= \boxed{m[\rho\omega(x_0 \cos \omega t + y_0 \sin \omega t) + \rho^2 \omega] \hat{k}}. \end{aligned}$$

11. The location of the center of mass is defined by $\vec{R} = (m_1 \vec{r}_1 + m_2 \vec{r}_2) / (m_1 + m_2)$. If $\vec{R} = 0$, we have

$$\vec{r}_1 = -(m_2 / m_1) \vec{r}_2.$$

If we let $\vec{r} = \vec{r}_2 - \vec{r}_1$, we get

$$\vec{r}_2 - \vec{r} = -(m_2 / m_1) \vec{r}_2, \text{ which reduces to } \vec{r}_2 = m_1 \vec{r} / (m_1 + m_2).$$

For the momenta, we have

$$\vec{p}_1 = m_1 \vec{v}_1 = m_1 d\vec{r}_1 / dt \text{ and } \vec{p}_2 = m_2 \vec{v}_2 = m_2 d\vec{r}_2 / dt.$$

That the center of mass is at rest means that $\vec{P} = \vec{p}_1 + \vec{p}_2 = 0$, or $\vec{p}_1 = -\vec{p}_2$.

Using this result, the total angular momentum of the system is

$$\vec{L} = (\vec{r}_1 \times \vec{p}_1) + (\vec{r}_2 \times \vec{p}_2) = (\vec{r}_2 - \vec{r}_1) \times \vec{p}_2, \text{ which we can write as}$$

$$\vec{L} = (\vec{r}_2 - \vec{r}_1) \times (m_2 \vec{v}_2) = (\vec{r}_2 - \vec{r}_1) \times (m_2 d\vec{r}_2 / dt). \text{ In terms of } \vec{r}, \text{ this becomes}$$

$$\vec{L} = \boxed{\vec{r} \times [m_2 m_1 / (m_1 + m_2)] d\vec{r} / dt = \vec{r} \times \vec{\mu} d\vec{r} / dt}.$$

This is the angular momentum of a particle of mass μ at a position \vec{r} .

12. (a) We use the parallel-axis theorem:

$$I = I_{\text{CM}} + Md^2 = 9.8 \times 10^{37} \text{ kg} \cdot \text{m}^2 + (6.0 \times 10^{24} \text{ kg})(1.5 \times 10^{11} \text{ m})^2 \\ = \boxed{1.4 \times 10^{47} \text{ kg} \cdot \text{m}^2}.$$

- (b) The angular momentum is from the orbital motion, as a point mass, and the rotation of the earth:

$$L = Md^2\omega_{\text{Sun}} + I_{\text{CM}}\omega_{\text{Earth}} \\ = (1.4 \times 10^{47} \text{ kg} \cdot \text{m}^2)2\pi / [(365 \text{ d/yr})(24 \text{ h/d})(3600 \text{ s/h})] + \\ (9.8 \times 10^{37} \text{ kg} \cdot \text{m}^2)2\pi / [(24 \text{ h/d})(3600 \text{ s/h})] \\ = \boxed{2.7 \times 10^{40} \text{ kg} \cdot \text{m}^2/\text{s}}.$$

- (c) fractional difference = $[(Md^2\omega_{\text{Sun}} + I_{\text{CM}}\omega_{\text{Earth}}) - Md^2\omega_{\text{Sun}}] / (Md^2\omega_{\text{Sun}} + I_{\text{CM}}\omega_{\text{Earth}})$
 $\approx I_{\text{CM}}\omega_{\text{Earth}} / Md^2\omega_{\text{Sun}} = I_{\text{CM}}T_{\text{Sun}} / Md^2T_{\text{Earth}}$
 $= (9.8 \times 10^{37} \text{ kg} \cdot \text{m}^2)(365 \text{ d}) / [(1.4 \times 10^{47} \text{ kg} \cdot \text{m}^2)(1 \text{ d})]$
 $= \boxed{2.6 \times 10^{-7}}.$

The angular speed of rotation of Earth is 365 times larger than that of its rotation about the Sun, but the Sun-Earth distance is $\approx 10^5$ greater than Earth's radius. Because the rotational inertia depends on the square of the distance, the fractional difference is very small.

13. (a) Because each mass is the same distance from the axis, the rotational inertia of the system is

$$I = 4m(\frac{1}{2}L\sqrt{2})^2 = 2mL^2 = 2(0.1 \text{ kg})(0.20 \text{ m})^2 = \boxed{8.0 \times 10^{-3} \text{ kg} \cdot \text{m}^2}.$$

The angular momentum of the system is

$$\vec{L} = I\vec{\omega} = (8.0 \times 10^{-3} \text{ kg} \cdot \text{m}^2)(8 \text{ rad/s})\hat{k} \\ = \boxed{(6.4 \times 10^{-2} \text{ kg} \cdot \text{m}^2/\text{s})\hat{k} \text{ along } \vec{\omega} \text{-direction}}.$$

- (b) With the coordinate system shown on the diagram, for the mass along the x-axis, we have

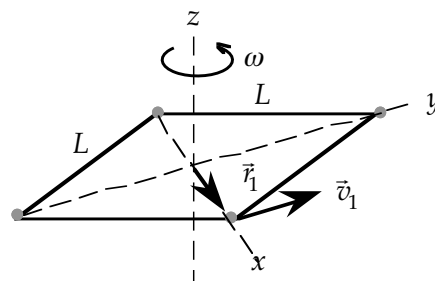
$$\vec{r}_1 = \frac{1}{2}L\sqrt{2}\hat{i}, \text{ and } \vec{v}_1 = \vec{\omega} \times \vec{r}_1 = (\omega\hat{k}) \times (\frac{1}{2}L\sqrt{2}\hat{i}) = \frac{1}{2}\omega L\sqrt{2}\hat{j}.$$

The angular momentum of this mass is

$$\vec{L}_1 = \vec{r}_1 \times m\vec{v}_1 = (\frac{1}{2}L\sqrt{2})m(\frac{1}{2}\omega L\sqrt{2})(\hat{i} \times \hat{j}) = \frac{1}{2}mL^2\omega\hat{k} \\ = \frac{1}{2}(0.1 \text{ kg})(0.20 \text{ m})^2(8 \text{ rad/s})\hat{k} \\ = \boxed{(1.6 \times 10^{-2} \text{ kg} \cdot \text{m}^2/\text{s})\hat{k}}.$$

From the symmetry of the system, each of the other masses will have the same angular momentum, so the total is

$$\vec{L} = 4\vec{L}_1 = \boxed{(6.4 \times 10^{-2} \text{ kg} \cdot \text{m}^2/\text{s})\hat{k}}, \text{ the same as in part (a).}$$



14. For a point a distance
- D
- below the origin, we have

$$\vec{r}_1 = \frac{1}{2}L\sqrt{2}\hat{i} + D\hat{k}, \text{ and}$$

$$\vec{v}_1 = \vec{\omega} \times \vec{r}_1 = (\omega\hat{k}) \times (\frac{1}{2}L\sqrt{2}\hat{i} + D\hat{k}) = \frac{1}{2}\omega L\sqrt{2}\hat{j}.$$

The angular momentum of this mass is

$$\vec{L}_1 = \vec{r}_1 \times m\vec{v}_1 = (\frac{1}{2}L\sqrt{2}\hat{i} + D\hat{k}) \times m(\frac{1}{2}\omega L\sqrt{2}\hat{j}) \\ = \boxed{\frac{1}{2}mL^2\omega\hat{k} - \frac{1}{\sqrt{2}}Dm\omega L\hat{i}}.$$

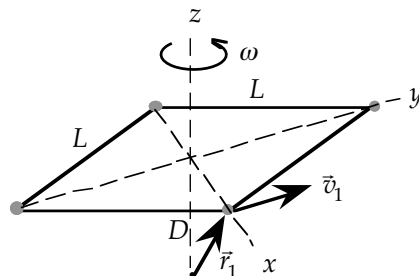
Note that the x-component points toward the axis.

From the symmetry of the system, each of the other masses will have a similar angular momentum with the same z-component.

The other component will have the same magnitude but point toward the axis. When we add the vectors the x- or y-components will cancel, so the total angular momentum is

$$\vec{L} = 4L_{1z}\hat{k} = 4(\frac{1}{2}mL^2\omega\hat{k}) = \boxed{(6.4 \times 10^{-2} \text{ kg} \cdot \text{m}^2/\text{s})\hat{k}},$$

which is the same as in Problem 13.



15. In each case we have $L = I\omega$, so we find the rotational inertia.

(a) All masses are equidistant from the axis, so we have

$$I = 3m[d/(2 \cos 30^\circ)]^2 = md^2.$$

$$L = md\omega d^2 \text{ along the axis of rotation.}$$

(b) Only one mass contributes to the rotational inertia, so we have

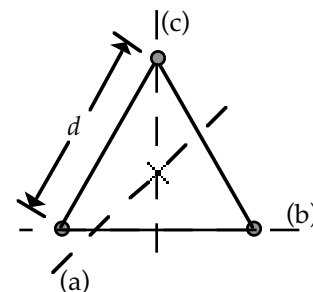
$$I = m(d \cos 30^\circ)^2 = \frac{3}{4}md^2.$$

$$L = \frac{3}{4}m\omega d^2 \text{ along the axis of rotation.}$$

(c) Two masses contribute to the rotational inertia, so we have

$$I = 2m(\frac{1}{2}d)^2 = \frac{1}{2}md^2$$

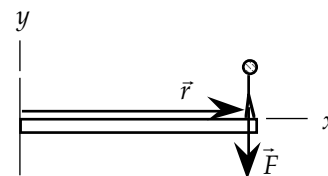
$$L = \frac{1}{2}m\omega d^2 \text{ along the axis of rotation.}$$



16.
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= (3.4 \text{ m})\hat{i} \times [-(72 \text{ kg})(9.8 \text{ m/s}^2)]\hat{j} = -(2.4 \times 10^3 \text{ N} \cdot \text{m})\hat{k}.$$

$$\tau = 2.4 \times 10^3 \text{ N} \cdot \text{m}.$$

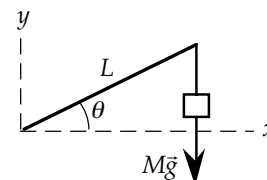


17.
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= (3\hat{i} - \hat{j} - 5\hat{k}) \text{ m} \times (2\hat{i} + 4\hat{j} + 3\hat{k}) \text{ N} = (17\hat{i} - 19\hat{j} + 14\hat{k}) \text{ N} \cdot \text{m}.$$

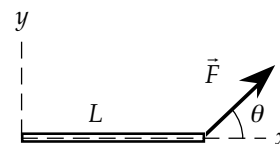
18. With the coordinate system shown, we find the torque from

$$\begin{aligned}\vec{\tau} &= \vec{r} \times \vec{F} \\ &= (L \cos \theta \hat{i} + L \sin \theta \hat{j}) \times (-Mg \hat{j}) \\ &= -MgL \cos \theta \hat{k} \\ &= -(18 \text{ kg})(9.8 \text{ m/s}^2)(2.2 \text{ m})(\cos 20^\circ) \hat{k} \\ &= -(3.6 \times 10^3 \text{ N} \cdot \text{m})\hat{k} \text{ (into page)}.\end{aligned}$$



19. With the coordinate system shown on the diagram, we find the torque from

$$\begin{aligned}\vec{\tau} &= \vec{r} \times \vec{F} \\ &= L\hat{i} \times (F \cos \theta \hat{i} + F \sin \theta \hat{j}) \\ &= LF \sin \theta \hat{k} \\ &= (1.0 \text{ m})(200 \text{ N}) \cos 45^\circ \hat{k} \\ &= (140 \text{ N} \cdot \text{m})\hat{k} \text{ (perpendicular to table)}.\end{aligned}$$



20. For the resultant force, we have

$$\sum \vec{F} = \vec{F} + (-\vec{F}) = 0.$$

For the resultant torque, we have

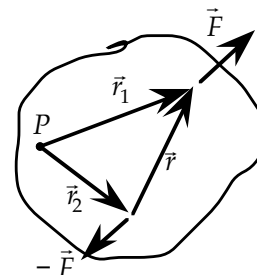
$$\vec{\tau} = \vec{r}_1 \times \vec{F} + \vec{r}_2 \times (-\vec{F}) = (\vec{r}_1 - \vec{r}_2) \times \vec{F}.$$

From the diagram, we see that

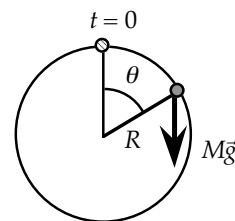
$$\vec{r}_2 + \vec{r} = \vec{r}_1, \text{ or } \vec{r}_1 - \vec{r}_2 = \vec{r}.$$

Thus the torque is

$$\vec{\tau} = \vec{r} \times \vec{F}, \text{ independent of the location of the point } P.$$



21. The location of the mass is given by $\theta = \omega t$.
The torque is along the axle, into the page. Its magnitude is
 $\tau = (R \sin \theta) Mg = \boxed{MgR \sin(\omega t)}$.



22. $\vec{\tau} = \hat{r} \times m\vec{g}$
 $= \{(v \cos \theta) t \hat{i} + [(v \sin \theta) t - \frac{1}{2}gt^2] \hat{j}\} \times (-mg \hat{j}) = \boxed{-(mgvt \cos \theta) \hat{k}}$.
 This is just mg times the perpendicular distance x to the line of mg .

23. The perpendicular distance from the axis A to the initial path of the ball is

$$r_i = d \sin[2(\pi/2 - \theta)] = d \sin(\pi - 2\theta) = d \sin(2\theta).$$

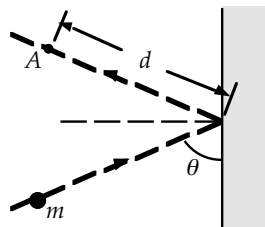
The initial angular momentum is

$$L_i = r_i mv = \boxed{mvd \sin(2\theta) \text{ up}}.$$

Because the final velocity passes through the axis, we have

$$L_f = \boxed{0}.$$

The wall exerts an impulsive force on the ball and thus an impulsive torque that changes the angular momentum.



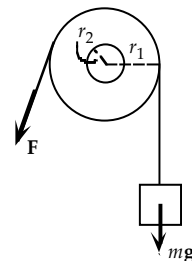
24. At a steady speed, $d\vec{L}/dt = 0$, so we have $\vec{\tau}_{\text{net}} = 0$. Both torques are along the axle, with that due to F out and that due to mg in, which we take as positive. Thus we have

$$\tau_{\text{net}} = 0;$$

$$mgr_1 - Fr_2 = 0, \text{ which gives}$$

$$F = mg(r_1/r_2)$$

$$= (200 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ cm})/(50 \text{ cm}) = \boxed{3.9 \times 10^2 \text{ N}}.$$

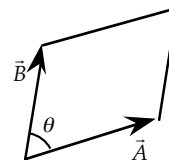


25. $\vec{A} \times \vec{B} = (2\hat{i} - 4\hat{j} + 5\hat{k}) \times (\hat{i} + 3\hat{j} - 2\hat{k})$
 $= [(-4)(-2) - (3)(5)]\hat{i} + [(5)(1) - (2)(-2)]\hat{j} + [(2)(3) - (-4)(1)]\hat{k} = \boxed{-7\hat{i} + 9\hat{j} + 10\hat{k}}.$

26. $|\vec{A} \times \vec{B}| = AB \sin \theta$.

From the diagram we see that $B \sin \theta$ is the height of the parallelogram. The area of the parallelogram is

$$\text{area} = (\text{base})(\text{height}) = (A)(B \sin \theta) = |\vec{A} \times \vec{B}|.$$



27. Before the children land on the merry-go-round, they have zero angular momentum about the axis. From the conservation of angular momentum of the system of merry-go-round and children, we have

$$L = I_1 \omega_1 = I_2 \omega_2;$$

$$(120 \text{ kg} \cdot \text{m}^2)(2.5 \text{ rad/s}) = [(120 \text{ kg} \cdot \text{m}^2) + 2(25 \text{ kg})(1.7 \text{ m})^2] \omega_2, \text{ which gives}$$

$$\omega_2 = \boxed{1.1 \text{ rad/s}}.$$

28. The person, before landing, has zero angular momentum about the axis. From the conservation of angular momentum of the system of disk and person, we have

$$L = I_1\omega_1 = I_2\omega_2;$$

$$\frac{1}{2}(38 \text{ kg})(1.7 \text{ m})^2(0.075 \text{ rad/s}) = [\frac{1}{2}(38 \text{ kg})(1.7 \text{ m})^2 + (71 \text{ kg})(0.9 \text{ m})^2]\omega_2, \text{ which gives}$$

$$\omega_2 = \boxed{0.037 \text{ rad/s}}.$$

29. If r_{\perp} is the perpendicular distance from the parked car to the path of the fire truck, the magnitude of the angular momentum is

$$L = r_{\perp}mv.$$

Because r_{\perp} does not change, we have

$$L_{10}/L_0 = r_{\perp}mv_{10}/r_{\perp}mv_0 = v_{10}/v_0 = 2, \text{ so } v_{10} = 2(15 \text{ m/s}) = \boxed{30 \text{ m/s}}.$$

30. The angular momentum of the skater is conserved:

$$L = I_1\omega_1 = I_2\omega_2; (I_{\text{skater}} + 2mr_1^2)\omega_1 = (I_{\text{skater}} + 2mr_2^2)\omega_2;$$

$$[2.3 \text{ kg} \cdot \text{m}^2 + 2(3\text{kg})(0.8 \text{ m})^2](0.7 \text{ rev/s}) = [2.3 \text{ kg} \cdot \text{m}^2 + 2(3 \text{ kg})(0.4 \text{ m})^2]\omega_2, \text{ which gives}$$

$$\omega_2 = \boxed{1.3 \text{ rev/s}}.$$

31. We choose counterclockwise for the positive rotation. Using the speeds with respect to the ground, from the conservation of angular momentum of the system of turntable and bug, we have

$$L = 0 = I_{\text{turntable}}\omega_{\text{turntable}} + I_{\text{bug}}\omega_{\text{bug}} = \frac{1}{2}m_{\text{turntable}}R^2\omega_{\text{turntable}} + m_{\text{bug}}R^2\omega_{\text{bug}}; \text{ which gives}$$

$$\omega_{\text{turntable}} = -(2m_{\text{bug}}/m_{\text{turntable}})\omega_{\text{bug}}.$$

Because the angular velocities are constant, the angular displacements, $\theta = \omega t$, are related by

$$\theta_{\text{turntable}} = -(2m_{\text{bug}}/m_{\text{turntable}})\theta_{\text{bug}}.$$

When the bug makes one full circle with respect to the turntable, we have

$$\theta_{\text{turntable}} - \theta_{\text{bug}} = 2\pi;$$

$$\theta_{\text{turntable}} - [-(m_{\text{turntable}}/2m_{\text{bug}})\theta_{\text{turntable}}] = 2\pi, \text{ which gives}$$

$$\theta_{\text{turntable}} = 4\pi m_{\text{bug}}/(m_{\text{turntable}} + 2m_{\text{bug}})$$

$$= 4\pi(0.002 \text{ kg})/[(0.24 \text{ kg}) + 2(0.002 \text{ kg})] = \boxed{0.103 \text{ rad } (0.016 \text{ rev})}.$$

32. We use energy conservation to find the speed of the point mass before it strikes the bar:

$$v_1 = (2gh)^{1/2} = [2(9.8 \text{ m/s}^2)(1.1 \text{ m})]^{1/2} = 4.64 \text{ m/s}.$$

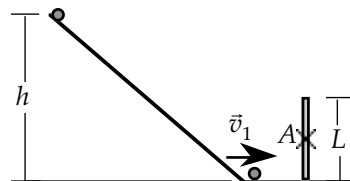
From the conservation of angular momentum of the system of mass and bar about the pivot point A during the collision, we have

$$L_A = mv_1(\frac{1}{2}L) = [(1/12)ML^2 + m(\frac{1}{2}L)^2]\omega;$$

$$(0.017 \text{ kg})(4.64 \text{ m/s})(0.10 \text{ m}) = [(1/12)(0.2 \text{ kg})(0.2 \text{ m})^2 +$$

$$(0.017 \text{ kg})(0.1 \text{ m})^2]\omega, \text{ which gives}$$

$$\omega = \boxed{9.4 \text{ rad/s}}.$$



33. Label the clay 1 and the wheel 2. The initial speed of the clay just before hitting the wheel is

$$v_1 = (2gh)^{1/2} = [2(9.8 \text{ m/s}^2)(0.75 \text{ m})]^{1/2} = 3.834 \text{ m/s}.$$

Relative to the center of the wheel the angular momentum of the clay-wheel system just before their collision is

$$L_i = m_1 v_1 R, \text{ with } R \text{ the radius of the wheel.}$$

After the collision the rotational inertia of the system is $I_f = m_1 R^2 + \frac{1}{2}m_2 R^2$, and so the angular speed ω of the system is obtained from conservation of angular momentum:

$$L_i = m_1 v_1 R = L_f = I_f \omega = (m_1 R^2 + \frac{1}{2}m_2 R^2)\omega, \text{ or}$$

$$\omega = m_1 v_1 / [(m_1 + \frac{1}{2}m_2)R]$$

$$= (0.100 \text{ kg})(3.834 \text{ m/s}) / [(0.100 \text{ kg} + 10 \text{ kg}/2)(0.50 \text{ m})]$$

$$= \boxed{0.15 \text{ rad/s}}.$$

34. For the circular motion we can write

$$\omega = \omega_0 + \alpha t;$$

$$6 \text{ rad/s} = 4.5 \text{ rad/s} + (0.3 \text{ rad/s}^2)t, \text{ which gives}$$

$$t = \boxed{5.0 \text{ s}}.$$

The work done by the torque increases the kinetic energy of the flywheel:

$$W = \Delta K = \frac{1}{2}I(\omega^2 - \omega_0^2) = \frac{1}{2}MR^2(\omega^2 - \omega_0^2)$$

$$= \frac{1}{2}(680 \text{ kg})(1.2 \text{ m})^2[(6 \text{ rad/s})^2 - (4.5 \text{ rad/s})^2]$$

$$= \boxed{7.7 \times 10^3 \text{ J}}.$$

35. From
- $P = \tau\omega$
- we have

$$\tau = P/\omega = (240 \text{ hp})(746 \text{ W/hp})/[(3200 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min})]$$

$$= \boxed{5.3 \times 10^2 \text{ N} \cdot \text{m along the axis}}.$$

36. We choose the reference level for gravitational potential energy at the initial position. The kinetic energy will be the translational energy of the center of mass and the rotational energy about the center of mass. Because there is no work done by friction while the cylinder is rolling, for the work-energy theorem we have

$$W_{\text{net}} = \Delta K + \Delta U;$$

$$0 = (\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0) + Mg(0 - \ell \sin \theta).$$

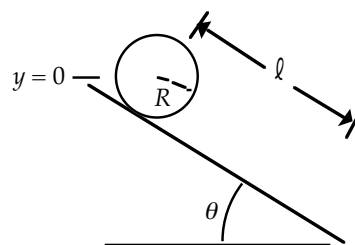
Because the cylinder is rolling, $v = R\omega$. The rotational inertia is $\frac{1}{2}MR^2$.

Thus we get

$$\frac{1}{2}Mv^2 + \frac{1}{2}(\frac{1}{2}MR^2)(v^2/R^2) = Mg\ell \sin \theta, \text{ which gives}$$

$$v = (4g\ell \sin \theta/3)^{1/2}, \text{ and } \omega = (4g\ell \sin \theta/3)^{1/2}/R.$$

We obtain the same result as in Section 9-7, but more directly.



37. The work decreases the kinetic energy of the flywheel:

$$W = \Delta K = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$$

$$-1200 \text{ J} = \frac{1}{2}(0.033 \text{ kg} \cdot \text{m}^2)[\omega_f^2 - (490 \text{ rad/s})^2], \text{ which gives}$$

$$\omega_f = \boxed{409 \text{ rad/s}}.$$

38. The component of the pulling force
- \vec{F}
- that is parallel to the direction of motion of the roller is
- $F \cos \theta$
- , where
- $F = 55 \text{ N}$
- and
- $\theta = 45^\circ$
- . The work done by that force after it pulls the roller through a distance
- d
- is
- $W = Fd \cos \theta$
- , which result in a change in the kinetic energy of the roller:

$$W = Fd \cos \theta = \Delta K = \frac{1}{2}MV_{\text{cm}}^2 + \frac{1}{2}I\omega^2.$$

But for pure rolling $\omega = V_{\text{cm}}/R$, and for a solid cylinder $I = \frac{1}{2}MR^2$; so

$$I\omega^2 = \frac{1}{2}MR^2(V_{\text{cm}}/R)^2 = \frac{1}{2}MV_{\text{cm}}^2, \text{ whereupon}$$

$$\Delta K = \frac{3}{4}MV_{\text{cm}}^2. \text{ Equate this with } W \text{ to obtain}$$

$$V_{\text{cm}} = (\frac{4}{3}Fd \cos \theta / M)^{1/2} \\ = [\frac{4}{3}(55 \text{ N})(2 \text{ m})(\cos 30^\circ) / (150 \text{ kg})]^{1/2} = \boxed{0.92 \text{ m/s}}.$$

39. From the result of Problem 36, we have

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}MR^2)(\frac{4}{3}g\ell \sin \theta / R^2) = \frac{1}{3}Mg\ell \sin \theta \\ = \frac{1}{3}(0.2 \text{ kg})(9.8 \text{ m/s}^2)(0.80 \text{ m}) \sin 15^\circ = \boxed{0.14 \text{ J}}.$$

40. (a)
- $I = \frac{1}{2}MR^2 = \frac{1}{2}\rho VR^2$
-
- $= \frac{1}{2}(8 \text{ g/cm}^3)\pi(120 \text{ cm})^2(45 \text{ cm})(1.2 \text{ m})^2(10^{-3} \text{ kg/g})$
-
- $= \boxed{1.2 \times 10^4 \text{ kg} \cdot \text{m}^2}.$

- (b) The work done by the flywheel comes from the decrease of its kinetic energy:

$$W = -\Delta K = -(0 - \frac{1}{2}I\omega_0^2) \\ = \frac{1}{2}(1.17 \times 10^4 \text{ kg} \cdot \text{m}^2)[(260 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min})]^2 \\ = \boxed{4.3 \times 10^6 \text{ J}}.$$

41. For Newton's second law, both force and acceleration are central, so we have

$$F = -kr = -mv^2/r, \text{ which gives } v = \sqrt{k/m} r.$$

The Bohr quantization condition is

$$L = mvr = n\hbar, n = 1, 2, \dots$$

We combine these to get $mvr = m\sqrt{k/m} r^2 = n\hbar$, which gives

$$r_n = (n^2\hbar^2/mk)^{1/4}, n = 1, 2, \dots$$

For v we get $v_n = (n^2\hbar^2k/m^3)^{1/4}, n = 1, 2, \dots$

From this we can get the kinetic energy:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(n^2\hbar^2k/m^3)^{1/2}; \quad K_n = \sqrt{k/m} n\hbar/2, n = 1, 2, \dots$$

42. For the potential energy $U(r) = Cr$, we find the force:

$F = -dU/dr = -C$. For Newton's second law, both force and acceleration are central, so we have

$$F = -C = -mv^2/r, \text{ which gives } v = \sqrt{Cr/m}.$$

The Bohr quantization condition is

$$L = mvr = n\hbar, n = 1, 2, \dots$$

We combine these to get $mvr = m\sqrt{Cr/m} r = \sqrt{Cr^3/m} = n\hbar$, which gives $r_n = (n^2\hbar^2/Cm)^{1/3}, n = 1, 2, \dots$

For v we get $v_n = (n\hbar C/m^2)^{1/3}, n = 1, 2, \dots$. From this we can get the kinetic energy:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(n\hbar C/m^2)^{2/3}; \quad K_n = \left[\frac{1}{2}(n^2\hbar^2 C^2/m)^{1/3}, n = 1, 2, \dots\right].$$

43. The permissible energies of the hydrogen atom are

$$E_n = -(13.6 \text{ eV})/n^2; \quad E_1 = -13.6 \text{ eV}, \quad E_2 = -3.4 \text{ eV}, \quad E_3 = -1.5 \text{ eV}, \quad E_4 = -0.85 \text{ eV}, \text{ etc.}$$

There is no energy level 2.0 eV above the lowest state, i. e., at -11.6 eV .

Possible excitation energies are $E_n - E_1 = [13.6 - (13.6/n^2)] \text{ eV}, n = 2, 3, \dots$

From the above values, possible excitation energies are $[10.2 \text{ eV}, 12.1 \text{ eV}, 12.75 \text{ eV}, \dots]$.

44. (a) The angular momentum of a uniform sphere is

$$L = I\omega = \frac{2}{5}(mr^2\omega), \text{ which gives}$$

$$\omega = \frac{5}{2}L/mr^2 = \frac{5}{2}(0.5 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s}) / (1.67 \times 10^{-27} \text{ kg})(1.3 \times 10^{-15} \text{ m})^2 = [4.4 \times 10^{22} \text{ rad/s}].$$

The frequency of the rotation is

$$f = \omega/2\pi = (4.4 \times 10^{22} \text{ rad/s}) / 2\pi = [7.0 \times 10^{21} \text{ Hz}].$$

- (b) The speed of the circular motion of the outermost portion is

$$v = r\omega = (1.3 \times 10^{-15} \text{ m})(4.4 \times 10^{22} \text{ rad/s}) = [5.7 \times 10^7 \text{ m/s}], \text{ which is } 0.19c.$$

- (c) The kinetic energy of rotation is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}L^2/I = \frac{1}{2}(0.5 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s})^2 / [\frac{2}{5}(1.67 \times 10^{-27} \text{ kg})(1.3 \times 10^{-15} \text{ m})^2] = [1.1 \times 10^{-12} \text{ J}].$$

45. We choose the coordinate system shown, so the initial angular momentum of the top lies in the xz -plane and makes an angle ϕ with the z -axis:

$$\vec{L} = I\vec{\omega} = I\omega \sin \phi \hat{i} + I\omega \cos \phi \hat{k}.$$

With ℓ the distance of the center of mass along the symmetry axis from the tip, the torque comes from the weight: $\vec{\tau} = Mg\ell \sin \phi \hat{j}$.

When we use Newton's second law for a time Δt , we get

$$\Delta \vec{L} = \vec{\tau} \Delta t = Mg\ell \Delta t \sin \phi \hat{j}, \text{ which gives a new angular momentum:}$$

$$\vec{L} = I\omega \sin \phi \hat{i} + Mg\ell \Delta t \sin \phi \hat{j} + I\omega \cos \phi \hat{k}.$$

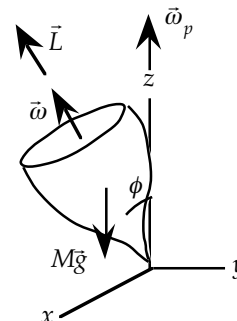
The z -component has not changed, while the component in the xy -plane has turned away from the x -axis through an angle:

$$\Delta \theta = (Mg\ell \Delta t \sin \phi) / (I\omega \sin \phi) = Mg\ell \Delta t / I\omega.$$

As the top precesses, the torque continues to be perpendicular to the plane

formed by \vec{L} and the z -axis. Thus, the z -component will continue to be constant and the rate of precession is

$$\omega_p = \Delta \theta / \Delta t = [Mg\ell / I\omega \text{ along the } z\text{-axis}].$$



46. With the initial angular momentum up, she wants to tilt the axle and thus the angular momentum away from her. Because $\Delta L = \tau \Delta t$, initially the torque must be radially out. As she continues the motion, the torque will have to be perpendicular to the axle in the vertical plane. During the reversal of the direction, the angular momentum of the system of student, stool, and wheel is conserved:

$$\vec{L} = I\omega\hat{k} = I^*\omega^*\hat{k} - I\omega\hat{k}, \text{ which gives}$$

$$\omega^* = \frac{2I\omega}{I^*}.$$

47. We find the center of mass relative to the pivot:

$$R_{CM} = (m_1 r_1 + m_2 r_2) / (m_1 + m_2)$$

$$= [(0.8 \text{ kg})(-0.16 \text{ m}) + (1 \text{ kg})(0.10 \text{ m})] / (0.8 \text{ kg} + 1 \text{ kg})$$

$$= -0.016 \text{ m}.$$

The rotational inertia of the mass about the spin axis is zero.

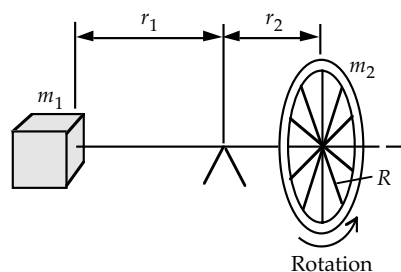
We can treat this as a top, with a precessional rate of

$$\omega_p = MgR_{CM} / I\omega$$

$$= MgR_{CM} / m_2 R^2 \omega$$

$$= (1.8 \text{ kg})(9.8 \text{ m/s}^2)(0.016 \text{ m}) / (1 \text{ kg})(0.10 \text{ m})^2 (10 \text{ rad/s})$$

$$= \boxed{2.8 \text{ rad/s}}.$$



48. For the projectile motion, we find the position and velocity components:

$$x = v_{0x}t = v_0 t; \quad y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = h + 0 - \frac{1}{2}gt^2;$$

$$v_x = \text{constant} = v_0; \quad v_y = v_{0y} - gt = 0 - gt.$$

When we express these as vectors, we have

$$\vec{r} = v_0 t \hat{i} + (h - \frac{1}{2}gt^2) \hat{j}; \quad \vec{p} = m\vec{v} = mv_0 \hat{i} - mgt \hat{j}.$$

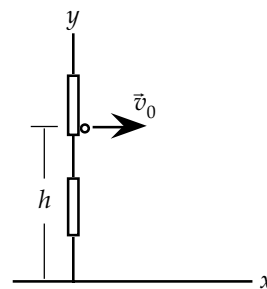
The angular momentum is

$$\vec{L} = \vec{r} \times \vec{p} = [v_0 t \hat{i} + (h - \frac{1}{2}gt^2) \hat{j}] \times (mv_0 \hat{i} - mgt \hat{j})$$

$$= -mgtv_0 t^2 \hat{k} - mv_0(h - \frac{1}{2}gt^2) \hat{k} = -mv_0(h + \frac{1}{2}gt^2) \hat{k}$$

$$= -(0.110 \text{ kg})(4.5 \text{ m/s})[6.0 \text{ m} + \frac{1}{2}(9.8 \text{ m/s}^2)t^2] \hat{k}$$

$$= \boxed{-[3.0 \text{ kg} \cdot \text{m}^2/\text{s} + (2.4 \text{ kg} \cdot \text{m}^2/\text{s}^3)t^2] \hat{k} \text{ (in to the page)}}.$$



49. We choose the reference level for gravitational potential energy at the initial position. The kinetic energy will be the translational energy of the center of mass and the rotational energy about the center of mass. Because there is no work done by friction while the cylinder is rolling, for the work-energy theorem we have

$$W_{\text{net}} = \Delta K + \Delta U;$$

$$0 = (\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0) + Mg(0 - \ell \sin \theta).$$

Because the cylinder is rolling, $v = R\omega$.

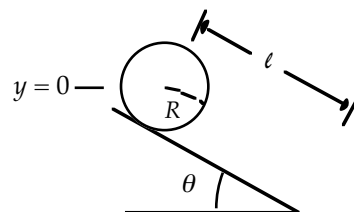
The rotational inertia is $\frac{1}{2}MR^2$. Thus we get

$$\frac{1}{2}M(R\omega)^2 + \frac{1}{2}(\frac{1}{2}MR^2)\omega^2 = Mg\ell \sin \theta, \text{ which we write as}$$

$$\omega^2 = \frac{4}{3}g\ell (\sin \theta) / R^2$$

$$= \frac{4}{3}(9.8 \text{ m/s}^2)(1.5 \text{ m})(\sin 28^\circ) / (0.042 \text{ m})^2, \text{ which gives}$$

$$\omega = \boxed{72 \text{ rad/s}}.$$



50. We find an expression for the speed of the block after it travels a distance d down the plane, starting from rest:

$$v^2 = v_0^2 + 2ax = 0 + 2ad = 2(0.1 \text{ m/s}^2)d = (0.2 \text{ m/s}^2)d.$$

Because the speed of the thread is the tangential speed of the axle, for the angular speed of the cylinder we have $\omega = v/r$, or

$$\omega^2 = v^2/r^2 = (0.2 \text{ m/s}^2)d / (0.005 \text{ m})^2 = (8.0 \times 10^3 \text{ rad}^2/\text{m} \cdot \text{s}^2)d.$$

The rotational inertia of the cylinder is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.5 \text{ kg})(0.04 \text{ m})^2 = 4.0 \times 10^{-4} \text{ kg} \cdot \text{m}^2.$$

We choose the reference level for gravitational potential energy at the initial position of the mass.

For the work-energy theorem applied to the system of cylinder and mass, we have

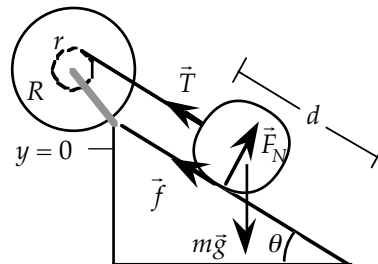
$$W = \Delta K + \Delta U;$$

$$- \mu_k mg \cos \theta d = (\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - 0) + mg(0 - d \sin \theta);$$

$$- \mu_k (1 \text{ kg})(9.8 \text{ m/s}^2) \cos 30^\circ d =$$

$$\frac{1}{2}(1 \text{ kg})(0.2 \text{ m/s}^2)d + \frac{1}{2}(4.0 \times 10^{-4} \text{ kg} \cdot \text{m}^2)(8.0 \times 10^3 \text{ rad}^2/\text{m} \cdot \text{s}^2)d - (1 \text{ kg})(9.8 \text{ m/s}^2) \sin 30^\circ d.$$

Note that the distance d cancels out, so we get $\mu_k = \boxed{0.38}$.



51. We treat the child as a point mass moving on a radial line of the platform. For the system of child and platform, angular momentum is conserved:

$$L = I_{\text{platform}}\omega_0 = (I_{\text{platform}} + m_{\text{child}}R^2)\omega_1;$$

$$(450 \text{ kg} \cdot \text{m}^2)(0.8 \text{ rad/s}) = [450 \text{ kg} \cdot \text{m}^2 + (32 \text{ kg})(2 \text{ m})^2]\omega_1, \text{ which gives}$$

$$\omega_1 = \boxed{0.62 \text{ rad/s}}.$$

The change in energy is

$$\begin{aligned} \Delta K &= \frac{1}{2}(I_{\text{platform}} + m_{\text{child}}R^2)\omega_1^2 - \frac{1}{2}I_{\text{platform}}\omega_0^2 \\ &= \frac{1}{2}[450 \text{ kg} \cdot \text{m}^2 + (32 \text{ kg})(2 \text{ m})^2](0.62 \text{ rad/s})^2 - \frac{1}{2}(450 \text{ kg} \cdot \text{m}^2)(0.8 \text{ rad/s})^2 \\ &= \boxed{-33 \text{ J}}. \end{aligned}$$

The work was done by the force of friction between the child and the platform, which is necessary to enable the child to walk.

52. Due to the absence of the net external force both the linear and angular momenta of the system, which consists of the board (1) and the mass (2), are conserved. After the collision the board (with the mass attached) will move forward and rotate at the same time.

The initial linear momentum of the system before the collision is $p_i = m_1 v_{1i}$, and afterwards $p_f = (m_1 + m_2)v_{\text{cm}}$, so from $p_i = p_f$ we know that, after the collision, the center of mass of the system moves at a speed of

$$v_{\text{cm}} = m_2 v_{2i} / (m_1 + m_2) = (30 \text{ kg}) v_{1i} / (20 \text{ kg} + 30 \text{ kg}) = \boxed{0.60 v_{1i}},$$

in the same direction as the initial velocity of the mass.

The system also spins about its center of mass. The rate of spin can be found from conservation of angular momentum. To find the exact value we need to know the direction of motion of the mass before the collision relative to the orientation of the board.

53. Assuming that the ball hits the door perpendicularly, then the magnitude of the change in momentum for the ball as a result of the collision is

$$\Delta p = m(v_i + v_f) = (0.035 \text{ kg})(45 \text{ m/s} + 35 \text{ m/s}) = 2.8 \text{ kg} \cdot \text{m/s}.$$

The corresponding angular impulse delivered by the ball to the door is $l\Delta p$, with $l = 0.85 \text{ m}$. Set

$$l\Delta p = \Delta L = I\omega = (\frac{1}{3}Ml^2)\omega \text{ to obtain}$$

$$\begin{aligned} \omega &= 3\Delta p / Ml = 3(2.8 \text{ kg} \cdot \text{m/s}) / [(3.0 \text{ kg})(0.85 \text{ m})] \\ &= \boxed{3.3 \text{ rad/s}}. \end{aligned}$$

54. The bullet passes through the wheel so quickly that it leaves before the wheel turns. For this collision, angular momentum of the bullet-wheel system is conserved:

$$L = mv_i d = mv_f d + I\omega, \text{ which we write as}$$

$$\frac{1}{2}MR^2\omega = ma(v_i - v_f);$$

$$\frac{1}{2}(3.0 \text{ kg})(0.18 \text{ m})^2\omega = (1.5 \times 10^{-2} \text{ kg})(0.14 \text{ m})(350 \text{ m/s} - 270 \text{ m/s}),$$

which gives

$$\omega = \boxed{3.4 \text{ rad/s (clockwise)}}.$$

The angular momentum of the wheel is

$$L = \frac{1}{2}MR^2\omega = \frac{1}{2}(2.0 \text{ kg})(0.18 \text{ m})^2(3.4 \text{ rad/s})$$

$$= \boxed{0.17 \text{ kg} \cdot \text{m}^2/\text{s (clockwise)}}.$$

The kinetic energy of the wheel is

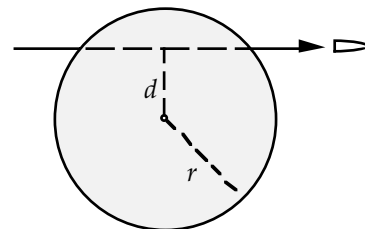
$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}[\frac{1}{2}(2.0 \text{ kg})(0.18 \text{ m})^2](3.4 \text{ rad/s})^2 = \boxed{0.28 \text{ J}}.$$

The change in the kinetic energy of the bullet is

$$\Delta K_{\text{bullet}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}(1.5 \times 10^{-2} \text{ kg})[(270 \text{ m/s})^2 - (350 \text{ m/s})^2]$$

$$= \boxed{-3.7 \times 10^2 \text{ J}}.$$

The total kinetic energy is not conserved. Negative work is done by the drag forces as the bullet passes through the wheel.



55. (a) For the system of the two blocks and pulley, no work will be done by nonconservative forces. The rope ensures that each block has the same speed v and the angular speed of the pulley is $\omega = v/R$. We choose the reference level for gravitational potential energy at the floor.

The rotational inertia of the pulley is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2 \text{ kg})(0.1 \text{ m})^2 = 0.01 \text{ kg} \cdot \text{m}^2.$$

For the work-energy theorem we have

$$W_{\text{net}} = \Delta K + \Delta U;$$

$$0 = (\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 - 0) + m_1g(0 - h) + m_2g(h - 0);$$

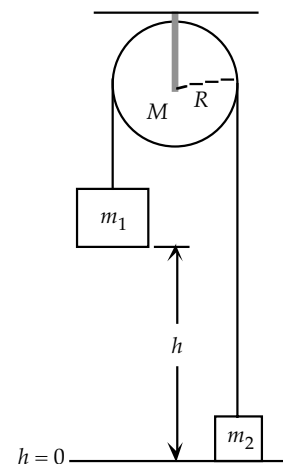
$$\frac{1}{2}(4 \text{ kg})v^2 + \frac{1}{2}(1 \text{ kg})v^2 + \frac{1}{2}(0.01 \text{ kg} \cdot \text{m}^2)(v/0.1 \text{ m})^2 = (4 \text{ kg} - 1 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}), \text{ which gives}$$

$$v = \boxed{4 \text{ m/s}}.$$

- (b) Because the motion has constant acceleration, we have

$$y = \frac{1}{2}(v + v_0)t;$$

$$1.5 \text{ m} = \frac{1}{2}(3.8 \text{ m/s} + 0)t, \text{ which gives } t = \boxed{0.8 \text{ s}}.$$



56. (a) We choose the coordinates shown on the force diagrams. Note that we take the positive direction in the direction of the acceleration for each object.

We write $\Sigma F_y = ma_y$ for the larger mass:

$$m_1g - T_1 = m_1a.$$

We write $\Sigma F_y = ma_y$ for the smaller mass:

$$T_2 - m_2g = m_2a.$$

We write $\Sigma \tau = I\alpha$ for the pulley about its axle:

$$T_1R - T_2R = MR^2\alpha.$$

Because the string does not slip on the pulley, we have $a = R\alpha$.

When we combine these equations to eliminate T_1 , T_2 and α , we get

$$a = (m_1 - m_2)g / (m_1 + m_2 + M)$$

$$= (1.2 \text{ kg} - 0.85 \text{ kg})(9.8 \text{ m/s}^2) / (1.2 \text{ kg} + 0.85 \text{ kg} + 0.45 \text{ kg})$$

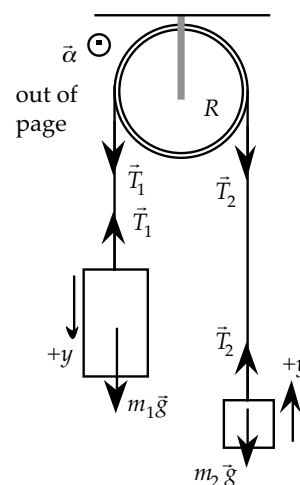
$$= \boxed{1.4 \text{ m/s}^2}.$$

- (b) For the motion of the larger mass, we have

$$y = y_0 + v_0t + \frac{1}{2}a_yt^2;$$

$$1.6 \text{ m} = 0 + 0 + \frac{1}{2}(1.4 \text{ m/s}^2)t^2, \text{ which gives}$$

$$t = \boxed{1.5 \text{ s}}.$$



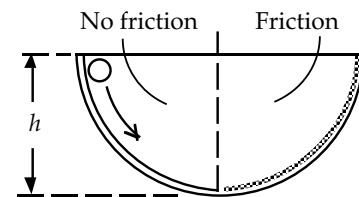
57. While the ball is sliding down the smooth side of the bowl, the kinetic energy of translation increases as the potential energy decreases. If the ball starts rolling immediately at the bottom, the distance in which sliding changes to rolling is very short and the work done by friction can be neglected. There will be both translational and rotational kinetic energy as the ball starts rolling up the other side. While the ball is rolling up the side, friction does no work. Thus we use the work-energy theorem from the release point to the point where the ball momentarily comes to rest:

$$W_{\text{net}} = \Delta K + \Delta U;$$

$$0 = (0 - 0) + Mg(h' - h), \text{ which gives}$$

$$h' = h.$$

With our assumption of no work done by the friction forces, the initial and final potential energies must be the same.



58. (a) Each horizontal section of the door can be considered to be a rod, so we have

$$I_d = \frac{1}{3}M\ell^2 = \frac{1}{3}(15 \text{ kg})(1.2 \text{ m})^2 = \boxed{7.2 \text{ kg} \cdot \text{m}^2}.$$

- (b) During the collision, angular momentum of the door-mud ball system about an axis through the hinges is conserved:

$$L = mv\ell = (I_d + m\ell^2)\omega;$$

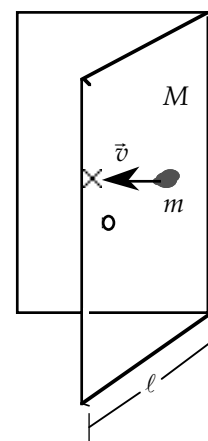
$$(0.3 \text{ kg})(12 \text{ m/s})(1.2 \text{ m}) = [7.2 \text{ kg} \cdot \text{m}^2 + (0.3 \text{ kg})(1.2 \text{ m})^2]\omega, \text{ which gives}$$

$$\omega = \boxed{0.57 \text{ rad/s}}.$$

- (c) $K_f/K_i = \frac{1}{2}(I_d + m\ell^2)\omega^2 / \frac{1}{2}mv^2$

$$= [7.2 \text{ kg} \cdot \text{m}^2 + (0.3 \text{ kg})(1.2 \text{ m})^2](0.566 \text{ rad/s})^2 / [(0.3 \text{ kg})(12 \text{ m/s})^2]$$

$$= \boxed{0.057}.$$



59. We convert the speed: $(80 \text{ km/h}) / (3.6 \text{ ks/h}) = 22.2 \text{ m/s}$.

- (a) If we ignore the change in friction force, the horizontal linear momentum is conserved:

$$m_1 v_1 = (m_1 + m_2) v_2$$

$$(75 \text{ kg} + 450 \text{ kg})(22 \text{ m/s}) = (75 \text{ kg} + 450 \text{ kg} + 75 \text{ kg})v_2, \text{ which gives } v_2 = \boxed{19.4 \text{ m/s}}.$$

- (b) If we ignore the rotational energy of the wheels, the fraction of the kinetic energy that is lost is

$$-\Delta K/K = [\frac{1}{2}m_1 v_1^2 - \frac{1}{2}(m_1 + m_2)v_2^2] / \frac{1}{2}m_1 v_1^2 = 1 - [(m_1 + m_2)/m_1](v_2/v_1)^2$$

$$= 1 - [(75 \text{ kg} + 450 \text{ kg} + 75 \text{ kg}) / (75 \text{ kg} + 450 \text{ kg})][(19.4 \text{ m/s}) / (22.2 \text{ m/s})]^2$$

$$= \boxed{0.127}.$$

- (c) We still have conservation of linear momentum, so $v_2 = 19.4 \text{ m/s}$.

For the rolling wheels we have $\omega = v/r$. Thus the fraction of the kinetic energy that is lost is

$$-\Delta K/K = 1 - [\frac{1}{2}(m_1 + m_2)v_2^2 + 2(\frac{1}{2}I\omega_2^2)] / [\frac{1}{2}m_1 v_1^2 + 2(\frac{1}{2}I\omega_1^2)]$$

$$= 1 - [(m_1 + m_2 + 2I/r^2) / (m_1 + 2I/r^2)](v_2/v_1)^2$$

$$= 1 - \{[600 \text{ kg} + 2(3 \text{ kg} \cdot \text{m}^2) / (0.5 \text{ m})^2] / [525 \text{ kg} + 2(3 \text{ kg} \cdot \text{m}^2) / (0.5 \text{ m})^2]\}[(19.4 \text{ m/s}) / (22.2 \text{ m/s})]^2$$

$$= \boxed{0.132}.$$

60. From the work-energy theorem, we have

$$W_{\text{net}} = \Delta K = \frac{1}{2}I\omega^2 - 0.$$

$$\text{Thus } P = W/\Delta t = \Delta K/\Delta t = \frac{1}{2}I\omega^2/\Delta t;$$

$$1.6 \times 10^3 \text{ W} = \frac{1}{2}(1.2 \text{ kg} \cdot \text{m}^2)[(17,000 \text{ rev/min})(2\pi \text{ rad/rev}) / (60 \text{ s/min})]^2 / \Delta t, \text{ which gives}$$

$$\Delta t = 1.2 \times 10^3 \text{ s} = \boxed{20 \text{ min}}.$$

61. Because mass M is stationary, the tension in the string, which provides the centripetal acceleration of the mass m , is $T = Mg$. Thus, for $\Sigma F_r = ma_r$ for the mass m , we have

$$Mg = mv_1^2/r_1 = mr_1\omega_1^2;$$

$$M(9.8 \text{ m/s}^2) = (0.2 \text{ kg})(0.8 \text{ m})(40 \text{ rad/s})^2, \text{ which gives } M = \boxed{26 \text{ kg}}.$$

Because the mass M is increased slowly, we can assume that $T = Mg$ at all times. For the mass m , there are no torques acting about the center, so we have conservation of angular momentum:

$$mv_1r_1 = mv_2r_2, \text{ or } r_1^2\omega_1 = r_2^2\omega_2;$$

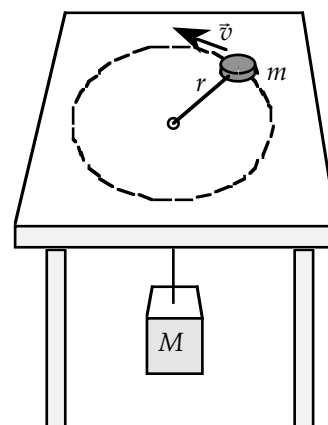
$$(0.8 \text{ m})^2(40 \text{ rad/s}) = (0.7 \text{ m})^2\omega_2, \text{ which gives } \omega_2 = \boxed{52 \text{ rad/s}}.$$

For $\Sigma F_r = ma_r$ for the mass m , we have

$$M'g = mr_2\omega_2^2;$$

$$M'(9.8 \text{ m/s}^2) = (0.2 \text{ kg})(0.7 \text{ m})(52 \text{ rad/s})^2, \text{ which gives } M' = 38 \text{ kg}.$$

The increase in M is $38 \text{ kg} - 26 \text{ kg} = \boxed{12 \text{ kg}}.$



62. If we take \vec{r} and \vec{p} to be in the xy -plane, we have

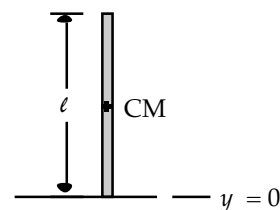
$$\begin{aligned} (\vec{r} \times \vec{p}) \cdot (\vec{r} \times \vec{p}) &= (xp_y - yp_x)\hat{k} \cdot (xp_y - yp_x)\hat{k} = (xp_y - yp_x)^2 \\ &= x^2p_y^2 - 2xyp_xp_y + y^2p_x^2 \\ &= (x^2 + y^2)p_y^2 + (x^2 + y^2)p_x^2 - y^2p_y^2 - 2xyp_xp_y - x^2p_x^2 \\ &= r^2p^2 - (xp_x + yp_y)^2 = r^2p^2 - (\vec{r} \cdot \vec{p})^2. \end{aligned}$$

For the kinetic energy we have

$$K = p^2/2m = [(\vec{r} \times \vec{p}) \cdot (\vec{r} \times \vec{p})/r^2 + (\vec{r} \cdot \vec{p})^2/r^2]/2m$$

$$= \vec{L} \cdot \vec{L} / 2mr^2 + (rp_r)^2 / 2mr^2;$$

$$K = \boxed{L^2/2mr^2 + p_r^2/2m}.$$



63. Because of the band, the tangential speed of the shaft must equal the tangential speed of the flywheel:

$$r_s\omega_s = r_w\omega_w;$$

$$(0.05 \text{ m})(1400 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min}) = (0.35 \text{ m})\omega_w, \text{ which gives } \omega_w = 20.9 \text{ rad/s}.$$

The work done increases the kinetic energy of the flywheel:

$$W_{\text{net}} = \Delta K = \frac{1}{2}(\frac{1}{2}Mr_w^2)\omega_w^2 - 0 = \frac{1}{2}[\frac{1}{2}(300 \text{ kg})(0.35 \text{ m})^2](20.9 \text{ rad/s})^2 = \boxed{4.0 \times 10^3 \text{ J}}.$$

64. (a) If the contact point does not move, no work is done by the friction force. With the reference level for potential energy at the table, we use energy conservation:

$$K_i + U_i = K_f + U_f;$$

$$0 + \frac{1}{2}Mg\ell = \frac{1}{2}(\frac{1}{3}M\ell^2)\omega^2 + 0, \text{ which gives } \omega = \boxed{(3g/\ell)^{1/2}}, \text{ clockwise}.$$

- (b) The angular momentum as the rod hits is

$$L = I\omega = (\frac{1}{3}M\ell^2)(3g/\ell)^{1/2} = (\frac{1}{3}M^2g\ell^3)^{1/2}, \text{ clockwise}.$$

- (c) The kinetic energy is $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{3}M\ell^2)(3g/\ell) = \boxed{\frac{1}{2}Mg\ell}.$

65. For $\Sigma F_r = ma_r$, we have

$$F = K/r^2 = Mv^2/r, \text{ which gives } v^2 = K/Mr.$$

- (a) The angular momentum is

$$L = Mvr = Mv(K/Mv^2), \text{ which gives } v = \boxed{K/L}.$$

- (b) $r = K/(Mv^2) = K/(MK^2/L^2) = \boxed{L^2/MK}.$

- (c) $T = 2\pi r/v = 2\pi(L^2/MK)/(K/L) = \boxed{2\pi L^3/MK^2}.$

- (d) $a = v^2/r = (K/L)^2/(L^2/MK) = \boxed{MK^3/L^4}.$

66. The velocity of the mass is

$\vec{v} = d\vec{r}/dt = d(\hat{i} A \cos \omega_1 t + \hat{j} B \cos \omega_2 t)/dt = -\hat{i} A \omega_1 \sin \omega_1 t - \hat{j} B \omega_2 \sin \omega_2 t$,
and its angular momentum is

$$\begin{aligned}\vec{L} &= \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) \\ &= m(\hat{i} A \cos \omega_1 t + \hat{j} B \cos \omega_2 t) \times (-\hat{i} A \omega_1 \sin \omega_1 t - \hat{j} B \omega_2 \sin \omega_2 t) \\ &= m[(A \cos \omega_1 t)(-B \omega_2 \sin \omega_2 t) - (B \cos \omega_2 t)(-A \omega_1 \sin \omega_1 t)]\hat{k} \\ &= \boxed{mAB(\omega_1 \sin \omega_1 t \cos \omega_2 t - \omega_2 \cos \omega_1 t \sin \omega_2 t)\hat{k}}. \quad \vec{L} \text{ is in the } z\text{-direction.}\end{aligned}$$

To make L a constant, set

$$\begin{aligned}d\vec{L}/dt &= mAB \hat{k} \, d(\omega_1 \sin \omega_1 t \cos \omega_2 t - \omega_2 \cos \omega_1 t \sin \omega_2 t)/dt \\ &= mAB \hat{k} (\omega_1^2 \cos \omega_1 t \cos \omega_2 t - \omega_1 \omega_2 \sin \omega_1 t \sin \omega_2 t + \omega_1 \omega_2 \sin \omega_1 t \sin \omega_2 t - \omega_2^2 \cos \omega_1 t \cos \omega_2 t) \\ &= mAB \hat{k} (\omega_1^2 - \omega_2^2) \cos \omega_1 t \cos \omega_2 t \\ &= 0,\end{aligned}$$

which gives $\boxed{\omega_1 = \omega_2}$, whereupon $\vec{L} = mAB\omega\hat{k}$.

67. Assume the hurricane to be a uniform cylinder of air, 100 km in radius, 10 km high, with average air speed of 100 km/h (about 30 m/s) and average density of
- 1 kg/m^3
- . The mass of the hurricane is

$$m \approx \pi R^2 h \approx \pi(100,000 \text{ m})^2 (10,000 \text{ m}) \approx 3 \times 10^{14} \text{ kg, and its kinetic energy is}$$

$$K \approx \frac{1}{2}mv^2 \approx \frac{1}{2}(3 \times 10^{14} \text{ kg})(30 \text{ m/s})^2 \approx \boxed{1 \times 10^{17} \text{ J}}.$$

To find the angular momentum, consider a thin cylindrical shell of the hurricane, height h , between radii r and $r + dr$. The mass of the shell is $dm = \rho dV = \rho(2\pi r h dr)$, and for the shell

$$dL \approx vr dm = 2\pi\rho h v r^2 dr.$$

Integrate over r from 0 to R :

$$\begin{aligned}L &= \int dL \approx 2\pi\rho h v \int r^2 dr = \frac{2}{3}\pi\rho h v R^3 = (\frac{2}{3}vR)(\rho \pi R^2 h) = \frac{2}{3}mvR \\ &\approx \frac{2}{3}(3 \times 10^{14} \text{ kg})(30 \text{ m/s})(100,000 \text{ m}) = \boxed{6 \times 10^{20} \text{ kg} \cdot \text{m}^2/\text{s}}.\end{aligned}$$

In comparison, the spin angular momentum of Earth is $7 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}$, from Chapter 9.

68. For the inelastic collision, angular momentum about the contact point
- A
- is conserved:

$$L_A = mv\ell = (\frac{1}{3}M\ell^2 + m\ell^2)\omega_0,$$

which gives the initial angular speed for the falling;

$$\omega_0 = 3mv/[(M + 3m)\ell] = 3mv/[(5m + 3m)\ell] = 3v/8\ell.$$

After the collision, the rotational inertia about the contact point is

$$I_A = \frac{1}{3}M\ell^2 + m\ell^2 = 8m\ell^2/3.$$

If the contact point does not move, no work is done by the friction force.

For the falling motion, we use energy conservation, with the reference level for potential energy at the table:

$$K_i + U_i = K_f + U_f;$$

$$\frac{1}{2}I_A\omega_0^2 + (\frac{1}{2}Mg\ell) + mg\ell = \frac{1}{2}I_A\omega^2 + 0$$

$$\frac{1}{2}(8m\ell^2/3)(3v/8\ell)^2 + \frac{1}{2}(5m)g\ell + mg\ell = \frac{1}{2}(8m\ell^2/3)\omega^2 + 0,$$

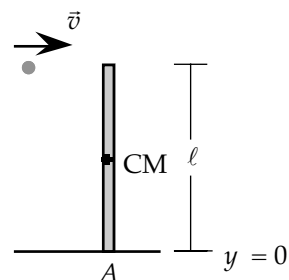
which gives $\omega = \boxed{(9v^2/16 + 21g\ell/2)^{1/2}/2\ell}$, clockwise.

The angular momentum as the rod hits is

$$\begin{aligned}L &= I\omega = (8m\ell^2/3)(9v^2/16 + 21g\ell/2)^{1/2}/2\ell \\ &= \boxed{(4m\ell/3)(9v^2/16 + 21g\ell/2)^{1/2}}, \text{ clockwise.}\end{aligned}$$

The kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \boxed{m\ell(3v^2/4 + 14g\ell)}.$$



69. If we call the length of the string L , the angle that the top has turned when the string comes off is $\theta = L/R$. With a constant force (and thus constant torque), the accelerations will be constant.

We write $\Sigma\tau = I\alpha$ about the center of mass of the top:

$$FR = \frac{1}{2}MR^2\alpha, \text{ which gives } \alpha = 2F/MR.$$

For the angular motion we find the time of the accelerated motion from

$$\theta = \theta_0 + \frac{1}{2}\alpha t^2;$$

$$L/R = \frac{1}{2}(2F/MR)t^2, \text{ which gives } t = \boxed{(LM/F)^{1/2}}.$$

- (a) We write $\Sigma F = ma$ for the center of mass:

$$F = Ma, \text{ which gives } a = F/M.$$

The speed when the string drops off is

$$v = v_0 + at = 0 + (F/M)(LM/F)^{1/2} = (FL/M)^{1/2}$$

$$= \sqrt{(0.6 \text{ N})(1 \text{ m}) / (0.10 \text{ kg})} = \boxed{2.4 \text{ m/s}}.$$

- (b) We find the angular speed from

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$= 0 + 2(2F/MR)(L/R) = 4FL/MR^2$$

$$= 4(0.6 \text{ N})(1 \text{ m}) / (0.10 \text{ kg})(0.02 \text{ m})^2, \text{ which gives}$$

$$\omega = \boxed{2.4 \times 10^2 \text{ rad/s}}.$$

70. The cylinder will roll about the contact point A.

We write $\Sigma\tau = I\alpha$ about the point A:

$$F(R-h) + F_{N1}\sqrt{R^2 - (R-h)^2} - Mg\sqrt{R^2 - (R-h)^2} = I_A\alpha.$$

When the cylinder does roll over the curb, contact with the ground is lost and $F_{N1} = 0$. Thus we get

$$F = (I_A\alpha + Mg\sqrt{R^2 - (R-h)^2}) / (R-h)$$

$$= [I_A\alpha / (R-h)] + [Mg\sqrt{2Rh - h^2} / (R-h)].$$

The minimum force occurs when $\alpha = 0$:

$$F_{\min} = Mg\sqrt{h(2R-h)} / (R-h).$$

