

# CHAPTER 8    Linear Momentum, Collisions, and the Center of Mass

## Answers to Understanding the Concepts Questions

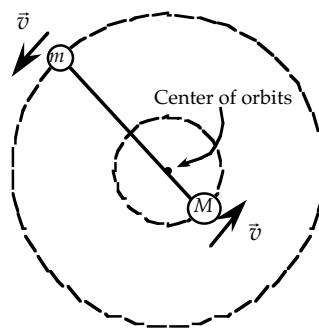
1. The comet moves under the influence of the gravitational force of the Sun. We can think of the Sun-comet system as a two-body system that undergoes a collision; Newton's third law guarantees that net momentum is conserved throughout. The net momentum is the momentum of the comet plus that of the Sun. If the momentum of the comet is in one direction coming in and another going out, the corresponding change in momentum must be compensated by an equal and opposite momentum change for the Sun. The Sun is so much more massive than any comet that the change in motion of the Sun due to its momentum change would probably not be perceptible to our instruments, just as the motion of Earth is not perceptible in a collision with a falling apple.
2. The collision is essentially elastic. Balls 2 through 5 receive an impulse from ball 1 and immediately pass it along to ball 5, so they do not move as a result. Ball 5 has the same mass as ball 1, so according to the discussion in elastic collisions between two identical masses in the text, ball 1 and 5 simply exchange their speeds upon collision. Therefore ball 1 stops, while ball 5 acquires the speed of ball 1 just before collision and rises to the same initial height from which ball 1 is released.
3. Let's assume that the table is horizontal and frictionless so the two-puck system experiences no net external force. Then according to Newton's second law its center of mass undergoes no acceleration. Since the two pucks eventually stop, the velocity of the center of mass is zero after the collision and so it must be zero before the collision. In fact the two pucks move in towards each other in opposite directions before the collision, so the center of mass of the system can indeed be at rest both before and after the collision.
4. The simplest object to visualize is a horseshoe, for which symmetry makes it clear that the center of mass is somewhere within the open portion. If we take a symmetrical hollow sphere the center of mass is at the center, in the hollow portion. While you might say that that point is still within the sphere, imagine now drilling a tunnel to the hollow from the outside and gradually increasing the size of the tunnel. At some point the tunnel will be large enough so that you would no longer say that the center of mass is actually within the hollowed sphere.
5. No. The motion of the center of mass is subject to the influence of external forces only, which in this case is essentially just the gravitational force. As a result the center of mass behaves like a projectile and follows a parabolic path. It cannot make a midair loop.
6. (a) Not necessarily. They are moving in opposite directions, which means that they could be on a collision course — but only if their velocities are aligned in the same straight line (i.e., collinear), which may not be the case. Even if their velocities are aligned, they could still be headed directly away from each other.  
(b) Yes. If their velocities are not collinear they would be moving in two parallel planes. (However, there always exists a plane that accommodates both of these velocities.)  
(c) Yes.  
(d) Not necessarily. All we know is that the two velocities have opposite directions, which does not guarantee that they are aligned — they could be just parallel.

7. The center of mass of an object curved like a horseshoe lies within the open part of the object. That is true for a curved human body as well. If the jumper can manipulate his or her body so that it is “draped” over the bar, then the center of mass can be below the bar even if a part of the body is over the bar.
8. The conservation of linear momentum requires that  $\vec{p}_{1i} = \vec{p}_{1f} + \vec{p}_{2f}$ . Note that  $\vec{p}_{1f} + \vec{p}_{2f}$  is in the plane formed by  $\vec{p}_{1f}$  and  $\vec{p}_{2f}$ , so  $\vec{p}_{1i}$ , being equal to  $\vec{p}_{1f} + \vec{p}_{2f}$ , must also lie in that plane. Thus  $\vec{p}_{1i}$ ,  $\vec{p}_{1f}$  and  $\vec{p}_{2f}$  must indeed be in the same plane. If there are three outgoing particles, then  $\vec{p}_{1i} = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f}$ . Consider the plane formed by  $\vec{p}_{1f}$  and  $\vec{p}_{2f}$  in which lies  $\vec{p}_{1f} + \vec{p}_{2f}$ . There is no reason why  $\vec{p}_{3f}$  has to lie in that plane. In fact, it is entirely possible for  $\vec{p}_{3f}$  to have a component that is perpendicular to that plane, as long as  $\vec{p}_{1i}$  has the same component. So with three outgoing particles the momenta do not necessarily lie in the same plane.
9. By symmetry the center of mass of the circle is at the origin. Draw an  $x$ -axis originated from the center of the circle, pointing toward the point mass. The center of mass of the system lies on the  $x$ -axis, with  $x_{\text{cm}} = m_{\text{point}} R / (m_{\text{circle}} + m_{\text{point}}) < R$ . It is within the circle.
10. Momentum is conserved only in the system of vase and Earth; the downward motion of the vase is accompanied by a tiny upward motion of Earth. However, the net horizontal momentum of the vase is indeed conserved if the floor is smooth, so that pieces do not “catch” on the floor. If the vase falls straight down, with no initial horizontal motion, then the net horizontal momentum of the pieces of the vase must add to zero.
11. Yes. The racket transfers part of its forward momentum to the tennis ball but it still has some momentum left to continue its forward motion, albeit at a reduced speed.
12. It is a good assumption to take the collision of the racket and ball to be elastic, roughly independent of the tension in the strings, for the typical range of stringing tension. More power but less control is associated with looser stringing. The ball remains in contact with the strings for a longer period and the player can push on the ball for a longer period — note that the racket and ball do not form an isolated system, because the player’s muscles act on the racket. In contrast, more control but less power is associated with tighter stringing. This time the contact period is shorter and the player has less time in which variation in the directionality of the shot can come in. Incidentally, if the racket is too loosely strung, then the elasticity of the collision is lost, and so is power.
13. In principle, the total momentum of the system consisting of the pendulum and Earth must be conserved. The reason why you don’t see Earth moving back and forth as a result like the pendulum is the enormous disparity between their masses. The more massive an object, the slower it moves for the same amount of momentum.
14. The adult and the child must exert roughly the same magnitude of torque about the pivot of the see-saw, or the see-saw would simply tilt towards one side and not bounce up and down. Since the adult weighs considerably more his/her lever arm must be considerably less than that of the child, meaning that he/she should sit much closer to the midpoint of the see-saw (where the pivot is). So the correct answer is (a).
15. There is no absolute meaning to “short” in the context of collisions; the physical principles are the same for all periods over which the force acts. For us it is more a matter of when the time is so short that our ordinary senses cannot see a finite time interval. Scientific instruments can of course greatly improve our ability to see what is happening during the time over which “impulsive” forces act.
16. Unlike baseball catchers, cricket players draw back their hands upon catching the ball so that the force of impact acts over a longer time interval, thereby reducing the impact force.

17. Assume that all collisions are elastic. The large ball hits the ground first and rebounds with the same speed  $v$  as it hits the ground. The small ball, meanwhile, is heading downward with about the same speed  $v$  (as it is dropped from the same height), and as it collides head-on with the upward-moving large one its upward speed after the collision would almost be  $3v$ , as you can check by using the elastic collision formula with one mass much greater than the other. This is much greater than  $v$ , since the large ball moves upward and delivers a greater upward impulse on the small one than the stationary ground would. Since the height it attains is proportional to the squared value of its initial upward speed, the small ball would now reach a height of  $3^2 = 9$  times the height it would have attained just by bouncing off the ground with a speed  $v$ ! So if you dropped the balls from about 1 m above the floor the small one would now rebound to nearly 9 m, which is greater than most floor-to-ceiling distances.
18. The acceleration of the more massive car is less than that of the less massive car in a collision. It is the acceleration that breaks bones or otherwise injures, so that the more massive car is the safer. Of course other factors play a role, such as the ease with which the car is crumpled. Energy that goes into crumpling a fender is better for you than energy that goes into crumpling a part of you!
19. As the gun fires it delivers a forward impulse on the ammunition, which in turn delivers a backward impulse of the same magnitude to the gun via Newton's third law, causing the gun to roll back.
20. As the car crumbles it absorbs more of the impulse delivered to it, rather than passing it along to the occupant.
21. Suppose the shell is fired to the right. Then, to conserve momentum in the closed system of car and shell, the car will recoil to the left. When the cannon ball hits the right-hand wall of the car (and assuming it does not pass through) the car will stop: since the initial total momentum is zero, the final total momentum must also be zero. In the process the car will have moved a little to the left, showing an outside observer that something has happened inside.
22. No. As the pigeons fly upward they must beat down the air inside the cage in order to overcome gravitational pull. So, while the pigeon themselves are not pushing down the cage with their feet they are in fact still doing so — through the air.
23. If the parachute is moving quickly, then in a fixed time interval it will collide with more air molecules than it would if it were moving slowly. Thus the momentum transfer in that interval will be greater at higher speeds, and this corresponds to a greater drag force on the chute at higher speeds.
24. Suppose that the speed of each ball is  $v$  relative to the pool table. To the observer in the new reference frame at rest with respect to ball 1, ball 2 is moving towards him with a speed  $2v$ . After the collision, ball 2 stops and ball 1 recoils backwards (i.e., in the same direction as ball 2 before the collision) with speed  $2v$ . (It is interesting to note that, regardless of which reference frame we use, two identical masses always *exchange* their velocities as a result of the elastic collision between them.)
25. The force exerted by the cushion on the billiard ball is normal (perpendicular to the side of the pool table). As a result, the component of the momentum of the ball along the side of the table does not change, while the component perpendicular to the side of the table reverses its direction but keeps its magnitude after the elastic collision. This means that the outgoing momentum of the ball must also make a  $45^\circ$  from the side of the table. If the collision is inelastic (with energy loss), then the magnitude of the velocity is somewhat less after the collision, while its component along the side of the table is still essentially the same since the force of impact is perpendicular to the side. As a result the outgoing velocity of the ball makes an angle that is less than  $45^\circ$  with the side of the table.

## Solutions to Problems

1. (a)  $p = mv = (40 \times 10^{-3} \text{ kg})(110 \text{ km/h})(10^3 \text{ m/km})/(3.6 \times 10^3 \text{ s/h}) = \boxed{1.2 \text{ kg} \cdot \text{m/s}}$ .  
 (b)  $p = mv = (145 \times 10^{-3} \text{ kg})(35 \text{ m/s}) = \boxed{5.1 \text{ kg} \cdot \text{m/s}}$ .  
 (c)  $p = mv = (72 \text{ kg})(22 \text{ mi/h})(1.6 \text{ km/mi})(10^3 \text{ m/km})/(3.6 \times 10^3 \text{ s/h}) = \boxed{7.0 \times 10^2 \text{ kg} \cdot \text{m/s}}$ .  
 (d)  $p = mv = (95 \text{ kg})(100 \text{ m})/(12.5 \text{ s}) = \boxed{7.6 \times 10^2 \text{ kg} \cdot \text{m/s}}$ .
2. Because the motion has constant acceleration, with down positive we find the velocity from  
 $v = v_0 + at = 0 + (9.8 \text{ m/s}^2)(1.6 \text{ s}) = 16 \text{ m/s}$ .  
 The momentum is  $p = mv = (1.65 \text{ kg})(16 \text{ m/s}) = \boxed{26 \text{ kg} \cdot \text{m/s}, \text{ downward}}$ .
3. (a)  $p = mv = (70 \text{ kg})(6 \text{ m/s}) = \boxed{4.2 \times 10^2 \text{ kg} \cdot \text{m/s}}$ .  
 (b)  $p = mv = (10^5 \text{ kg})(60 \text{ m/s}) = \boxed{6.0 \times 10^6 \text{ kg} \cdot \text{m/s}}$ .  
 (c)  $p = mv = (1100 \text{ kg})(25 \text{ mi/h})(1.6 \times 10^3 \text{ m/mi})/(3.6 \times 10^3 \text{ s/h}) = \boxed{1.2 \times 10^4 \text{ kg} \cdot \text{m/s}}$ .  
 (d)  $p = mv = (1.67 \times 10^{-27} \text{ kg})(2 \times 10^5 \text{ m/s}) = \boxed{3.3 \times 10^{-22} \text{ kg} \cdot \text{m/s}}$ .  
 (e) Because the motion has constant acceleration, with down positive, we find the speed from  
 $v^2 + v_0^2 + 2a \Delta y = 0 + 2(9.8 \text{ m/s}^2)(0.10 \text{ m})$ , which gives  $v = 1.4 \text{ m/s}$ .  
 $p = mv = (10 \times 10^{-3} \text{ kg})(1.4 \text{ m/s}) = \boxed{1.4 \times 10^{-2} \text{ kg} \cdot \text{m/s}}$ .
4. The central force provides the centripetal acceleration, so we can write  
 $F = mv^2/R$ , or  $v = \sqrt{FR/m}$ .  
 (a)  $p = mv = \boxed{\sqrt{mFR}}$ .  
 (b) With  $F = Kv$ , we have  
 $mv = \sqrt{mRKv}$ , from which we get  
 $R = (mv)^2 / Kmv = \boxed{p/K}$ .
5. (a) During the firing we use momentum conservation:  
 $p_{ri} + p_{bi} = p_{rf} + p_{bf}$   
 $0 + 0 = (7 \text{ kg})v_{rf} + (10 \times 10^{-3} \text{ kg})(700 \text{ m/s})$ , from which we get  $v_{rf} = \boxed{-1.0 \text{ m/s}}$ .  
 (b) The energy transmitted to the shoulder comes from the decrease in kinetic energy of the rifle:  
 $\Delta E = - (0 - \frac{1}{2}mv^2) = \frac{1}{2}(7 \text{ kg})(1.0 \text{ m/s})^2 = \boxed{3.5 \text{ J}}$ .
6. Because the velocities have opposite directions, momentum conservation gives us  
 $p_m + p_M = 0 = MV - mv$ , which gives  $V/v = m/M$ .  
 The force of attraction provides the centripetal acceleration for each mass:  
 $F = mv^2/r = MV^2/R$ . Thus  
 $R/r = MV^2/mv^2 = m/M$ .  
 (a) For circular motion the angular speed is  
 $\omega_M = V/R = (mv/M)/(mr/M) = v/r = \omega_m$ .  
 (b) From above,  $\boxed{r/R = M/m}$ .



7. (a)  $\vec{p} = m_1 \vec{v}_1 + m_2 \vec{v}_2$   
 $= (2.4 \text{ kg})[(-2.0\hat{i} - 3.5\hat{j}) \text{ m/s}] + (1.6 \text{ kg})[(1.8\hat{i} - 1.5\hat{j}) \text{ m/s}] = \boxed{-(1.9\hat{i} + 10.8\hat{j}) \text{ kg} \cdot \text{m/s}}$

(b) Because momentum is conserved, we have

$$\vec{p} = (-1.9\hat{i} - 10.8\hat{j}) \text{ kg} \cdot \text{m/s} = (2.4 \text{ kg})(2.5\hat{i} \text{ m/s}) + (1.6 \text{ kg})\vec{v}_2', \text{ which gives}$$

$$\vec{v}_2' = \boxed{-(4.9\hat{i} + 6.8\hat{j}) \text{ m/s}}$$

(c) Because mass is conserved, we have  $m_2' = 2.4 \text{ kg} + 1.6 \text{ kg} - 2.1 \text{ kg} = 1.9 \text{ kg}$ .

Because momentum is conserved, we have

$$\vec{p} = (-1.9\hat{i} - 10.8\hat{j}) \text{ kg} \cdot \text{m/s} = (2.1 \text{ kg})\vec{v}_1' + (1.9 \text{ kg})[(-2.5\hat{j} + 1.3\hat{k}) \text{ m/s}], \text{ which gives}$$

$$\vec{v}_1' = \boxed{-(0.90\hat{i} + 2.9\hat{j} + 1.2\hat{k}) \text{ m/s}}$$

(d) The initial total kinetic energy is

$$K_1 + K_2 = \frac{1}{2}(2.4 \text{ kg})[(-2.0 \text{ m/s})^2 + (-3.5 \text{ m/s})^2] + \frac{1}{2}(1.6 \text{ kg})[(1.8 \text{ m/s})^2 + (-1.5 \text{ m/s})^2]$$

$$= \boxed{24 \text{ J}}$$

For part (b) the final total kinetic energy is

$$K_1' + K_2' = \frac{1}{2}(2.4 \text{ kg})(2.5 \text{ m/s})^2 + \frac{1}{2}(1.6 \text{ kg})[(-4.9 \text{ m/s})^2 + (-6.8 \text{ m/s})^2] = \boxed{64 \text{ J}}$$

Because this is greater than the initial kinetic energy, some internally stored energy was transformed into kinetic energy.

For part (c) the final total kinetic energy is

$$K_1' + K_2' = \frac{1}{2}(2.1 \text{ kg})[(-0.90 \text{ m/s})^2 + (-2.9 \text{ m/s})^2 + (-1.2 \text{ m/s})^2] +$$

$$\frac{1}{2}(1.9 \text{ kg})[(-2.5 \text{ m/s})^2 + (1.3 \text{ m/s})^2] = \boxed{19 \text{ J}}$$

Because this is less than the initial kinetic energy, some energy was stored as potential or lost due to internal friction.

8. (a) We can treat this as two successive one-dimensional collisions, with momentum and kinetic energy conserved.

For the first collision, we have

$$\text{momentum: } mv_0 + 0 = mv_1 + mv_2;$$

$$\text{kinetic energy: } \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2.$$

Combining these equations, we get  $v_1 = 0$  and  $v_2 = v_0$ . The velocities are exchanged.

The second collision is a repeat of the first one. Thus the first and second billiard balls are at rest and the third moves with the velocity of the original ball.

(b) Again we have momentum and kinetic energy conserved, but for a two-dimensional collision. The interaction between the original ball and each of the others will be along the lines joining the centers. From the symmetry of the collision, the original ball will rebound straight back and the two struck balls will have velocities of equal magnitude along their respective lines of centers.

Using the coordinate system shown, we have

$$x\text{-momentum: } mv_0 + 0 = mv_1 + 2mv_2 \cos 30^\circ;$$

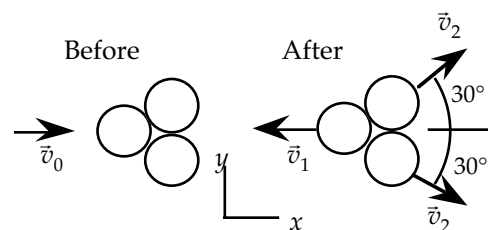
$$y\text{-momentum: } 0 + 0 = 0 + mv_2 \sin 30^\circ - mv_2 \sin 30^\circ;$$

$$\text{kinetic energy: } \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_1^2 + 2(\frac{1}{2}mv_2^2).$$

When we combine these equations, we get  $v_1 = -0.20v_0$  and  $v_2 = 0.69v_0$ .

Thus the first ball recoils straight back with speed  $0.20v_0$ ;

the other two move at speed  $0.69v_0$  — one at  $30^\circ$  above the original line and the other  $30^\circ$  below.



9. For this one-dimensional collision, we have  $J = p_f - p_i$ , from which we get

$$p_f = 10 \text{ N} \cdot \text{s} - 4.0 \text{ kg} \cdot \text{m/s} = 6.0 \text{ kg} \cdot \text{m/s}. \text{ Thus the final speed is}$$

$$v = p_f/m = (6.0 \text{ kg} \cdot \text{m/s})/0.15 \text{ kg} = \boxed{40 \text{ m/s}}.$$

10. For this one-dimensional collision, we have  $J = p_f - p_i$ . We assume that the final momentum of the arrow is very small, so we get

$$J_{\text{arrow}} = 0 - (0.032 \text{ kg})[(160 \text{ km/h})/(3600 \text{ s/h})] = \boxed{-1.4 \text{ kg} \cdot \text{m/s}}.$$

The impulse on the block is  $J_{\text{block}} = -J_{\text{arrow}} = +1.4 \text{ kg} \cdot \text{m/s}$ .

For the block we have

$$J_{\text{block}} = M(v_b - 0); \quad 1.4 \text{ kg} \cdot \text{m/s} = (3.5 \text{ kg})v_b, \text{ which gives}$$

$$v_b = \boxed{0.41 \text{ m/s in the direction of the arrow's initial motion}}.$$

11. For this one-dimensional collision, we have  $J = p_f - p_i$ . For the ball we get

$$J_{\text{ball}} = (0.145 \text{ kg})(-45 \text{ m/s} - 36 \text{ m/s}) = \boxed{-12 \text{ kg} \cdot \text{m/s, opposite to the original motion}}.$$

The average force is

$$F_{\text{av}} = J_{\text{ball}}/\Delta t = (12 \text{ kg} \cdot \text{m/s})/(7.0 \times 10^{-4} \text{ s}) = \boxed{1.7 \times 10^4 \text{ N, opposite to the original motion}}.$$

12. (a) For the vertical motion, with up positive, we find the speed at the ground from

$$v^2 = v_0^2 + 2a \Delta y = 0 + 2(-9.8 \text{ m/s}^2)(-3.2 \text{ m}), \text{ which gives } v = 7.9 \text{ m/s}.$$

Because the ball rebounds to the same height, the speed leaving the ground is the same.

Thus the impulse is

$$J = \Delta p = (0.440 \text{ kg})[7.9 \text{ m/s} - (-7.9 \text{ m/s})] = \boxed{7.0 \text{ kg} \cdot \text{m/s, up}}.$$

(b)  $F_{\text{av}} = J/\Delta t = (7.0 \text{ kg} \cdot \text{m/s})/(0.008 \text{ s}) = \boxed{9 \times 10^2 \text{ N, up}}.$

13. For the vertical motion, with up positive, we find the speed at the ground from

$$v^2 = v_0^2 + 2a \Delta y = 0 + 2(-9.8 \text{ m/s}^2)(-11 \text{ m}), \text{ which gives } v = 15 \text{ m/s}.$$

If the person does not bounce, the impulse on the person is

$$J_{\text{person}} = \Delta p = (80 \text{ kg})[0 - (-15 \text{ m/s})] = 1.2 \times 10^3 \text{ kg} \cdot \text{m/s}. \text{ Then}$$

$$J_{\text{net}} = -J_{\text{person}} = \boxed{-1.2 \times 10^3 \text{ kg} \cdot \text{m/s, down}}.$$

Once the person hits the net, we find the time to stop from

$$\Delta y = v_{\text{av}} \Delta t; \quad -0.70 \text{ m} = \frac{1}{2}[0 + (-15 \text{ m/s})] \Delta t, \text{ which gives } \Delta t = 0.093 \text{ s}. \text{ Thus}$$

$$F_{\text{av}} = J_{\text{person}}/\Delta t = (1.2 \times 10^3 \text{ kg} \cdot \text{m/s})/(0.093 \text{ s}) = \boxed{1.3 \times 10^4 \text{ N, up}}.$$

14. (a)  $J = \Delta p \approx (0.5 \text{ kg})(2 \text{ m/s}) \approx \boxed{1 \text{ kg} \cdot \text{m/s}}.$

- (b) We can estimate the speed of the football after the kick from the distance kicked.

For projectile motion we have

$$R = (2 v_0^2 \sin \theta \cos \theta)/g. \text{ If we assume } \theta = 45^\circ \text{ and } R = 50 \text{ m, we get}$$

$$v_0^2 = (50 \text{ m})(9.8 \text{ m/s}^2)/[2(0.7)(0.7)], \text{ which gives } v_0 \approx 22 \text{ m/s}. \text{ Thus}$$

$$J = \Delta p \approx (1 \text{ kg})(22 \text{ m/s}) \approx \boxed{22 \text{ kg} \cdot \text{m/s}}.$$

15. The required vertical jump is  $6'7'' - 3'10'' = [(2 + 9/12) \text{ ft}](0.305 \text{ m/ft}) = 0.84 \text{ m}$ . With up as positive, we find the initial speed for this jump from

$$v^2 = v_0^2 + 2a \Delta y;$$

$$0 = v_0^2 + 2(-9.8 \text{ m/s}^2)(0.84 \text{ m}), \text{ which gives } v_0 = 4.1 \text{ m/s}.$$

Thus the required impulse on the jumper is

$$J = \Delta p = mv_0 - 0 = (55 \text{ kg})(4.1 \text{ m/s}) = \boxed{2.2 \times 10^2 \text{ kg} \cdot \text{m/s, up}}.$$

16. (a) For this one-dimensional motion, we have  $J = p_f - p_i$ . For the satellite we get

$$J_{\text{satellite}} = (850 \text{ kg})(0.45 \text{ m/s} - 0) = \boxed{3.8 \times 10^2 \text{ kg} \cdot \text{m/s}}.$$

- (b) The average force is

$$F_{\text{av}} = J_{\text{satellite}}/\Delta t = (3.8 \times 10^2 \text{ kg} \cdot \text{m/s})/(0.85 \text{ s}) = \boxed{4.5 \times 10^2 \text{ N}}.$$

17. We find the speed falling or rising through a height  $h$  from energy conservation:

$$\frac{1}{2}mv^2 = mgh.$$

The downward speed as the ball hits the ground is

$$v_1 = \sqrt{2gh_1} = \sqrt{2(9.8 \text{ m/s}^2)(2.0 \text{ m})} = 6.3 \text{ m/s}.$$

The upward speed as the ball leaves the ground is

$$v_2 = \sqrt{2gh_2} = \sqrt{2(9.8 \text{ m/s}^2)(1.4 \text{ m})} = 5.2 \text{ m/s}.$$

The average force is

$$F_{\text{av}} = \Delta p / \Delta t = [m(v_2 - v_1)] / \Delta t = (0.260 \text{ kg})[(5.2 \text{ m/s}) - (-6.3 \text{ m/s})] / (0.004 \text{ s}) = \boxed{7.5 \times 10^2 \text{ N, up}}.$$

18. (a)  $J = \Delta p = (0.05 \text{ kg})(100 \text{ m/s} - 0) = \boxed{5 \text{ kg} \cdot \text{m/s}}.$   
 (b)  $F_{\text{av}} = J / \Delta t = (5 \text{ kg} \cdot \text{m/s}) / (0.02 \text{ s}) = \boxed{250 \text{ N}}.$   
 (c) For a linear function,  $F_{\text{av}} = \frac{1}{2}(F_0 + F_f) = \frac{1}{2}F_0$ . Thus  
 $F_0 = 2F_{\text{av}} = 2(250 \text{ N}) = \boxed{500 \text{ N}}.$

19. (a) We find the assumed constant acceleration from

$$v^2 = v_0^2 + 2a \Delta x;$$

$$0 = (45 \text{ m/s})^2 + 2a_{\text{av}}(0.25 \text{ m}), \text{ which gives } a_{\text{av}} = 4.1 \times 10^3 \text{ m/s}^2.$$

By applying Newton's second law, we get

$$F_{\text{av}} = ma_{\text{av}} = (0.14 \text{ kg})(4.1 \times 10^3 \text{ m/s}^2) = \boxed{5.7 \times 10^2 \text{ N}}.$$

- (b) The work done is  $W = F_{\text{av}} \Delta x = (5.7 \times 10^2 \text{ N})(0.25 \text{ m}) = \boxed{1.4 \times 10^2 \text{ J}}.$

- (c) The impulse exerted changes the momentum of the ball:

$$F_{\text{av}} \Delta t = \Delta p, \text{ from which we get}$$

$$\Delta t = \Delta p / F_{\text{av}} = (0.14 \text{ kg})(0 + 45 \text{ m/s}) / (5.7 \times 10^2 \text{ N}) = \boxed{0.011 \text{ s}}.$$

20. (a) For the vertical motion, with down positive, we find Lois' speed at the ground from

$$v^2 = v_0^2 + 2a \Delta y = 0 + 2(9.8 \text{ m/s}^2)(65 \text{ m}), \text{ which gives } v = 36 \text{ m/s}.$$

The impulse required to stop her is

$$J = \Delta p = (52.5 \text{ kg})(0 - 36 \text{ m/s}) = \boxed{-1.9 \times 10^3 \text{ kg} \cdot \text{m/s, up}}.$$

- (b) If the force is constant, the acceleration is constant; so

$$\Delta y' = \frac{1}{2}(v_0 + v) \Delta t; \quad 1.0 \text{ m} = \frac{1}{2}(36 \text{ m/s} + 0) \Delta t, \text{ from which we get } \Delta t = \boxed{0.056 \text{ s}}.$$

- (c) We find the average force from

$$F_{\text{av}} = J / \Delta t = (-1.87 \times 10^3 \text{ kg} \cdot \text{m/s}) / (0.056 \text{ s}) = \boxed{-3.3 \times 10^4 \text{ N, up}}.$$

For Lois,  $mg = 5.1 \times 10^2 \text{ N}$ , so  $F_{\text{av}} = \boxed{67 mg}$ . Ouch!

21. For a perfectly elastic collision, the ball will rebound to the same height above the step; the speed as it leaves a step will be the same as when it hits the step. For the first step:  $h_1 = 0.60 \text{ m}$ ; for the second step:  $h_2 = 0.80 \text{ m}$ ; for the  $N$ th step:  $h_N = 0.60 \text{ m} + (N - 1)0.20 \text{ m}$ .

For the fall and rise we use energy conservation to get  $v_N = \sqrt{2gh_N}$ .

For the impulse at the  $N$ th step we have

$$J_N = \Delta p = m[v_N - (-v_N)]$$

$$= 2mv_N = 2m\sqrt{2gh_N}$$

$$= 2(0.150 \text{ kg})\sqrt{2g[0.60 \text{ m} + (N - 1)(0.20 \text{ m})]}$$

$$= 0.30\sqrt{2g(0.40 + 0.20N)} \text{ kg} \cdot \text{m/s} = 0.60\sqrt{g(0.20 + 0.10N)} \text{ kg} \cdot \text{m/s}.$$

22. The initial speed of the ball is found from its range,  $R = v_0^2 \sin 2\theta_0 / g$ , to be  
 $v_0 = (gR / \sin 2\theta_0)^{1/2} = [(9.8 \text{ m/s}^2)(175 \text{ m}) / \sin (2 \times 30^\circ)]^{1/2} = 44.5 \text{ m/s}$ . Estimating the mass of the golf ball to be  $m = 50 \text{ g} = 0.05 \text{ kg}$ , we obtain  
 $J = \Delta p = mv_0 \approx (0.05 \text{ kg})(44.5 \text{ m/s}) = \boxed{2 \text{ kg} \cdot \text{m/s}}$  (in the direction of the initial velocity).
23. The speed  $v$  with which each ball hits the ground is  $v = (2gh)^{1/2} = [2(9.8 \text{ m/s}^2)(1.5 \text{ m})]^{1/2} = 5.42 \text{ m/s}$ . Taking up as the positive  $y$ -direction, then before the impact with the ground  $v_i = -5.42 \text{ m/s}$ , and after the impact  $v_f = +5.42 \text{ m/s}$  for the superball, and  $v_f = 0$  for the other ball. The impulse delivered by the floor to the superball is then  
 $J = \Delta p = m\Delta v = m(v_f - v_i) = (0.120 \text{ kg})[5.42 \text{ m/s} - (-5.42 \text{ m/s})] = +1.3 \text{ kg} \cdot \text{m/s}$ , up; while that delivered to the other ball is  
 $J' = (0.120 \text{ kg})[0 - (-5.42 \text{ m/s})] = +0.65 \text{ kg} \cdot \text{m/s}$ , up. The impulse received by the floor is  
 $-J = \boxed{-1.3 \text{ kg} \cdot \text{m/s}}$ , with the negative sign indicating that the direction of the impulse is **down**. Similarly, the impulse the floor receives from the other ball is  
 $-J' = \boxed{-0.65 \text{ kg} \cdot \text{m/s}}$ , directed **down**.
24. For this one-dimensional motion, we use a coordinate system on the ice and take the positive direction from the brother to the sister.
- (a) When the sister slides the rock, we use momentum conservation:  
 $p_{si} + p_{ri} = p_{s1} + p_{r1}$   
 $0 + 0 = (40 \text{ kg})v_{s1} + (2.0 \text{ kg})(-1.0 \text{ m/s})$ , which gives  
 $v_{s1} = \boxed{0.050 \text{ m/s}}$ .
- (b) When the brother receives the rock, we use momentum conservation:  
 $p_{b0} + p_{r1} = p_{b1} + p_{r2}$   
 $0 + (2.0 \text{ kg})(-1.0 \text{ m/s}) = (40 \text{ kg} + 2.0 \text{ kg})v_{b1}$ , which gives  
 $v_{b1} = \boxed{-0.048 \text{ m/s}}$ .
- (c) When the brother slides the rock, we use momentum conservation:  
 $p_{b1} + p_{r2} = p_{b2} + p_{r3}$   
 $(40 \text{ kg} + 2.0 \text{ kg})[(-1/21) \text{ m/s}] = (40 \text{ kg})v_{b2} + (2.0 \text{ kg})[(-1/21) + 1.0] \text{ m/s}$ , which gives  
 $v_{b2} = [(-1/21) - (1/20)] \text{ m/s} = \boxed{-0.098 \text{ m/s}}$ .
- (d) When the sister receives the rock, we use momentum conservation:  
 $p_{s1} + p_{r3} = p_{s2} + p_{r2}$   
 $(40 \text{ kg})[(1/20) \text{ m/s}] + (2.0 \text{ kg})[1.0 - (1/21)] \text{ m/s} = (40 \text{ kg} + 2.0 \text{ kg})v_{s2}$ , which gives  
 $v_{s2} = (41/21^2) \text{ m/s} = \boxed{0.093 \text{ m/s}}$ .
- (e) For each throw, the thrower's speed increases by  $\Delta v = (1/20) \text{ m/s}$ . Because the speed of the rock is  $1.0 \text{ m/s}$  with respect to the thrower, eventually the rock will not have a velocity toward the other child with respect to the ice greater than the velocity of that child and thus will not reach that child. The game does not go on forever.
25. For this one-dimensional motion, we take the direction of the first cart for the positive direction. For this perfectly inelastic collision, we use momentum conservation:  
 $mv_1 + Mv_2 = (M + m)V$   
 $(250 \text{ g})(1.2 \text{ m/s}) + (500 \text{ g})(-0.80 \text{ m/s}) = (750 \text{ g})V$ , which gives  
 $V = \boxed{-0.13 \text{ m/s}}$  (opposite to the direction of the first cart).
26. For this perfectly inelastic collision, we use momentum conservation:  
 $m_0v_0 + 0 = (m_0 + m)v$  to get  $v/v_0 = m_0/(m_0 + m)$ .  
 For the fractional change in kinetic energy, we write  
 $\Delta K / K_i = [(m_0 + m)v^2 - m_0v_0^2] / m_0v_0^2 = [(m_0 + m)/m_0] (v/v_0)^2 - 1$   
 $= [(m_0 + m)/m_0][m_0/(m_0 + m)]^2 - 1 = \boxed{-m/(m_0 + m)}$ .



27. For this one-dimensional motion, we take the direction of the snow ball for the positive direction.

For this perfectly inelastic collision, we use momentum conservation:

$$mv + 0 = (M + m)V;$$

$$(0.400 \text{ kg})(10 \text{ m/s}) = (5.40 \text{ kg})V, \text{ which gives } V = \boxed{0.74 \text{ m/s}}.$$

28. At the peak height the momentum is zero. If the first fragment moves straight down immediately after the explosion, the second fragment must move straight up. We use momentum conservation, with up positive:

$$P_i = P_{1f} + P_{2f}$$

$$0 = (1.1 \text{ kg})(-15 \text{ m/s}) + (2.7 \text{ kg})v_{2f}, \text{ which gives } v_{2f} = \boxed{6.1 \text{ m/s, straight up}}.$$

29. For this one-dimensional motion, we take the direction of the first object for the positive direction.

For this perfectly inelastic collision, we use momentum conservation:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)V;$$

$$m_1v + m_2(-v) = (m_1 + m_2)\frac{1}{2}v; \text{ which gives } m_1/m_2 = \boxed{3}.$$

30. Because momentum for the two-vehicle system is conserved, we can write

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f, \text{ where } m_1 = m_2 \text{ is the mass of each vehicle. Thus}$$

$$\vec{v}_f = \frac{1}{2}(\vec{v}_1 + \vec{v}_2) = \boxed{6.0 \text{ mi/h (north)} + 6.0 \text{ mi/h (west)}}.$$

Since  $\vec{v}_1$  and  $\vec{v}_2$  are perpendicular to each other the magnitude of the final velocity is

$$v_f = (v_1^2 + v_2^2)^{1/2} = [(6.0 \text{ mi/h})^2 + (6.0 \text{ mi/h})^2]^{1/2} = 8.5 \text{ mi/h, and } \vec{v}_f \text{ is in the northwest direction. So}$$

$$\vec{v}_f = \boxed{8.5 \text{ mi/h (northwest)}}.$$

31. Label the Moon with subscript  $m$  and the asteroid with  $a$ . Then from conservation of linear momentum

$$m_m\vec{v}_{mi} + m_a\vec{v}_{ai} = (m_m + m_a)\vec{v}_f, \text{ or}$$

$$\vec{v}_f = (m_m\vec{v}_{mi} + m_a\vec{v}_{ai}) / (m_m + m_a) \approx \vec{v}_{mi} + m_a\vec{v}_{ai} / m_m,$$

where we noted that  $m_m \gg m_a$  so  $m_m / (m_m + m_a) \approx 1$ . The change in velocity for the moon as a result of the impact is then

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_{mi} \approx (\vec{v}_{mi} + m_a\vec{v}_{ai} / m_m) - \vec{v}_{mi} = m_a\vec{v}_{ai} / m_m.$$

The fractional change in speed for the Moon is then  $\Delta v / v_{mi} \approx m_a v_{ai} / m_m v_{mi}$ . Here  $m_a \approx \frac{4}{3}\pi(0.75 \times 10^3 \text{ m})^3 (7.1 \times 10^3 \text{ kg/m}^3) = 1.3 \times 10^{13} \text{ kg}$ ;  $m_m = 7.36 \times 10^{22} \text{ kg}$ ; and  $v_{mi}$  can be estimated from the orbital radius of the Moon,  $R = 3.8 \times 10^5 \text{ km}$ , and the period of the Moon,  $T = 27.3 \text{ days}$ , as  $v_{mi} = 2\pi R / T = 2\pi (3.8 \times 10^8 \text{ m}) / (27.3 \times 86400 \text{ s}) = 0.99 \text{ km/s}$ . Thus

$$\Delta v / v_{mi} \approx m_a v_{ai} / m_m v_{mi} = (1.3 \times 10^{13} \text{ kg})(25 \text{ km/s}) / [(7.36 \times 10^{22} \text{ kg})(0.99 \text{ km/s})] = \boxed{4 \times 10^{-9}}.$$

32. The  $^{241}\text{Am}$  nucleus is at rest before emitting the alpha particle, so the alpha particle and  $^{237}\text{Np}$  nucleus must move in opposite directions, which makes this a one-dimensional situation.

(a) Because momentum is conserved, we can write

$$m_{\text{Am}}v_{\text{Am}} = m_{\text{He}}v_{\text{He}} + m_{\text{Np}}v_{\text{Np}};$$

$$0 = (4.0m_0)v_{\text{He}} + (237m_0)v_{\text{Np}}, \text{ which gives } v_{\text{Np}} = -(4.0/237)v_{\text{He}}.$$

The released energy must be the kinetic energies of the emitted products:

$$Q = \frac{1}{2}m_{\text{He}}v_{\text{He}}^2 + \frac{1}{2}m_{\text{Np}}v_{\text{Np}}^2;$$

$$9.6 \times 10^{-13} \text{ J} = \frac{1}{2}(4.0m_0)v_{\text{He}}^2 + \frac{1}{2}(237m_0)(4/237)^2v_{\text{He}}^2 = 2.034m_0v_{\text{He}}^2, \text{ so } v_{\text{He}} = 1.7 \times 10^7 \text{ m/s}.$$

$$\text{Thus } v_{\text{Np}} = \boxed{2.8 \times 10^5 \text{ m/s}}.$$

(b) The kinetic energies are

$$K_{\text{He}} = \frac{1}{2}(4.0)(1.66 \times 10^{-27} \text{ kg})(1.7 \times 10^7 \text{ m/s})^2 = \boxed{9.6 \times 10^{-13} \text{ J}};$$

$$K_{\text{Np}} = \frac{1}{2}(237)(1.66 \times 10^{-27} \text{ kg})(2.8 \times 10^5 \text{ m/s})^2 = \boxed{1.6 \times 10^{-14} \text{ J}}.$$

Note that almost all of the energy is carried off by the alpha particle.

33. We let  $V$  be the speed of the block and bullet immediately after the collision and before the pendulum swings.

For this perfectly inelastic collision, we use momentum conservation:

$$mv + 0 = (M + m)V, \text{ which gives } V/v = m/(M + m).$$

- (a) The fractional change in the kinetic energy is

$$\Delta K / K_i = [(M + m)V^2 - mv^2] / mv^2 \\ = [(M + m)/m](V/v)^2 - 1 = -M/(m + M),$$

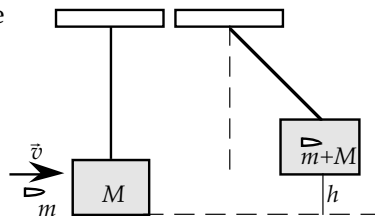
so the fraction lost =  $\boxed{M/(m + M)}$ .

- (b) For the pendulum motion we use energy conservation:

$$\frac{1}{2}(M + m)V^2 = (m + M)gh.$$

We combine this with the result from momentum conservation to get

$$V = \sqrt{2gh} = mv/(m + M), \text{ which gives } v = [(m + M)/m]\sqrt{2gh}.$$



34. From the result for Problem 33, we have

$$v = [(m + M)/m]\sqrt{2gh} = [(0.018 \text{ kg} + 1.8 \text{ kg})/(0.018 \text{ kg})] [2(9.8 \text{ m/s}^2)(0.045 \text{ m})]^{1/2} = \boxed{95 \text{ m/s}}.$$

35. We use a coordinate system with the positive direction opposite to the direction the persons jump.

- (a) For the first jump, with velocities relative to the ice, we use momentum conservation:

$$p_{1i} + p_{2i} + p_{si} = p_{1f} + p_{2f} + p_{sf};$$

$$0 + 0 + 0 = m_1(-v) + (m_2 + M)v', \text{ from which we get } v' = m_1v/(m_2 + M).$$

For the second jump, with velocities relative to the ice, we use momentum conservation:

$$p_2' + p_s' = p_{2f} + p_{sf}$$

$$(m_2 + M)v' = m_2(v' - v) + Mv_f, \text{ from which we get}$$

$$v_f = (Mv' + m_2v)/M.$$

Using the result from the first jump, we get  $v_f = \boxed{v[m_2^2 + M(m_1 + m_2)]/[M(m_2 + M)]}$  forward.

- (b) For the first jump, with velocities relative to the ice, we use momentum conservation:

$$p_{1i} + p_{2i} + p_{si} = p_{1f} + p_{2f} + p_{sf};$$

$$0 + 0 + 0 = m_1v' + m_2(-v) + Mv', \text{ from which we get } v' = m_2v/(m_1 + M).$$

For the second jump, with velocities relative to the ice, we use momentum conservation:

$$p_1' + p_s' = p_{1f} + p_{sf};$$

$$(m_1 + M)v' = m_1(v' - v) + Mv_f, \text{ from which we get}$$

$$v_f = (Mv' + m_1v)/M.$$

Using the result from the first jump, we get  $v_f = \boxed{v[m_1^2 + M(m_1 + m_2)]/[M(m_1 + M)]}$  forward.

- (c) When they jump together, we use momentum conservation:

$$p_{1i} + p_{2i} + p_{si} = p_{1f} + p_{2f} + p_{sf};$$

$$0 + 0 + 0 = (m_1 + m_2)(-v) + Mv_f, \text{ from which we get } v_f = \boxed{v(m_1 + m_2)/M}$$
 forward.

36. (a) Because momentum is conserved, we can write

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_3 + m_2\vec{v}_4;$$

$$(0.30 \text{ kg})(1.4 \text{ m/s})\hat{i} + (0.15 \text{ kg})(-2.5 \text{ m/s})\hat{i} = (0.30 \text{ kg})v_3\hat{i} + (0.15 \text{ kg})v_4\hat{i}.$$

Because the collision is elastic, the relative velocity does not change:

$$\vec{v}_1 - \vec{v}_2 = -(\vec{v}_3 - \vec{v}_4), \text{ or } 1.4\hat{i} \text{ m/s} - (-2.5\hat{i} \text{ m/s}) = -(v_3\hat{i} - v_4\hat{i}).$$

Combining these two equations, we get

$$\boxed{v_3 = -1.2 \text{ m/s} \text{ and } v_4 = +2.7 \text{ m/s}}.$$

- (b)  $K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\ = \frac{1}{2}(0.30 \text{ kg})(1.4 \text{ m/s})^2 + \frac{1}{2}(0.15 \text{ kg})(-2.5 \text{ m/s})^2 \\ = \boxed{0.76 \text{ J}}.$

37. (a) Because momentum is conserved, we can write

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4;$$

$$(0.4 \text{ kg})(3.0 \text{ m/s}) + (0.8 \text{ kg})(0) = (0.4 \text{ kg})v_3 + (0.8 \text{ kg})(1.6 \text{ m/s}), \text{ which gives}$$

$$v_3 = \boxed{-0.20 \text{ m/s}}.$$

- (b) We find the kinetic energy before and after the collision:

$$K_i = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}(0.4 \text{ kg})(3.0 \text{ m/s})^2 = 1.8 \text{ J};$$

$$K_f = \frac{1}{2}m_1 v_3^2 + \frac{1}{2}m_2 v_4^2 = \frac{1}{2}(0.4 \text{ kg})(-0.2 \text{ m/s})^2 + \frac{1}{2}(0.8 \text{ kg})(1.6 \text{ m/s})^2 = 1.03 \text{ J}.$$

Because kinetic energy is lost, the collision is **inelastic**.

The maximum kinetic energy is lost when the collision is perfectly inelastic.

For momentum conservation of this type of collision, we write

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f, \text{ which gives}$$

$$v_f = [(0.4 \text{ kg})(3.0 \text{ m/s}) + (0.8 \text{ kg})(0)] / (0.4 \text{ kg} + 0.8 \text{ kg}) = 1.0 \text{ m/s}.$$

Thus the maximum kinetic energy change is

$$\Delta K_{\text{max}} = \frac{1}{2}(1.2 \text{ kg})(1.0 \text{ m/s})^2 - 1.80 \text{ J} = -1.20 \text{ J}.$$

The percentage of this that is lost in the actual collision is

$$\Delta K / \Delta K_{\text{max}} = [(1.03 \text{ J} - 1.80 \text{ J}) / (-1.20 \text{ J})](100\%) = \boxed{64\%}.$$

38. (a) For this perfectly inelastic collision, we use momentum conservation:

$$mv + 0 = (m + M)V, \text{ which gives}$$

$$V = mv / (m + M) = (70 \text{ g})(450 \text{ m/s}) / (70 \text{ g} + 2600 \text{ g}) = \boxed{12 \text{ m/s}}.$$

- (b) For this elastic collision, the small mass of the bullet means that it appears to bounce from a stationary wall. Thus the velocity of the bullet changes direction only.

For momentum conservation we write:

$$mv + 0 = m(-v) + MV_{\text{block}}, \text{ which gives}$$

$$V_{\text{block}} = 2(70 \text{ g})(450 \text{ m/s}) / (2600 \text{ g}) = \boxed{24 \text{ m/s in the original direction of the bullet}}.$$

39. When the large mass rebounds perfectly elastically from the wall, its kinetic energy does not change, so its velocity reverses direction with the same magnitude.

For the elastic collision of the two masses, we use momentum conservation:

$$m v_1 + M v_2 = m v_3 + M v_4;$$

$$(0.126 \text{ kg})(0.875 \text{ m/s}) + (9.66 \text{ kg})(-0.875 \text{ m/s}) = (0.126 \text{ kg})v_3 + (9.66 \text{ kg})v_4.$$

Because the collision is elastic, the relative speed does not change:

$$v_1 - v_2 = -(v_3 - v_4), \text{ or}$$

$$0.875 \text{ m/s} - (-0.875 \text{ m/s}) = -(v_3 - v_4).$$

Combining these two equations, we get

$$v_4 = -0.84 \text{ m/s} \quad \text{and} \quad v_3 = -2.59 \text{ m/s}, \text{ so the return speed is } \boxed{2.59 \text{ m/s}}.$$

40. In a time  $\Delta t$  the number of bullets hitting the wall will be  $N = 60 \Delta t$ .

- (a) If the bullets stop in the wall, the momentum change will be

$$\Delta p = N[0 - (0.020 \text{ kg})(300 \text{ m/s})] = -360 \Delta t.$$

The average force exerted on the the wall is the reaction to the force on the bullets:

$$F_{\text{wall}} = -\Delta p / \Delta t = -(-360 \Delta t) / \Delta t = \boxed{360 \text{ N}}.$$

- (b) If the bullets rebound elastically, the velocity reverses direction, so  $v' = -v$ . Thus

$$\Delta p = N[(0.020 \text{ kg})(-300 \text{ m/s}) - (0.020 \text{ kg})(300 \text{ m/s})] = -720 \Delta t.$$

The average force exerted on the the wall is the reaction to the force on the bullets:

$$F_{\text{wall}} = -\Delta p / \Delta t = -(-720 \Delta t) / \Delta t = \boxed{720 \text{ N}}.$$

41. Because the tension does no work, when a pendulum of mass  $m$  swings through an angle  $\theta$ , we use energy conservation to find the speed at the bottom:

$$v = \sqrt{2gh} = \sqrt{2g(L - L \cos \theta)}.$$

The speed of the lighter mass just before the collision is

$$v_1 = \sqrt{2gh} = \sqrt{2g(L - L \cos \theta_1)} = \sqrt{2(9.8 \text{ m/s}^2)(1.00 \text{ m})(1 - \cos 80^\circ)} = 4.0 \text{ m/s}.$$

For the elastic collision of the two masses, we use momentum conservation:

$$mv_1 + Mv_2 = mv_3 + Mv_4;$$

$$(0.400 \text{ kg})(4.0 \text{ m/s}) + (0.600 \text{ kg})(0) = (0.400 \text{ kg})v_3 + (0.600 \text{ kg})v_4.$$

Because the collision is elastic, the relative speed does not change:

$$v_1 - v_2 = -(v_3 - v_4), \text{ or } 4.0 \text{ m/s} - 0 = -(v_3 - v_4).$$

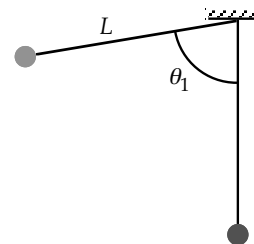
Combining these two equations, we get

$$v_3 = -0.80 \text{ m/s}. \text{ The negative sign indicates a rebound.}$$

We find the angle the lighter mass swings after the collision from

$$v_3 = \sqrt{2g(L - L \cos \theta_3)}; \quad 0.80 \text{ m/s} = \sqrt{2(9.8 \text{ m/s}^2)(1.00 \text{ m})(1 - \cos \theta_3)},$$

which gives  $\theta_3 = 15^\circ$ .



42. (a) We use energy conservation to find the speed at the bottom:

$$v = \sqrt{2gh} = \sqrt{2gL}.$$

The speed of the lighter mass just before the collision is

$$v_1 = [2(9.8 \text{ m/s}^2)(0.95 \text{ m})]^{0.5} = 4.3 \text{ m/s}.$$

For the elastic collision of the two masses, we use momentum conservation:

$$mv_1 + Mv_2 = mv_3 + Mv_4;$$

$$(0.37 \text{ kg})(4.3 \text{ m/s}) + (0.56 \text{ kg})(0) = (0.37 \text{ kg})v_3 + (0.56 \text{ kg})v_4.$$

Because the collision is elastic, the relative speed does not change:

$$v_1 - v_2 = -(v_3 - v_4), \text{ or } 4.3 \text{ m/s} - 0 = -(v_3 - v_4).$$

Combining these two equations, we get

$$v_3 = -0.9 \text{ m/s} \text{ and } v_4 = 3.4 \text{ m/s}. \text{ The negative sign indicates a rebound.}$$

- (b) After the collision, we find the height to which the mass  $m$  rebounds from

$$h_3 = v_3^2 / 2g = (-0.9 \text{ m/s})^2 / [2(9.8 \text{ m/s}^2)] = 0.04 \text{ m}.$$

43. (a) Because both objects fall the same distance  $h$ , we find their speed just before they hit the floor from

$$v_1 = \sqrt{2gh}.$$

- (b) If the ball rebounds elastically, its velocity reverses direction, so  $v_2 = -v_1 = -\sqrt{2gh}$ .

The speed of each object will be  $\sqrt{2gh}$ .

- (c) For the elastic collision of the two objects, we use momentum conservation:

$$mv_1 + Mv_2 = mv_3 + Mv_4;$$

$$m\sqrt{2gh} + M(-\sqrt{2gh}) = mv_3 + Mv_4.$$

Because the collision is elastic, the relative speed does not change:

$$v_1 - v_2 = -(v_3 - v_4), \text{ or } \sqrt{2gh} - (-\sqrt{2gh}) = -(v_3 - v_4).$$

Combining these two equations, we get

$$v_3 = -[(3M - m)/(m + M)]\sqrt{2gh} = [(m - 3M)/(m + M)]\sqrt{2gh}, \text{ up.}$$

- (d) We find the rebound height for the marble from

$$|v_3| = \sqrt{2gh'}; \quad |(m - 3M)/(m + M)|\sqrt{2gh} = \sqrt{2gh'}, \text{ which gives}$$

$$h' = [(m - 3M)/(m + M)]^2 h.$$

- (e) If  $M \gg m$ , we get  $h' = 9h$ .

44. For the elastic collision of the two masses, we use momentum conservation:

$$m(0) + Mv_i = mv + Mv_f;$$

Because the collision is elastic, the relative speed does not change:

$$0 - v_i = -(v - v_f).$$

Combining these two equations, we get

$$v = 2v_i/(1 + m/M) \text{ and } v_f = 2v_i/(1 + m/M) - v_i.$$

The fractional change of the large mass is

$$(v_i - v_f)/v_i = 1 - [2/(1 + m/M) - 1] \approx 1 - 2(1 - m/M) + 1 = \boxed{2m/M}.$$

45. For the collision of the two masses, we use momentum conservation:

$$m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} = m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (1.2 \text{ kg})(2.4 \text{ m/s})\hat{i} + (0.80 \text{ kg})(-3.6 \text{ m/s})\hat{i} = 0.$$

The total momentum of the system is 0, so we have

$$\vec{v}_{1f} = -(m_2/m_1) \vec{v}_{2f} = -[(0.80 \text{ kg})/(1.2 \text{ kg})](1.8 \text{ m/s}) \text{ } 60^\circ \text{ above the } x\text{-axis, which gives}$$

$$\vec{v}_{1f} = \boxed{1.2 \text{ m/s } 60^\circ \text{ below the } -x\text{-axis}}.$$

Because both speeds decrease, the total kinetic energy decreases. The collision is not elastic.

46. Because the bodies have equal but opposite momenta, the total momentum must be zero before and after the collision:

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A(v_1 \hat{i} + v_2 \hat{j}) + m_B \vec{v}_B';$$

$$m v \hat{i} + m(-v \hat{i}) = 0 = m(v_1 \hat{i} + v_2 \hat{j}) + m \vec{v}_B'; \text{ which gives}$$

$$\vec{v}_B' = \boxed{-v_1 \hat{i} - v_2 \hat{j}}.$$

47. For the collision of the two masses, we use momentum conservation:

$$m \vec{v} + 0 = m \vec{v}_1 + m \vec{v}_2;$$

$$(2.50 \text{ m/s}) \hat{i} + 0 = [(0.50 \text{ m/s}) \hat{i} + (-1.00 \text{ m/s}) \hat{j}] + \vec{v}_2, \text{ which gives}$$

$$\vec{v}_2 = \boxed{(2.00 \hat{i} + 1.00 \hat{j}) \text{ m/s}}.$$

To test whether the collision is elastic, we find the kinetic energy before and after:

$$K_i = \frac{1}{2} m (2.50 \text{ m/s})^2 = 3.13 \text{ m};$$

$$K_f = \frac{1}{2} m [(0.50 \text{ m/s})^2 + (-1.00 \text{ m/s})^2 + (2.00 \text{ m/s})^2 + (-1.00 \text{ m/s})^2] = 3.13 \text{ m}.$$

Because the kinetic energy is conserved, the collision is elastic.

Because the masses are the same, we could also show that the final velocities are perpendicular:

$$\vec{v}_1 \cdot \vec{v}_2 = [(0.50 \text{ m/s}) \hat{i} + (-1.00 \text{ m/s}) \hat{j}] \cdot [(2.00 \text{ m/s}) \hat{i} + (-1.00 \text{ m/s}) \hat{j}] = 0.$$

48. For an elastic collision, kinetic energy is conserved, so we have

$$K_f = K_i; \quad \frac{1}{2} m v_3^2 + \frac{1}{2} m v_4^2 = \frac{1}{2} m v_1^2 + 0;$$

$$(2.3 \text{ m/s})^2 + v_4^2 = (3.5 \text{ m/s})^2, \text{ which gives}$$

$$v_4 = \boxed{2.6 \text{ m/s}}.$$

49. From the discussion in the text, if the final velocities of the two equal masses are perpendicular, the collision is elastic. Kinetic energy is conserved, so we have

$$K_f = K_i; \quad \frac{1}{2} m v_3^2 + \frac{1}{2} m v_4^2 = \frac{1}{2} m v_1^2 + 0;$$

$$(1.5 \text{ m/s})^2 + v_4^2 = (3.0 \text{ m/s})^2, \text{ which gives } v_4 = \boxed{2.6 \text{ m/s}}.$$

50. We will take up and to the right as the positive directions.

(a) For the perfectly inelastic collision of the two masses, we use momentum conservation:

$$m_a v_{ai} + m_b v_{bi} = (m_a + m_b) \vec{v};$$

$$(2.0 \text{ kg})(5.0 \text{ m/s})\hat{i} + (3.0 \text{ kg})(-5.0 \text{ m/s})\hat{i} = (2.0 \text{ kg} + 3.0 \text{ kg})\vec{v}, \text{ which gives } \vec{v} = \boxed{-(1.0 \text{ m/s})\hat{i}}.$$

(b) The change in the kinetic energy is

$$\begin{aligned} \Delta K &= K_f - K_i = \frac{1}{2}(m_a + m_b)v^2 - \frac{1}{2}(m_a v_{ai}^2 + m_b v_{bi}^2) \\ &= \frac{1}{2}(2.0 \text{ kg} + 3.0 \text{ kg})(1.0 \text{ m/s})^2 - \frac{1}{2}[(2.0 \text{ kg})(5.0 \text{ m/s})^2 + (3.0 \text{ kg})(5.0 \text{ m/s})^2] = -60 \text{ J}. \end{aligned}$$

Thus,  $\boxed{60 \text{ J}}$  of kinetic energy is lost.

(c) For the perfectly inelastic collision of the two masses, we use momentum conservation:

$$m_a \vec{v}_{ai} + m_b \vec{v}_{bi} = (m_a + m_b) \vec{v};$$

$$(2.0 \text{ kg})(-5.0 \text{ m/s})\hat{j} + (3.0 \text{ kg})(-5.0 \text{ m/s})\hat{i} = (2.0 \text{ kg} + 3.0 \text{ kg})\vec{v}, \text{ which gives}$$

$$\vec{v} = \boxed{-(3.0\hat{i} + 2.0\hat{j})\text{ m/s}}.$$

(b) The change in the kinetic energy is

$$\begin{aligned} \Delta K &= K_f - K_i = \frac{1}{2}(m_a + m_b)v^2 - \frac{1}{2}(m_a v_{ai}^2 + m_b v_{bi}^2) \\ &= \frac{1}{2}(2.0 \text{ kg} + 3.0 \text{ kg})[(3.0 \text{ m/s})^2 + (2.0 \text{ m/s})^2] - \frac{1}{2}[(2.0 \text{ kg})(5.0 \text{ m/s})^2 + (3.0 \text{ kg})(5.0 \text{ m/s})^2] \\ &= -30 \text{ J}. \end{aligned}$$

Thus,  $\boxed{30 \text{ J}}$  of kinetic energy is lost.

51. We will take  $x$  to the right and  $y$  up. Because the speed of the bullet is so high, we ignore the effect of gravity and assume that it travels in a straight line until it hits the block. The time for the collision to take place is so small that we can ignore the effect of gravity on the system until after the collision.

For the perfectly inelastic collision of the two masses, we use momentum conservation:

$$x\text{-direction: } m_{\text{block}}v_{\text{block},i} - m_{\text{bullet}}v_{\text{bullet},i} \cos 60^\circ = (m_{\text{block}} + m_{\text{bullet}})v_x;$$

$$(0.80 \text{ kg})(10 \text{ m/s}) - (0.0050 \text{ kg})(550 \text{ m/s}) \cos 60^\circ = (0.805 \text{ kg})v_x, \text{ which gives } v_x = 8.2 \text{ m/s}.$$

$$y\text{-direction: } 0 - m_{\text{bullet}}v_{\text{bullet},i} \sin 60^\circ = (m_{\text{block}} + m_{\text{bullet}})v_y;$$

$$0 + (0.0050 \text{ kg})(550 \text{ m/s}) \sin 60^\circ = (0.805 \text{ kg})v_y, \text{ which gives } v_y = 3.0 \text{ m/s}.$$

The velocity of the block immediately after the collision is

$$\vec{v} = (8.2 \text{ m/s})\hat{i} + (3.0 \text{ m/s})\hat{j} = \boxed{8.7 \text{ m/s}, 20^\circ \text{ above the horizontal}}.$$

52. For the collision of the two masses, we use momentum conservation:

$$m_A \vec{v}_1 + m_B \vec{v}_2 = m_A \vec{v}_3 + m_B \vec{v}_4;$$

$$(0.15 \text{ kg})[(-1.7 \text{ m/s})\hat{i} - (2.0 \text{ m/s})\hat{j}] + (0.22 \text{ kg})(3.6 \text{ m/s})\hat{i} = 0 + (0.22 \text{ kg})\vec{v}_4, \text{ which gives}$$

$$\vec{v}_4 = (2.4 \text{ m/s})\hat{i} - (1.4 \text{ m/s})\hat{j}.$$

The kinetic energy of the second puck is

$$K_4 = \frac{1}{2}m_B v_4^2 = \frac{1}{2}(0.22 \text{ kg})[(2.4 \text{ m/s})^2 + (1.4 \text{ m/s})^2] = \boxed{0.85 \text{ J}}.$$

53. For the collision we use momentum conservation:

$$x\text{-direction: } m_{\text{car}}v_{\text{car},i} + 0 = m_{\text{car}}v_{\text{car},f} \cos \theta_{\text{car}} + m_{\text{truck}}v_{\text{truck},f} \cos \theta_{\text{truck}},$$

$$(1000 \text{ kg})v_{\text{car},i} =$$

$$(1000 \text{ kg})v_{\text{car},f} \cos 60^\circ + (1500 \text{ kg})(21.6 \text{ m/s}) \cos 33.7^\circ;$$

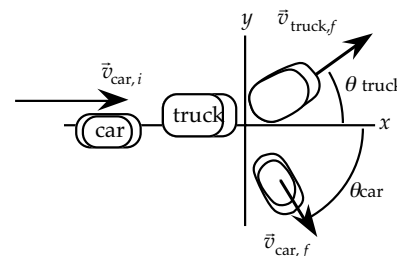
$$y\text{-direction: } 0 + 0 = -m_{\text{car}}v_{\text{car},f} \sin \theta_{\text{car}} + m_{\text{truck}}v_{\text{truck},f} \sin \theta_{\text{truck}},$$

$$0 = -(1000 \text{ kg})v_{\text{car},f} \sin 60^\circ + (1500 \text{ kg})(21.6 \text{ m/s}) \sin 33.7^\circ.$$

When we solve these two equations for the two unknowns, we get

$$v_{\text{car},f} = 20.8 \text{ m/s}, \text{ and } v_{\text{car},i} = 37.4 \text{ m/s} = 84 \text{ mi/h}.$$

**The driver of the car was speeding!**



54. (a) From the relationship between kinetic energy and momentum,  $K = p^2/2m$ , we have  $p = \sqrt{2mK}$ . For the collision of the two particles, we use momentum conservation, with  $\phi$  as the angle of the  $\alpha$ -particle below the  $x$ -axis:

$$p_{pi}\hat{i} + 0 = p_{pf}(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) + p_\alpha(\cos \phi \hat{i} - \sin \phi \hat{j}); \quad (b)$$

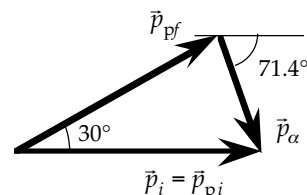
$$\sqrt{2mK} \hat{i} = \sqrt{2mK_p} (\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) + \sqrt{8mK_\alpha} (\cos \phi \hat{i} - \sin \phi \hat{j}).$$

Because kinetic energy is conserved, we have

$$K = K_p + K_\alpha$$

By combining this with the two component momentum equations, we get

$$\tan \phi = 2.97 \quad \text{or} \quad \phi = 71.4^\circ; \quad (c) \quad K_p = 0.935K \quad \text{and} \quad K_\alpha = 0.065K.$$



55. We choose the origin at the heavier sphere. The center of mass will lie along the line joining them:

$$X = \Sigma m_i x_i / \Sigma m_i = (0 + m_2 L) / (m_1 + m_2)$$

$$= (478 \text{ g})(2.24 \text{ m}) / (895 \text{ g} + 478 \text{ g}) = 0.78 \text{ m from the heavier sphere}.$$

56. We find the center of mass from

$$\vec{R} = \Sigma m_i \vec{r}_i / \Sigma m_i$$

$$= [(0.40 \text{ kg})0 + (0.40 \text{ kg})(0.35 \text{ m})\hat{j}] + (0.25 \text{ kg})[(0.15 \text{ m})\hat{i} + (0.58 \text{ m})\hat{j}] / (0.15 \text{ kg} + 0.40 \text{ kg} + 0.25 \text{ kg})$$

$$= (0.047\hat{i} + 0.36\hat{j}) \text{ m}.$$

57.  $X = \Sigma m_i x_i / M = [(1.6 \text{ kg})(0) + (1.8 \text{ kg})(1.2 \text{ m}) + (2.3 \text{ kg})(0)] / 5.7 \text{ kg}$   
 $= 0.38 \text{ m}$ , which is the same as in Example 8-12, since the origin was moved in the  $y$ -direction.  
 $Y = \Sigma m_i y_i / M = [(1.6 \text{ kg})(-1.1 \text{ m}) + (1.8 \text{ kg})(-1.1 \text{ m}) + (2.3 \text{ kg})(0)] / 5.7 \text{ kg}$   
 $= -0.66 \text{ m}$ , which is 1.1 m less than the result in Example 8-12.

58. For part (a):

$$\text{before and after: } v_{CM} = mv/3m = \frac{1}{3}v, \text{ in the initial direction of the first ball;}$$

For part (b):

$$\text{before: } v_{CM} = mv/3m = \frac{1}{3}v, \text{ in the initial direction of the first ball;}$$

$$\text{after: } v_{CM} = [m(-0.20v) + m(0.69v \cos 30^\circ) + m(0.69v \cos 30^\circ)] / 3m$$

$$= \frac{1}{3}v, \text{ in the initial direction of the first ball.}$$

In both cases,  $v_{CM} = \frac{1}{3}v$ , before and after, in the initial direction of the first ball.

59. Because the system of the children and the ball is isolated, the total momentum remains constant. Because the initial momentum is zero, the final speed of the center of mass will be zero.

60. The linear mass density of the iron bar is  $M/L = 7.5 \text{ kg}/0.75 \text{ m} = 10 \text{ kg/m}$ .

Since the mass of the bar is uniformly distributed, the center of the bar is at

$$\vec{R}_1 = (0 \text{ m})\hat{i} + (0.375 \text{ m})\hat{j}, \text{ and the other point mass of } 1.5 \text{ kg} \text{ is at } \vec{R}_2 = (0.25 \text{ m})\hat{i} + (0.375 \text{ m})\hat{j}.$$

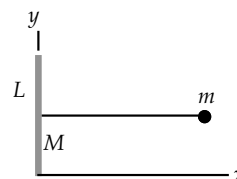
For the system of bar and point mass, we have

$$X = \Sigma m_i x_i / \Sigma m_i$$

$$= [(7.5 \text{ kg})(0) + (1.5 \text{ kg})(0.25 \text{ m})] / (7.5 \text{ kg} + 1.5 \text{ kg}) = 0.042 \text{ m};$$

$$Y = \Sigma m_i y_i / \Sigma m_i$$

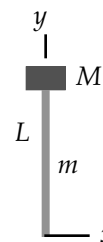
$$= [(7.5 \text{ kg})(0.375 \text{ m}) + (1.5 \text{ kg})(0.375 \text{ m})] / (7.5 \text{ kg} + 1.5 \text{ kg}) = 0.38 \text{ m}.$$



61. We ignore the dimensions of the iron block and treat it as a point mass. From the analysis at the beginning of Problem 56, we know that the center of mass of the handle is at its midpoint. We choose the origin at the bottom of the handle.

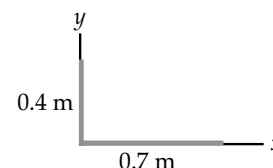
For the system, we have

$$\begin{aligned} X &= \Sigma m_i x_i / \Sigma m_i \\ &= [(4.0 \text{ kg})(0) + (1.8 \text{ kg})(0)] / (4.0 \text{ kg} + 1.8 \text{ kg}) = 0. \\ Y &= \Sigma m_i y_i / \Sigma m_i \\ &= [(4.0 \text{ kg})(1.2 \text{ m}) + (1.8 \text{ kg})(0.6 \text{ m})] / (4.0 \text{ kg} + 1.8 \text{ kg}) \\ &= 1.0 \text{ m from the bottom of the handle.} \end{aligned}$$



62. We choose the coordinate system shown in the diagram. The center of mass of each segment will be at the center of the segment. The linear mass density is  $\lambda = M / 1.1 \text{ m}$ .

$$\begin{aligned} X &= \Sigma m_i x_i / \Sigma m_i = [\lambda(0.25 \text{ m})(0) + \lambda(1.1 \text{ m})(0.55 \text{ m})] / [\lambda(1.35 \text{ m})] = 0.45 \text{ m}; \\ Y &= \Sigma m_i y_i / \Sigma m_i = [\lambda(0.25 \text{ m})(0.125 \text{ m}) + \lambda(1.1 \text{ m})(0)] / [\lambda(1.35 \text{ m})] = 0.023 \text{ m}. \end{aligned}$$



63. For a thin, spherical shell of uniform density, for every small element of mass  $dm$  there will be an identical element diametrically opposite. The center of mass of these two will be midway between, or at the center of the shell. The entire shell can be thought of as many such pairs, so the center of mass of the shell must be at the center. Another approach is to think of rotating the shell. If the center of mass were not at the center, it would move; however, because the shell would still look the same, the center of mass cannot move and must be at the center.

A solid sphere can be treated as a collection of shells. If the density depends only on distance from the center, each shell has uniform density and will have its center of mass at the center. Thus the center of mass of the sphere (all of the shells) will be at the center.

64. The uniform area mass density of the object is

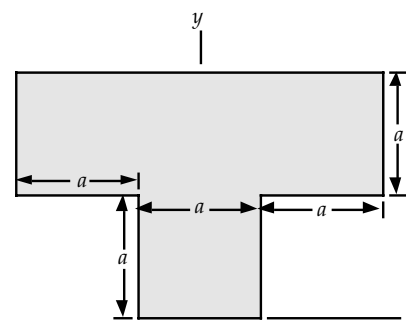
$$\sigma = M / A = M / 4a^2.$$

We choose the  $x$ -axis along the bottom of the T and the  $y$ -axis up through the center of the T.

By symmetry, we have  $X = 0$ .

For the  $y$ -position of the center of mass, we treat the object as two rectangles and integrate:

$$\begin{aligned} Y &= \frac{1}{M} \iint \alpha y \, dx \, dy = \frac{\sigma}{M} \left( \int_{-a/2}^{a/2} dx \int_0^a y \, dy + \int_{-3a/2}^{-a/2} dx \int_a^{2a} y \, dy \right) \\ &= \frac{\sigma}{M} \left( a \frac{y^2}{2} \Big|_0^a + 3a \frac{y^2}{2} \Big|_a^{2a} \right) = \frac{Ma}{4a^2 M} \left[ \frac{a^2}{2} \Big|_0^a + 3 \frac{(4a^2 - a^2)}{2} \Big|_a^{2a} \right] = \frac{5a}{4}. \end{aligned}$$



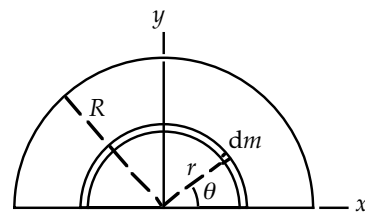


65. The uniform area mass density of the object is  $\sigma = M/A = 2M/\pi R^2$ .

The shape of the object suggests the polar coordinate system shown in the diagram.

By symmetry, we have  $X = 0$ .

For the  $y$ -position of the center of mass, we select a semi-circular arc at radius  $r$  of thickness  $dr$ . Within this arc we select a representative  $dm$  at the angle  $\theta$ , which has area  $r d\theta dr$  and mass  $dm = \sigma r d\theta dr$ . Then  $y = r \sin \theta$ . We find  $Y$  by integration over  $\theta$  and  $r$ :



$$\begin{aligned} Y &= \frac{1}{M} \int y \sigma dA = \frac{\sigma}{M} \int_0^R r dr \int_0^\pi r \sin \theta d\theta = \frac{\sigma}{M} \int_0^R r^2 dr \int_0^\pi \sin \theta d\theta \\ &= \frac{\sigma}{M} \left( \frac{R^3}{3} \right) (-\cos \theta) \Big|_0^\pi = -\frac{\sigma R^3}{3M} [(-1) - (+1)] \\ &= \frac{2\sigma R^3}{3M} = \frac{2}{3} \left( \frac{2M}{\pi R^2} \right) R^3 = \frac{4R}{3\pi} \text{ from the center of the arc, along the bisector.} \end{aligned}$$

66. Due to the symmetry nature of the problem the center of mass of the drilled cylinder is still on its axis of symmetry. We can effectively think of the drilled cylinder as a whole cylinder plus a “negative mass” that fills the drilled hole. Label the cylinder and the “negative mass” with subscripts 1 and 2, respectively, and denote the density of the cylinder as  $\rho$ . Then

$$m_2/m_1 = -\rho V_2/\rho V_1 = -V_2/V_1 = -\pi R_2^2 h_2 / \pi R_1^2 h_1 = -(5 \text{ cm})^2 (30 \text{ cm}) / [(15 \text{ cm})^2 (100 \text{ cm})] = -1/30.$$

Measured from the center of the top of the hole, then,

$$\begin{aligned} x_{\text{cm}} &= (m_1 x_1 + m_2 x_2) / (m_1 + m_2) = (x_1 + x_2 m_2 / m_1) / (1 + m_2 / m_1) \\ &= [50 \text{ cm} + 15 \text{ cm} (-1/30)] / [1 + (-1/30)] \\ &= 51 \text{ cm}, \end{aligned}$$

so the center of mass is 51 cm from the top of the hole, or  $100 \text{ cm} - 51 \text{ cm} = 49 \text{ cm}$  from the undrilled side, on the axis of the cylinder.

67.  $X = \Sigma m_i x_i / \Sigma m_i$   
 $= [(1 \text{ kg})(0) + (1 \text{ kg})(1 \text{ m}) + (2 \text{ kg})(1 \text{ m}) + (1 \text{ kg})(0 \text{ m})] / (1 \text{ kg} + 1 \text{ kg} + 2 \text{ kg} + 1 \text{ kg})$   
 $= \underline{0.6 \text{ m}};$   
 $Y = \Sigma m_i y_i / \Sigma m_i$   
 $= [(1 \text{ kg})(0) + (1 \text{ kg})(0 \text{ m}) + (2 \text{ kg})(1 \text{ m}) + (1 \text{ kg})(1 \text{ m})] / (1 \text{ kg} + 1 \text{ kg} + 2 \text{ kg} + 1 \text{ kg})$   
 $= \underline{0.6 \text{ m}}.$

If the 2-kg mass is replaced by a 1-kg mass then the system becomes symmetrical about the center of the square defined by the location of the four masses. The center of mass is at the center of the square, so

$$X = \underline{0.5 \text{ m}} \text{ and } Y = \underline{0.5 \text{ m}}.$$

68. We find the center of mass from the integral:

$$\vec{R} = (1/M) \iint \sigma \vec{r} dx dy.$$

If we divide the object into a number of masses  $M_i$ , this becomes

$$\vec{R} = (1/M) \iint \sigma \vec{r} dx dy = (1/M) \Sigma [M_i (1/M_i) \iint \sigma_i \vec{r} dx dy].$$

The center of mass of the  $i$ th section is  $\vec{R}_i = (1/M_i) \iint \sigma_i \vec{r} dx dy$ , so we have

$$\vec{R} = (1/M) \Sigma (M_i \vec{R}_i).$$

69. We find the total mass of the stick from the integral:

$$M = \int_0^L \lambda(x) dx = \int_0^L \left( \frac{K}{x^2 + a^2} \right) dx = \frac{K}{a} \tan^{-1} \left( \frac{L}{a} \right).$$

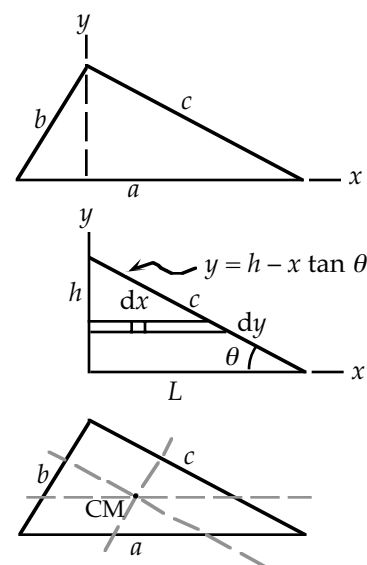
We find the center of mass from the integral:

$$\begin{aligned} X &= \frac{1}{M} \int_0^L \lambda(x)x dx = \frac{K}{M} \int_0^L \left( \frac{x}{x^2 + a^2} \right) dx = \frac{K}{2M} \ln(x^2 + a^2) \Big|_0^L \\ &= \frac{1}{2} \frac{\ln[(L^2 + a^2)/a^2]}{\tan^{-1}(L/a)}. \end{aligned}$$

70. We orient the triangle and choose the coordinate system shown in the diagram. We can treat the triangle as two right triangles, each of height  $h$ , as shown. To find the  $y$ -position of the center of mass of the triangle on the right, we select a horizontal strip at position  $y$  with height  $dy$ . The slope of the hypotenuse is  $-\tan \theta$ , using the angle shown on the diagram. The equation of the line in terms of the angle  $\theta$  is  $y = h - x \tan \theta$ . The horizontal strip lies between  $x = 0$  and  $x = (h - y)/\tan \theta$ . The area of the triangle is  $\frac{1}{2}Lh$ . We find  $Y$  by integration:

$$\begin{aligned} Y &= \frac{1}{M} \iint \sigma y dx dy = \frac{\sigma}{\sigma_1 Lh} \int_0^h y dy \int_0^{(h-y)/\tan \theta} dx \\ &= \frac{2}{Lh} \int_0^h y \left( \frac{h-y}{\tan \theta} \right) dy = \frac{2}{Lh \tan \theta} \left( \frac{h^2 h}{2} - \frac{h^3}{3} \right) \\ &= \frac{2}{Lh \tan \theta} \left( \frac{h^3}{6} \right) = \frac{h^2}{3L \tan \theta} = \frac{h}{3}. \end{aligned}$$

Because this result does not depend on the distance  $L$ , we get the same result for the triangle on the left. Thus the position of the center of mass of the original triangle is  $1/3$  of the distance from the side  $a$  to the opposite apex. By repeating the analysis with side  $b$  or side  $c$  at the bottom, we see that the center of mass can be found at the intersection of three lines drawn parallel to the sides and  $1/3$  of the distance to the opposite apex.



71. The thrust of the rocket is

$$m dv/dt = -u_{\text{ex}} dm/dt;$$

$$10 \times 10^6 \text{ N} = -(3.0 \times 10^3 \text{ m/s}) dm/dt, \text{ which gives } dm/dt = \boxed{-3.3 \times 10^3 \text{ kg/s}}.$$

72. If the rocket has mass  $m_0$  and speed  $v_0$  at  $t = 0$ , the speed at time  $t$  is given by

$$v - v_0 = u_{\text{ex}} \ln(m_0/m);$$

$$5900 \text{ m/s} - 5800 \text{ m/s} = (2600 \text{ m/s}) \ln(2.0 \times 10^4 \text{ kg}/m), \text{ which gives}$$

$$m = 19,200 \text{ kg}, \text{ so the amount of mass lost is } 20,000 \text{ kg} - 19,200 \text{ kg} = \boxed{8.0 \times 10^2 \text{ kg}}.$$

73. Because the speed change is very small, we can use the differential form of the equation of motion:

$$dv = -u_{\text{ex}}(dm/m);$$

$$10 \text{ m/s} = -(10^3 \text{ m/s})(dm/m), \text{ which gives } dm/m = -10^{-2}. \text{ Thus } \boxed{1\% \text{ of mass must be discarded}}.$$

74. Because both the discarded fuel and the rocket are acted on by gravity, which is assumed constant, we can superimpose the two motions and write

$$v = u_{\text{ex}} \ln(m_0/m_{\text{final}}) - gt$$

$$3400 \text{ m/s} = (2700 \text{ m/s}) \ln(m_0/m_{\text{final}}) - (9.8 \text{ m/s}^2)(120 \text{ s}), \text{ which gives } m_{\text{final}}/m_0 = \boxed{0.18}.$$

75. Because both the discarded fuel and the rocket are acted on by gravity, which is assumed constant, we can superimpose the two motions and write

$$v = u_{\text{ex}} \ln(m_0/m) - gt = u_{\text{ex}} \ln[m_0/(m_0 - m_{\text{fuel}})] - gt.$$

From the data given, we get

$$v = (2800 \text{ m/s}) \ln[1/(1 - 0.7)] - (9.8 \text{ m/s}^2)(90 \text{ s}) = \boxed{2.5 \times 10^3 \text{ m/s}}.$$

76. From the velocity  $v = u_{\text{ex}} \ln[m_0/(m_0 - m_{\text{fuel}})] - gt$ , we differentiate to get the acceleration:

$$\begin{aligned} a &= dv/dt = u_{\text{ex}} [(m_0 - m_{\text{fuel}})/m_0] d[m_0/(m_0 - m_{\text{fuel}})]/dt - g \\ &= [u_{\text{ex}}/(m_0 - m_{\text{fuel}})] dm_{\text{fuel}}/dt - g. \end{aligned}$$

At  $t = 0$ , no fuel has yet been burned:  $m_{\text{fuel}} = 0$ . Thus we have

$$a = 2.5g = (u_{\text{ex}}/m_0) (dm_{\text{fuel}}/dt)_0 - g, \text{ which gives}$$

$$(dm_{\text{fuel}}/dt)_0 = 3.5gm_0/u_{\text{ex}} = 3.5(9.8 \text{ m/s}^2)(6.0 \times 10^4 \text{ kg})/(2800 \text{ m/s}) = \boxed{7.4 \times 10^2 \text{ kg/s}}.$$

77. First, find the speed  $v_f$  of the bullet-block system just after the collision, from conservation of momentum:  $m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$  or

$$\begin{aligned} v_f &= (m_1v_{1i} + m_2v_{2i})/(m_1 + m_2) \\ &= [(0.010 \text{ kg})(450 \text{ m/s}) + (0.40 \text{ kg})(0)]/(0.010 \text{ kg} + 0.40 \text{ kg}) = 11 \text{ m/s}. \end{aligned}$$

The time of flight of the block is found from  $h = \frac{1}{2}gt^2$  to be  $t = (2h/g)^{1/2} = [2(1.0 \text{ m})/9.8 \text{ m/s}^2]^{1/2} = 0.45 \text{ s}$ , during which time the horizontal component of the block's velocity remains 11 m/s. So the horizontal range of the block is  $R = v_ft = (11 \text{ m/s})(0.45 \text{ s}) = \boxed{5.0 \text{ m}}$ .

78. As in the previous problem, we need to find the horizontal velocity of the bullet (1) after it collides with the block (2). Use conservation of linear momentum:  $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ . Here  $v_{1i} = 450 \text{ m/s}$  and  $v_{2i} = 0$ . To find  $v_{1f}$ , first get  $v_{2f}$  from  $v_{2f} = R_2/t = 0.55 \text{ m}/0.45 \text{ s} = 1.22 \text{ m/s}$ . (Note that the time of flight remains 0.45 s, as calculated in the previous problem, since  $h$  has not changed.) Thus

$$\begin{aligned} v_{1f} &= (m_1v_{1i} + m_2v_{2i} - m_2v_{2f})/m_1 \\ &= [(0.010 \text{ kg})(450 \text{ m/s}) + (0.40 \text{ kg})(0) - (0.40 \text{ kg})(1.22 \text{ m/s})]/0.010 \text{ kg} = 402 \text{ m/s}, \end{aligned}$$

and the range of the bullet after the collision is

$$R_1 = v_{1f}t = (402 \text{ m/s})(0.45 \text{ s}) = \boxed{0.18 \text{ km}}.$$

79. (a) In the reference frame of the center of mass of the student, we use conservation of momentum, assuming he spits one seed of mass  $m$ :

$$0 + 0 = Mv + m(-u),$$

which means that each time he spits one seed his speed increases by  $mu/M$  with respect to the ice. After  $n$  seeds he will have a speed

$$v_n = nmu/M \text{ and after 100 seeds he will have a speed of } v_f = 100mu/M.$$

If he spits  $n$  seeds with a speed of  $u/n$ , again in the reference frame of the center of mass of the student, from conservation of momentum we get

$$0 + 0 = Mv + nm(-u/n),$$

which means that each time he spits  $n$  seeds his speed increases by  $mu/M$  with respect to the ice. After spitting 100 seeds in groups of  $n$ , his speed with respect to the ice will be

$$v_f = (100/n)(mu/M) = (100mu/M)/n, \text{ which is less than before.}$$

Thus, it is better to spit out seeds one at a time.

- (b)  $v_{\text{max}} = 100mu/M = (100)(1 \times 10^{-3} \text{ kg})(3 \text{ m/s})/(50 \text{ kg}) = \boxed{6 \times 10^{-3} \text{ m/s}}.$

80. Because both the discarded fuel and the rocket are acted on by gravity, which is assumed constant, we can superimpose the two motions. Until the first stage is discarded, the speed of the rocket is given by
- $$v = u_{\text{ex}} \ln(m_0/m) - gt.$$

If we call the time of separation of the first stage  $t_1$ , the speed at that time is

$$v_1 = u_{\text{ex}} \ln[m_0/(m_0 - m_1)] - gt_1.$$

This is the initial speed for the second stage. Because the first stage is released, the mass at this time is  $m_0 - (m_1 + m_1')$ . The speed during the second stage is given by

$$v - v_1 = u_{\text{ex}} \ln[m_0 - (m_1 + m_1')/m] - g(t - t_1).$$

At final burnout, the mass is  $m_0 - (m_1 + m_1' + m_2)$ . When we substitute for  $v_1$ , the final speed is

$$v = u_{\text{ex}} \ln\{m_0(m_0 - m_1 - m_1')/[(m_0 - m_1 - m_1' - m_2)(m_0 - m_1)]\} - gt, \text{ where up is positive.}$$

81. We choose the positive direction to be the original direction of motion, which we take to the right.

(a)  $v_{\text{CM}} = (m_1 v_1 + m_2 v_2)/(m_1 + m_2)$

$$= [(48 \text{ kg})(6 \text{ m/s}) + (82 \text{ kg})(9 \text{ m/s})]/(48 \text{ kg} + 82 \text{ kg}) = \boxed{7.9 \text{ m/s (to the right)}}.$$

- (b) Because there are no outside forces, the velocity of the center of mass will not change:

$$v_{\text{CM}} = \boxed{7.9 \text{ m/s (to the right)}}.$$

- (c) With respect to the center of mass, we have

$$v_1' = v_1 - v_{\text{CM}} = 6 \text{ m/s} - 7.9 \text{ m/s} = \boxed{-1.9 \text{ m/s (to the left)}};$$

$$v_2' = v_2 - v_{\text{CM}} = 9 \text{ m/s} - 7.9 \text{ m/s} = \boxed{+1.1 \text{ m/s (to the right)}}.$$

- (d) We find the average force on the skaters from the momentum change:

$$F_{\text{av}} = \Delta p / \Delta t = (48 \text{ kg} + 82 \text{ kg})(0 - 7.9 \text{ m/s}) / 0.05 \text{ s} = -2.1 \times 10^4 \text{ N (to the left)}.$$

Therefore, the force on the barrier is the reaction to this force:  $\boxed{2.1 \times 10^4 \text{ N (to the right)}}.$

82. We find the speed falling or rising through a height  $h$  from energy conservation:  $\frac{1}{2}mv^2 = mgh$ .

The speed of the first car just before the collision is

$$v_1 = [2(9.8 \text{ m/s}^2)(1.8 \text{ m})]^{1/2} = 5.94 \text{ m/s}.$$

For the conservation of momentum during the collision we write

$$m_1 v_1 + 0 = (m_1 + m_2)v, \text{ which gives}$$

$$v = m_1 v_1 / (m_1 + m_2) = (16 \text{ Mg})(5.94 \text{ m/s}) / (16 \text{ Mg} + 8 \text{ Mg}) = 3.96 \text{ m/s}.$$

We find the height where they stop at

$$h = v^2 / 2g = (3.96 \text{ m/s})^2 / [2(9.8 \text{ m/s}^2)] = \boxed{0.80 \text{ m}}.$$

83. (a)

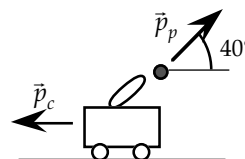
- (b) In the horizontal direction momentum is conserved:

$$0 = p_p \cos 35^\circ - p_c;$$

$$0 = (5 \text{ kg})(8800 \text{ m/s}) \cos 35^\circ - (800 \text{ kg})v_c, \text{ which gives}$$

$$v_c = \boxed{4.1 \text{ m/s recoil}}.$$

- (c) The recoiling cannon's component of momentum perpendicular to the ground remains zero because of the upward impulse provided by the ground.



84. We find the speed acquired by the sandbag during the collision by applying energy conservation to the swinging motion after collision:

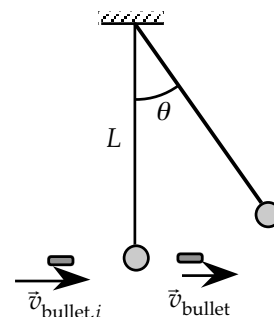
$$\frac{1}{2}Mv_{\text{sandbag}}^2 = Mgh = MgL(1 - \cos \theta);$$

$$\frac{1}{2}v_{\text{sandbag}}^2 = (9.8 \text{ m/s}^2)(1.2 \text{ m})(1 - \cos 40^\circ), \text{ which gives } v_{\text{sandbag}} = 2.4 \text{ m/s}.$$

For the conservation of momentum during the collision we write

$$m_{\text{bullet}}v_{\text{bullet},i} + M(0) = m_{\text{bullet}}v_{\text{bullet}} + Mv_{\text{sandbag}};$$

$$(0.0080 \text{ kg})(600 \text{ m/s}) = (0.0080 \text{ kg})(250 \text{ m/s}) + M(2.4 \text{ m/s}), \text{ so } M = \boxed{1.2 \text{ kg}}.$$



85. We find the speed falling or rising through a height  $h$  from energy conservation:  $\frac{1}{2}mv^2 = mgh$ .

(a) The speed of each mass just before the collision is

$$v_1 = \sqrt{2gh}.$$

We choose the positive direction at the bottom in the direction of the motion of the larger mass.

For the conservation of momentum during the collision we write

$$Mv_1 + m(-v_1) = Mv_3 + mv_4, \text{ with both final velocities assumed to be positive.}$$

Because the collision is elastic, the relative speed does not change:

$$v_1 - (-v_1) = -(v_3 - v_4) \quad \text{or} \quad 2v_1 = -v_3 + v_4.$$

Combining these two equations, we get

$$v_4 = [(3M - m)/(M + m)]v_1 = [(3M - m)/(M + m)]\sqrt{2gh}.$$

We find the rebound height for the marble from

$$v_4 = \sqrt{2gh'}; \quad [(3M - m)/(M + m)]\sqrt{2gh} = \sqrt{2gh'}, \text{ which gives } h' = [(3M - m)/(M + m)]^2 h.$$

The overshoot is

$$h' - h = \boxed{[8M(M - m)/(M + m)^2]h}.$$

(b) For the conservation of momentum during the perfectly inelastic collision, we write

$$Mv_1 + m(-v_1) = (M + m)v, \text{ which gives } v = [(M - m)/(M + m)]\sqrt{2gh} = \sqrt{2gh''}.$$

From this we get

$$h'' = [(M - m)/(M + m)]^2 h$$

and the "overshoot" is

$$h'' - h = \boxed{[-4mM/(M + m)^2]h}. \quad (\text{The combined masses will not reach the lip.})$$

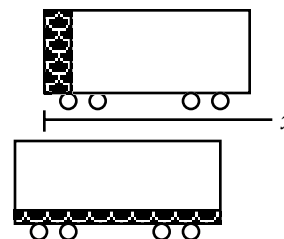
86. We choose the boxcar and the water as a system, with the origin at the end of the boxcar with the tank. Because there are no horizontal external forces, the horizontal momentum will always be zero and the center of mass of the system will not move horizontally.

Before the tank leaks, we find the center of mass from

$$\begin{aligned} X &= \Sigma m_i x_i / \Sigma m_i \\ &= [(1800 \text{ kg})(0.75 \text{ m}) + (4200 \text{ kg})(4.5 \text{ m})] / (1800 \text{ kg} + 4200 \text{ kg}) \\ &= 3.375 \text{ m.} \end{aligned}$$

After the tank empties, mass is distributed uniformly so the center of mass of mass of the system is 4.5 m from the left end of the boxcar. Therefore, the boxcar moves

$$4.5 \text{ m} - 3.375 \text{ m} = \boxed{1.1 \text{ m to the left}}.$$



87. This is a completely inelastic collision between the cowboy (1) and the steer (2), to which we apply conservation of momentum:  $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$ . Thus

$$v_f = (m_1 v_{1i} + m_2 v_{2i}) / (m_1 + m_2) = [(75 \text{ kg})(0) + (500 \text{ kg})(30 \text{ km/h})] / (75 \text{ kg} + 500 \text{ kg}) = \boxed{26 \text{ km/h}}.$$

88. We convert the speeds:  $v_p = 1600 \text{ km/h} = 445 \text{ m/s}$ ;  $v_0 = 35 \text{ km/h} = 9.7 \text{ m/s}$ .

(a) We find the time of flight of the projectile from

$$t = \Delta y / v_p = (30 \text{ m}) / (445 \text{ m/s}) = 0.067 \text{ s}.$$

The motion of the cyclist in the  $x$ -direction has constant velocity, so

$$x = x_0 + v_0 t;$$

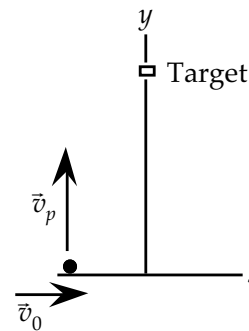
$$0 = x_0 + (9.7 \text{ m/s})(0.067 \text{ s}), \text{ which gives } x_0 = \boxed{-0.65 \text{ m}}.$$

(b) When the cyclist fires, momentum is conserved in the  $y$ -direction, so

$$0 = m_p v_p + m_c v_{cy} = (0.015 \text{ kg})(445 \text{ m/s}) + (65 \text{ kg})v_{cy},$$

which gives  $v'_{cy} = -0.103 \text{ m/s}$ .

The velocity of the cyclist is  $\vec{v} = \boxed{(9.7\hat{i} - 0.103\hat{j}) \text{ m/s}}$ , which has magnitude 9.7 m/s at an angle of  $0.61^\circ$  from the  $x$ -axis.



89. We know from the symmetry that the center of mass lies on a line between the center of the styrofoam and the center of the solid. We choose the center of the styrofoam as origin and  $y$  along the line joining the centers. Then  $X = 0$ .

A uniform sphere has its center of mass at its center. We can treat the system as three spheres:

a sphere of density  $\rho$  and radius  $R$  with  $Y_1 = 0$ ;

a sphere of density  $-\rho$  and radius  $\frac{1}{2}R$  with  $Y_2 = \frac{1}{2}R$ ;

a sphere of density  $5\rho$  and radius  $\frac{1}{2}R$  with  $Y_3 = \frac{1}{2}R$ .

The last two are equivalent to a sphere of density  $4\rho$  and radius  $\frac{1}{2}R$  with  $Y_4 = \frac{1}{2}R$ .

We find the center of mass from

$$Y = \Sigma m_i y_i / \Sigma m_i$$

$$= \{ \rho(\frac{4}{3}\pi R^3)(0) + 4\rho[\frac{4}{3}\pi(\frac{1}{2}R)^3](\frac{1}{2}R) \} / \{ \rho(\frac{4}{3}\pi R^3) + 4\rho[\frac{4}{3}\pi(\frac{1}{2}R)^3] \}. \text{ This reduces to}$$

$$Y = \boxed{R/6} \text{ from the center of the styrofoam sphere.}$$

90. Because the impulse is directed along the line joining the centers, we choose the  $x'y'$  coordinate system indicated on the diagram.

The angle between the two systems is  $\theta = \sin^{-1}[(1.0 \text{ cm}) / (6.0 \text{ cm})] = 9.6^\circ$ , and the unit vectors of the two systems transfer according to

$$\hat{i}' = \hat{i} \cos \theta - \hat{j} \sin \theta, \quad \hat{j}' = \hat{i} \sin \theta + \hat{j} \cos \theta.$$

The initial velocities are

$$\vec{v}_A = (1.5 \text{ m/s})\hat{i} = (1.5 \text{ m/s}) \cos \theta \hat{i}' + (1.5 \text{ m/s}) \sin \theta \hat{j}'$$

$$= (1.48 \text{ m/s})\hat{i}' + (0.25 \text{ m/s})\hat{j}'$$

$$\vec{v}_B = (-1.1 \text{ m/s})\hat{i} = (-1.1 \text{ m/s}) \cos \theta \hat{i}' + (-1.1 \text{ m/s}) \sin \theta \hat{j}'$$

$$= (-1.08 \text{ m/s})\hat{i}' + (-0.18 \text{ m/s})\hat{j}'.$$

The  $y'$ -components are not changed. In the  $x'$ -direction momentum is conserved, so we have

$$m(1.48 \text{ m/s}) + m(-1.08 \text{ m/s}) = mv_{A2x'} + mv_{B2x'}.$$

Because the collision is elastic, the relative speed in the  $x'$ -direction does not change:

$$v_{A1x'} - v_{B1x'} = -(v_{A2x'} - v_{B2x'}), \text{ or } 1.48 \text{ m/s} - (-1.08 \text{ m/s}) = -v_{A2x'} + v_{B2x'}.$$

Combining these two equations, we get

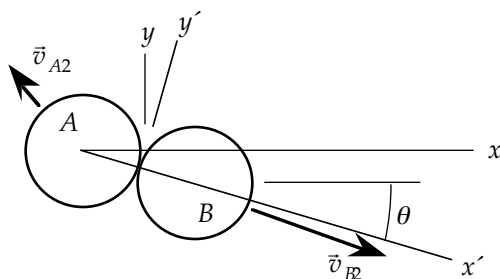
$$v_{A2x'} = -1.08 \text{ m/s}; \quad v_{B2x'} = 1.48 \text{ m/s}.$$

As expected for a perfectly elastic collision between equal masses, the velocities are exchanged.

When the final velocities are transformed back to the original system using the transformation formulas for the unit vectors, we get

$$\vec{v}_{A2} = (-1.08 \text{ m/s})\hat{i}' + (0.25 \text{ m/s})\hat{j}' = \boxed{(-1.0\hat{i} + 0.43\hat{j}) \text{ m/s}};$$

$$\vec{v}_{B2} = (1.48 \text{ m/s})\hat{i}' + (-0.18 \text{ m/s})\hat{j}' = \boxed{(1.4\hat{i} - 0.43\hat{j}) \text{ m/s}}.$$



91. We choose the positive direction in the direction of the slide. The velocity of the sled with respect to the ice is  $v_s$  and the velocity of the object with respect to the sled is  $v$ .

(a) Momentum is conserved for the system of students, sled, and object, so we have

$$0 = (m_1 + m_2 + m_s)v_s + M(v + v_s);$$

$$0 = (65 \text{ kg} + 65 \text{ kg} + 30 \text{ kg})v_s + (3 \text{ kg})(6 \text{ m/s} + v_s),$$

which gives  $v_s = -0.11 \text{ m/s}$  so the speed of the sled is  $\boxed{0.11 \text{ m/s}}$ .

(b) When the object is caught, we have

$$(m_1 + m_2 + m_s)v_s + M(v + v_s) = (m_1 + m_2 + m_s + M)v_f;$$

$$(160 \text{ kg})(-0.11 \text{ m/s}) + (3 \text{ kg})(6 \text{ m/s} - 0.11 \text{ m/s}) = (160 \text{ kg} + 3 \text{ kg})v_f,$$

which gives  $v_f = \boxed{0}$ , as expected.

(c) Because the object is moving at  $v$  with respect to the sled, we find the time of sliding from

$$\Delta t = L_s/v = (4 \text{ m})/(6 \text{ m/s}) = 0.67 \text{ s}.$$

In this time the sled moves

$$\Delta x_s = v_s \Delta t = (-0.11 \text{ m/s})(0.67 \text{ s}) = \boxed{-0.074 \text{ m}}.$$

(d) If we measure from the end where the object starts, the center of mass initially is

$$X_i = [(65 \text{ kg})(0) + (3 \text{ kg})(0) + (30 \text{ kg})(2 \text{ m}) + (65 \text{ kg})(4 \text{ m})]/(163 \text{ kg}) = 1.96 \text{ m}.$$

After the slide, we find the center of mass with respect to the sled and add the movement of the sled:

$$X_f = [(65 \text{ kg})(0) + (3 \text{ kg})(4 \text{ m}) + (30 \text{ kg})(2 \text{ m}) + (65 \text{ kg})(4 \text{ m})]/(163 \text{ kg}) - 0.074 \text{ m} = 1.96 \text{ m}.$$

Thus  $X_f - X_i = \boxed{0}$ ; the center of mass has not moved, as expected.

92. Rewrite  $P(t) = P(t + \Delta t)$  as  $P(t) = P(t + \Delta t) + mg \Delta t$ . This adds an additional term to the right-hand-side of Equation (8-64), which now reads

$$m\Delta v - (\Delta m)u_{ex} = mg \Delta t.$$

Taking  $\Delta t$  to be infinitesimally small and reversing the sign in front of  $dm/dt$ , as in the textbook, we get the modified version of Equation (8-65):

$$-u_{ex} \frac{dm}{dt} = m \frac{dv}{dt} + mg.$$

Divide each term by  $m$  and move all the terms to the left-hand-side:

$$-\frac{u_{ex}}{m} \frac{dm}{dt} - \frac{dv}{dt} - g = -u_{ex} \frac{d(\ln m)}{dt} - \frac{dv}{dt} - g = \frac{d}{dt}(-u_{ex} \ln m - v - gt) = 0,$$

which means

$$-u_{ex} \ln m - v - gt = \text{constant}.$$

To find the constant, set  $t = 0$ , when  $v = 0$  and  $m = m_0$ :  $-u_{ex} \ln m_0 = \text{constant}$ . Thus

$$-u_{ex} \ln m - v - gt = -u_{ex} \ln m_0, \text{ or}$$

$$v = u_{ex} (\ln m_0 - \ln m) - gt = u_{ex} \ln(m_0/m) - gt. \text{ This is Equation (8-70).}$$