

CHAPTER 7 Potential Energy and Conservation of Energy

Answers to Understanding the Concepts Questions

1. Drag forces are like friction in that they always oppose the motion and therefore depend on velocity (its direction, that is). They are therefore not conservative. When drag forces act, energy is lost to heating and turbulence in the medium responsible for the drag.
2. The total mechanical energy of a system is the sum of its kinetic and potential energies. Although the kinetic energy is never negative, the potential energy can be. In fact, by definition only the *difference* in potential energy between two states is physically significant; one is in fact allowed attach whatever constant to the potential energy without changing the difference. For example, if we set the gravitational potential energy of an object to be zero when it is on the 50th floor, its potential energy would be very negative when it is on the third floor, and the total mechanical energy may very well be negative.
3. It means that the value of the gravitational potential energy of the person at the bottom of the well is below that at the sea level; so, for example, positive work must be done on the person in order to bring him to the sea level.
4. The use of the conservation of energy in the analysis of motion depends on finding changes in the various terms of the energy. Changes in a particular energy term are independent of any constant in that term.
5. When dropped from rest the rubber ball has only gravitational potential energy. Ideally, if there is no loss of energy, the ball would return to the initial height, whereupon it regains all of its original potential energy back. In reality, of course, one would always expect some loss of mechanical energy, due to air friction as well as during the collision with the floor (in terms of heat and sound). So the ball will not be able to return to the same height since it does not have that much potential energy left. A ball throw down with a large initial speed has a significant amount of initial kinetic energy, which can be converted into gravitational potential energy, allowing it to attain a much greater height upon rebounding.
6. No. Consider, for example, the case when one stretches the spring by a certain length x and allow the spring to get back to the original length. As it is being stretched the force of the spring does an amount of work that is equal to $-\frac{1}{2}kx^2$. If k changes to k' when the spring starts to pull back, however, the work done by the spring as it gets back to its original length would be $+\frac{1}{2}k'x^2$. Obviously, the net work done by the force of the spring, $-\frac{1}{2}kx^2 + \frac{1}{2}k'x^2 = \frac{1}{2}(k' - k)x^2$, is no longer zero; so the force is therefore no longer conservative.
7. The conservation of energy is a principle that covers the entire physical world, including biological systems. In the case of sugar ingestion followed by exercise, there are many places where the chemical energy locked up in the sugar and the oxygen of the air that you breathe might go. This includes the energy of your motion, the heating of your muscles and blood and then of the surrounding air as you cool, and energy in the chemical products that are produced, including those contained in your expelled breath, perspiration, and waste products collected by your kidneys and other internal organs.

8. (a) Yes, $U = \frac{1}{2}kx^2$.
 (b) No. Since the drag force is always against the direction of motion of an object and therefore always does negative work on it, the net work W done by the drag force is negative, rather than zero, as the object makes moves along a loop and comes back to the starting point. This violates the condition that W be zero for a force to be conservative.
 (c) Assuming that the tabletop itself does not move, then as the object moves on the table the normal force exerted on it is always perpendicular to its direction of motion. Thus the work done by the normal force on the object always equals zero. So, it would be possible to define a potential energy associated with that force, except that the result would be trivial — $U = \text{constant}$.
 (d) Yes, since such a force would obviously do zero net work on an object as the object moves along a complete loop. The gravitational force near the surface of the earth provides a good example.
9. No. A real spring has internal friction, which is nonconservative. As a result, the total mechanical energy of a real spring-mass system is not quite conserved. Suspend a mass to a spring and set it into oscillation. If all the forces involved are conservative then the system would experience no reduction in mechanical energy, and the oscillation would go on forever. In reality, even if you do this experiment in a vacuum chamber (so as to take away the influence of air friction), the oscillation would still die down over time, die to the internal friction of the spring itself.
10. Yes. If the force is conservative then $F_x = -\partial U/\partial x$ and $F_y = -\partial U/\partial y$. Thus $\partial F_x/\partial y = \partial/\partial y(-\partial U/\partial x) = -\partial^2 U/\partial x\partial y$ and $\partial F_y/\partial x = \partial/\partial x(-\partial U/\partial y) = -\partial^2 U/\partial x\partial y$, and so $\partial F_x/\partial y = \partial F_y/\partial x$. Check to see if this equality holds. If it does then the force is conservative.
11. It is possible to test experimentally whether the potential energy mgh is converted to kinetic energy in projectile motion, and that the potential energy depends on height alone. The nonconservative effects of the atmosphere can be removed by conducting such experiments in a vacuum. More extensive tests are possible in observing motion of satellites far enough from Earth that the variation of the gravitational force, and the correspondingly modified potential energy, comes in.
12. Consider a skier of mass m sliding down a slope of vertical drop h , starting from rest. In the absence of friction there would be no energy loss, so $\Delta E = \Delta U + \Delta K = 0$, or $mgh - \frac{1}{2}mv^2 = 0$, which gives the final speed to be $v = (2gh)^{1/2}$. If the final speed turns out to be lower than that value then friction must have done an amount of negative work W : $W = \Delta E = \frac{1}{2}mv^2 - mgh < 0$. By measuring the final speed v and the height h we can then find W .
13. Yes. Both the air friction and the force of impact with the ground are nonconservative (although the force of impact would be conservative in an ideal situation where the collision is perfectly elastic). As a result the mechanical energy of the golf ball decreases, and it is unable to return to the same height from which it is released.
14. When friction is present, energy is lost as the motion continues. We can think of a small amount of energy removed from the total with each cycle. But the energy of the motion determines to what height along the slide the moving object goes. With every cycle, then, the maximum height reached decreases, and the back-and-forth motion becomes more and more reduced towards the minimum of the potential, that is, the minimum in the "bowl." Finally, the object will come to rest at the bottom. Certainly this is in accord with our experience.
15. As the object slides along the slope, the direction of the normal force is perpendicular to the slope, so it does zero work on the object and becomes irrelevant when we calculate the total work done on the object. The only force doing non-zero work on the object is the gravitational force, which is conservative. The system is therefore a conservative one. This can be verified by checking to make sure that $E_i = E_f$.
16. No. Energy is a scalar quantity and cannot be decomposed into components along different axis.

17. Zeros of the potential energy have absolutely nothing to do with zeros of the force. Zeros of the potential energy depend on arbitrary additive constants in that energy, not on any property of the force. These zeros have no physical significance. Zeros of the force are located at places where the derivative of the potential energy is zero; that is, at locations where the potential energy is not changing. Force zeros have a very real physical significance.
18. It depends on the way the object moves. If it slides along the incline then the force of friction between the object and the incline does work, and that changes the total mechanical energy of the system. Unless you take into consideration the change in energy due to the work done by the (nonconservative) friction, the speed of the object you obtained through energy considerations would not be correct. But if the object rolls along the incline without slipping, then friction does not do any work and the conservation of energy still applies — only now we need to also taken into consideration the kinetic energy due to the rotation of the object (see Chapter 9).
19. If the force of the rod were conservative, then as the ball comes back to where it started the net work done by that force would be zero, and the kinetic energy (and speed) of the ball would return to the same as before. But in our case the rod always does positive work on the ball, whose kinetic energy keeps on increasing as a result. So the force is not conservative.
20. Two types of energy sources have been used in mechanical clocks. In one, weights are raised, attaining a potential energy associated with gravity. The weights are allowed to fall slowly, so that forces such as the tension in the rope from which the weights hang can make the clock go. In the second type, springs are flexed (“wound”) in order to make the potential energy of a compressed spring available to be converted to the kinetic energy of the running clock.
21. (a) The source of the hot water provides additional energy to the system (which consists of the community of animals), so the total energy of the system is not likely to be conserved.
(b) Earth receives energy from the sun through radiation, but it also radiates energy out to the universe. If the rate of energy absorption from the sun equals that of its outward radiation then the total energy of Earth would be conserved, and that is crucial to the stability of the environment that supports life on this planet. So the total energy of the earth is fairly constant, at least over time intervals that are much shorter than geological scales.
(c) The system is sealed only in such a sense that no material exchange is permitted between it and the environment. This does not preclude it from receiving energy from the sun radiating through its transparent glass domes. So its total energy is not conserved.
(d) As the compressed air drives the piston it does work on the piston, resulting in a decrease in its own energy. So the energy of the compressed-air system is not conserved.
22. As the firework ascends it converts its kinetic energy into gravitational potential energy. The explosion (the noise of which you hear) releases chemical energy, part of which is turned into heat, sound, and light, and part is turned into the kinetic energy of the resulting fragments (which appear as dots of light). Each dot follows a projectile trajectory (approximately). As it moves downwards it converts gravitational potential energy into kinetic energy, while some of its energy is lost to air friction.
23. As long as there is no source of energy loss, the forces are entirely conservative, and with each bounce all the initial potential energy is converted to kinetic energy and then back to potential energy. As the marble reaches the top of its trajectory after a given bounce it has regained all the initial potential energy it had originally. Since this potential energy is proportional to its initial height, the marble will go back to its initial height with each bounce.

Solutions to Problems

1. (a) We choose the potential energy to be zero at the ground ($y = 0$), so $U = mgy$.

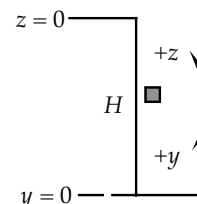
$$\text{Then } \Delta U = mg \Delta y = (10 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m}) = \boxed{0.20 \text{ kJ}}.$$

(b) $W_{\text{net}} = \Delta K = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(10 \text{ kg})(2.4 \text{ m/s})^2 = \boxed{29 \text{ J}}.$

$$W_F = \Delta K + \Delta U = 29 \text{ J} + 0.20 \text{ kJ} = \boxed{2.3 \text{ kJ}}.$$

2. From the diagram, we get $z = H - y$. Then

$$U = mgy = +mg(H - z) = \boxed{mg(30 \text{ m} - z)}.$$



3. We convert the speeds: $(95 \text{ mi/h})(1.61 \text{ km/mi})/(3.6 \text{ ks/h}) = 42.5 \text{ m/s}$; $120 \text{ mi/h} = 53.7 \text{ m/s}$.

We choose the potential energy to be zero at the ground ($y = 0$). Because the energy is conserved, we have

$$E = K_i + U_i = K_f + U_f; \quad \frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f;$$

$$\frac{1}{2}m(42.5 \text{ m/s})^2 + m(9.8 \text{ m/s}^2)(80 \text{ m}) = \frac{1}{2}m(53.7 \text{ m/s})^2 + m(9.8 \text{ m/s}^2)y_f, \text{ which gives}$$

$$y_f = \boxed{25 \text{ m}}.$$

4. We choose the potential energy to be zero at the bottom ($y = 0$). Because the energy is conserved, we have

$$E = K_i + U_i = K_f + U_f; \quad \frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f;$$

$$\frac{1}{2}mv_i^2 + m(9.8 \text{ m/s}^2)(45 \text{ m}) = \frac{1}{2}m(42 \text{ m/s})^2 + m(9.8 \text{ m/s}^2)(0), \text{ which gives}$$

$$v_i = \boxed{30 \text{ m/s}}.$$

5. The potential energy of the spring is zero when the spring is not compressed ($x = 0$). Because the energy is conserved, we have

$$E = K_i + U_i = K_f + U_f; \quad K_i + \frac{1}{2}kx_i^2 = K_f + \frac{1}{2}kx_f^2;$$

$$0 + \frac{1}{2}(5 \text{ N/m})(7 \times 10^{-2} \text{ m})^2 = K_f + 0, \text{ which gives}$$

$$K_f = \boxed{1.2 \times 10^{-2} \text{ J}}.$$

6. We find the force produced by the string by differentiating the potential energy:

$$F = -dU/dx = -d(bx^2 + cx^3)/dx = -(2bx + 3cx^2).$$

The force exerted by the archer is the reaction to this force:

$$F_{\text{archer}} = \boxed{2bx + 3cx^2};$$

$$F_{\text{string}} = \boxed{-2bx - 3cx^2}.$$

7. (a) A constant force is a conservative force. We use the reference point of $x = 0$ and obtain

$$U(x) = U(0) - \int_0^x F dx' = 0 - F \int_0^x dx' = -Fx = -(8 \text{ N})x = -8x \text{ J, with } x \text{ in m.}$$

(b) $E = K + U = \frac{1}{2}mv^2 - (8.0 \text{ N})x = \frac{1}{2}(5.0 \text{ kg})(+2.0 \text{ m/s})^2 - (8.0 \text{ N})(-1.0 \text{ m}) = \boxed{+18 \text{ J}}.$

- (c) Because energy is conserved, we have

$$E = K_2 + U_2 = \frac{1}{2}mv_2^2 - (8.0 \text{ N})x_2;$$

$$18 \text{ J} = \frac{1}{2}(5.0 \text{ kg})v_2^2 - (8.0 \text{ N})(3.0 \text{ m}), \text{ which gives}$$

$$v_2 = \boxed{4.1 \text{ m/s}}.$$

8. The potential energy is $U = \frac{1}{2}kx^2$, with $U = 0$ when $x = 0$. Then $3.77 \text{ J} = \frac{1}{2}(16.5 \text{ N/m})x^2$, which gives $x = \boxed{0.676 \text{ m}}$.

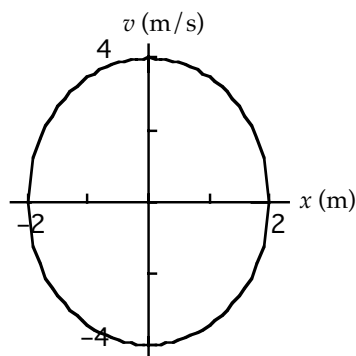
Because the energy is constant, the speed is maximum when $U = 0$:

$$E = K_1 + U_1 = K_2 + U_2 = 0 + 3.77 \text{ J};$$

$$\frac{1}{2}mv_{\text{max}}^2 + 0 = \frac{1}{2}m(1.71 \text{ m/s})^2, \text{ which gives}$$

$$m = \boxed{2.58 \text{ kg}}.$$

9. $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$;
 $16.0 = v^2 + 4.0x^2$.

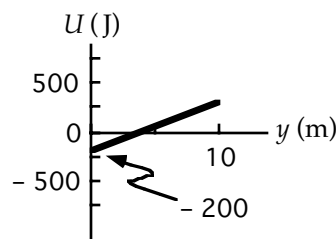
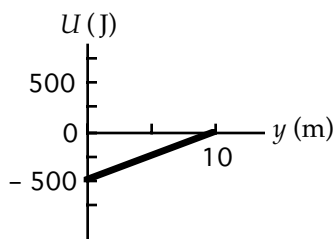
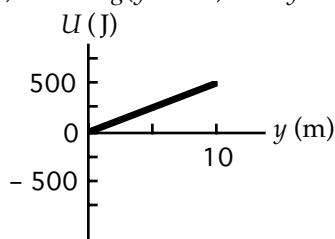


10. We take $y = 0$ at the ground.

(a) $U = mgy = (5 \text{ kg})(9.8 \text{ m/s}^2)y = 49y$.

(b) $U = mg(y - 10 \text{ m}) = 49y - 490$.

(c) $U = mg(y - 4 \text{ m}) = 49y - 200$.



11. Because the energy is conserved, we have

$$E = K_1 + U_1 = K_2 + U_2; \quad \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2;$$

$$\frac{1}{2}(0.528 \text{ kg})(3.85 \text{ m/s})^2 + 0 = 0 + \frac{1}{2}(26.7 \text{ N/m})x^2, \text{ which gives}$$

$$x = \boxed{0.541 \text{ m}}.$$

With a rough surface, there will be work done by the friction force, $f = -\mu_k N$, with $N = mg$. Thus

$$W_f = \Delta K + \Delta U;$$

$$-\mu_k mg x = (\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2) + (\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2);$$

$$-(0.411)(0.528 \text{ kg})(9.8 \text{ m/s}^2)x = [0 - \frac{1}{2}(0.528 \text{ kg})(3.85 \text{ m/s})^2] + [\frac{1}{2}(26.7 \text{ N/m})x^2 - 0].$$

The solutions of the quadratic equation are $x = 0.468 \text{ m}$, -0.627 m . From our expression for the work done by friction, which must be negative, we select the positive result: $x = \boxed{0.468 \text{ m}}$.

12. (a) With F being the applied force, when the bow is stretched we have $F - kx = 0$;

$$k = F/x = 65 \text{ N}/0.47 \text{ m} = \boxed{138 \text{ N/m}}.$$

- (b) Because the energy is conserved, we have

$$E = K_1 + U_1 = K_2 + U_2; \quad 0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + 0;$$

$$\frac{1}{2}(138 \text{ N/m})(0.47 \text{ m})^2 = \frac{1}{2}(40 \times 10^{-3} \text{ kg})v_2^2, \text{ which gives}$$

$$v_2 = \boxed{28 \text{ m/s}}.$$

13. We choose $y = 0$ at the relaxed position of the spring and denote the height of release by h and the magnitude of the compression of the spring by Δy .

Because the energy is conserved, we have

$$E = K + U_g + U_{\text{spring}} = \text{constant};$$

$$0 + mgh + 0 = 0 + mg(-\Delta y) + \frac{1}{2}k(-\Delta y)^2;$$

$$(15 \text{ kg})(9.8 \text{ m/s}^2)(6.0 \text{ m}) = (15 \text{ kg})(9.8 \text{ m/s}^2)(-\Delta y) + \frac{1}{2}(10^4 \text{ N/m})(\Delta y)^2.$$

This is a quadratic equation for Δy , from which we get $\Delta y = 0.43 \text{ m}$, -0.41 m .

From our choice of Δy as a magnitude, we select the positive value: $\Delta y = 0.43 \text{ m}$.

For the man, the numbers become

$$(60 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = (60 \text{ kg})(9.8 \text{ m/s}^2)(-\Delta y) + \frac{1}{2}(10^4 \text{ N/m})(\Delta y)^2, \text{ which gives } \Delta y = 0.48 \text{ m}.$$

The greater decrease in gravitational potential energy requires a greater spring potential energy.

14. From the vertical stretching, we can find the spring constant:

$$mg - k\Delta y = 0, \quad \text{or} \quad k = (0.70 \text{ kg})(9.8 \text{ m/s}^2) / (3.8 \times 10^{-2} \text{ m}) = 1.8 \times 10^2 \text{ N/m}.$$

- (a) During the compression, energy is conserved:

$$E = K + U = \text{constant}; \quad \frac{1}{2}mv_i^2 + 0 = 0 + \frac{1}{2}kx_f^2;$$

$$\frac{1}{2}(0.70 \text{ kg})(+2.2 \text{ m/s})^2 = \frac{1}{2}(1.8 \times 10^2 \text{ N/m})x_f^2, \text{ which gives } x_f = 0.14 \text{ m} = 14 \text{ cm}.$$

- (b) If we apply energy conservation from the initial motion until after the rebound, we have

$$0 + \frac{1}{2}mv_i^2 = 0 + \frac{1}{2}mv_f^2, \text{ thus } v_f = \pm v_i.$$

Because the mass is moving in the opposite direction, $v_f = 22 \text{ m/s}$ in negative x -direction.

15. We choose $x = 0$ at the equilibrium position.

- (a) Between the release and the equilibrium position, energy is conserved:

$$K_1 + U_1 = K_2 + U_2; \quad 0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + 0;$$

$$\frac{1}{2}(200 \text{ N/m})(5 \times 10^{-2} \text{ m})^2 = \frac{1}{2}(10 \times 10^{-3} \text{ kg})v_2^2, \text{ which gives } v_2 = 7.1 \text{ m/s}.$$

- (b) The work done by friction decreases the kinetic energy:

$$W_f = \Delta K + \Delta U = [0 - \frac{1}{2}(10 \times 10^{-3} \text{ kg})(7.1 \text{ m/s})^2] + (0 - 0) = -0.25 \text{ J}.$$

- (c) The normal force is mg , and the kinetic friction force opposes the motion:

$$W_f = -\mu_k mg \Delta x;$$

$$-0.25 \text{ J} = -\mu_k (10 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)(3.5 \text{ m}), \text{ which gives } \mu_k = 0.73.$$

16. The trampoline lets the person to raise his center of mass by an additional amount of $\Delta h = 2.5 \text{ m} - 1.0 \text{ m} = 1.5 \text{ m}$, corresponding to an additional gravitational potential energy of

$$\Delta U = mg\Delta h = (70 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = 1.0 \text{ kJ}.$$

This additional gravitational energy must come from the elastic potential energy of the trampoline. So the potential energy contained in the trampoline is 1.0 kJ.

17. The gravitational force exerted on a mass m at the height y is

$$F(y) = -mg(y) = -(g_0 - g'y), \text{ from which we find the potential energy:}$$

$$U = -\int F(y) dy = m \int (g_0 - g'y) dy = \boxed{mg_0 y - \frac{1}{2}mg'y^2}, \text{ with } U = 0 \text{ where } y = 0.$$

18. For a conservative one-dimensional force, we have $U(x) = U(s) - \int_s^x F(x') dx'$.

If we differentiate and use the fundamental relationship between integration and differentiation, we get

$$\frac{dU}{dx} = 0 - \frac{d}{dx} \int_s^x F(x') dx' = -F(x), \quad \text{or} \quad F(x) = -\frac{dU}{dx}.$$

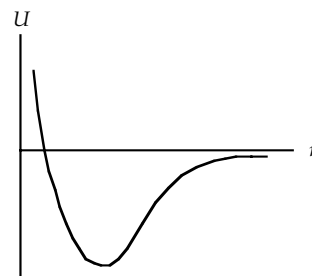
If $\frac{dU}{dx} > 0$, $F(x)$ is negative; if $\frac{dU}{dx} < 0$, $F(x)$ is positive.

19. Far away, $F = -dU/dx = 0$.

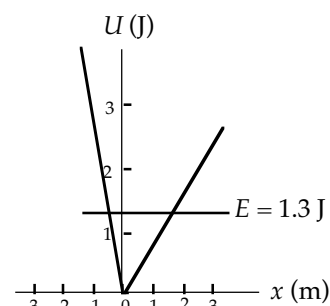
In the attractive region, dU/dx must be positive.

In the repulsive region, dU/dx must be negative.

At very small separations, $|dU/dx|$ must get very large.



20. (a) We choose $U = 0$ at $x = 0$. From the relation between force and potential energy, $F = -dU/dx$, we see that a constant force means a constant slope on the potential energy plot. For this force the slope is -2.5 N for negative x and $+0.8$ N for positive x .
- (b) At $x = -0.5$ m, the energy is only potential, with a value of 1.3 J. We show a line at constant energy of 1.3 J, which intercepts the plot at $x = 1.6$ m. Thus the particle travels **1.6 m** before stopping momentarily.



21. Refer to the potential energy plot in the solution to the previous problem. There is one stable equilibrium point, at $x = 0$, where U is at its minimum value.

The turning points are where $U = E$. For $x > 0$

$F(x) = -dU/dx = -0.8$ N, so $U(x) = (0.8 \text{ N})x$. Thus the right turning point x_1 satisfies

$$E = (0.8 \text{ N})x_1, \text{ or } x_1 = \boxed{E / (0.8 \text{ N})}.$$

For $x < 0$ $F(x) = -dU/dx = 2.5$ N, so $U(x) = -(2.5 \text{ N})x$. Thus the left turning point x_2 satisfies

$$E = -(2.5 \text{ N})x_2, \text{ or } x_2 = \boxed{-E / (2.5 \text{ N})}.$$

22. The kinetic energy of the mass is

$K = \frac{1}{2}mv^2 = E - U = E - 2.0 \text{ J} \sin \pi x$, where E is the total mechanical energy of the mass and can be found from its initial value: $E = \frac{1}{2}mv_0^2 + U(0) = \frac{1}{2}mv_0^2 - 2.0 \text{ J} \sin 0 = \frac{1}{2}mv_0^2$. The speed of the mass as a function of x is then

$$v = [v_0^2 - 4.0 \text{ J} (\sin \pi x) / (1.0 \text{ kg})]^{1/2}.$$

Plug in $v_0 = 0.71$ m/s and set $v = 0$ to obtain

$v(x) = [(0.71 \text{ m/s})^2 - 4.0 \text{ J} (\sin \pi x) / (1.0 \text{ kg})]^{1/2} = 0$, or $\sin \pi x = 0.126$. The two solutions to this equation that are adjacent to $x = 0$ are $x_1 = 0.040$ m and $x_2 = -1.04$ m. The motion of the mass is therefore restricted between x_1 and x_2 . The mass starts off at $x = 0$ with a positive velocity $v_0 = 0.71$ m/s, and as it moves along the $+x$ axis it slows down, reaching a turning point at x_1 . It then turns back and accelerates, passing through $x = 0$ with velocity $-v_0$. It continues beyond $x = 0$, its speed first increases and then decreases, and becomes zero as it reaches another turning point, x_2 . Afterwards the motion repeats itself as the mass oscillates between x_1 and x_2 .

For $v_0 = +3.0$ m/s we have

$v(x) = [(3.0 \text{ m/s})^2 - 4.0 \text{ J} (\sin \pi x) / (1.0 \text{ kg})]^{1/2}$, which never reaches zero. So the mass will continue to move along the positive x -axis with a speed that varies between $v_{\text{max}} = 3.6$ m/s and $v_{\text{min}} = 2.2$ m/s.

23. For the given force, $F(x) = +kx$, we find the potential energy:

$$U = -\int kx \, dx = -\frac{1}{2}kx^2, \text{ with } U = 0 \text{ when } x = 0.$$

From the release near the origin, we find the energy:

$$E = -\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = -\frac{1}{2}k(0)^2 + \frac{1}{2}m(0)^2 = 0.$$

- (a) When $x > 0$, $F > 0$ and the mass will move **to the right**; U decreases and K increases.

- (b) From $E = -\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = 0$, we get $v^2 = (k/m)x^2$, or $v = \boxed{(k/m)^{1/2}x}$.

- (c) When $x < 0$, $F < 0$ and the mass will move **to the left**;

$$U \text{ decreases and } K \text{ increases; } v = \boxed{-(k/m)^{1/2}x}.$$

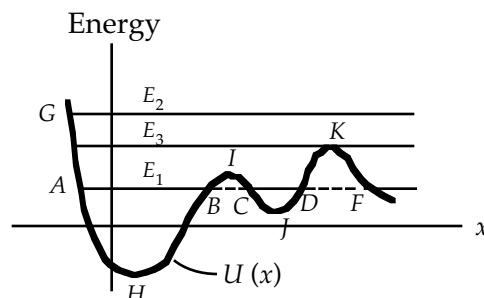
24. (a) For the energy E_1 , motion is restricted to

$$A < x < B, C < x < D, \text{ or } F < x.$$

For the energy E_2 , motion is restricted to

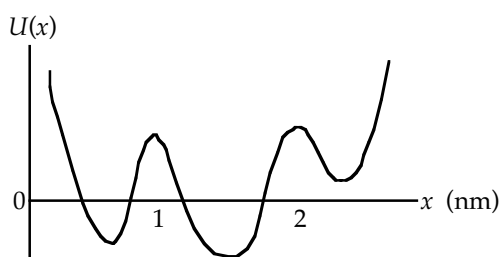
$$G < x.$$

- (b) The particle will remain at rest when $F = -dU/dx = 0$, which corresponds to H, I, J , or K .
 (c) Motion within turning points is possible for $E < E_3$. The positions will be in the valleys.
 (d) The equilibrium positions are where the slope is zero. Those at the bottom of the valleys, H and J , are stable; those at the peaks, I and K , are unstable.



25. From the problem statement we know that

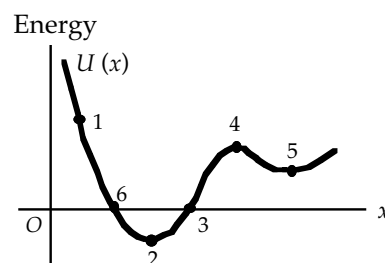
- (a) $U(x < 0) \rightarrow \infty$;
 (b) $U(x)$ has three local minima;
 (c) $U(x \rightarrow \infty) \rightarrow \infty$; and
 (d) there are peaks at $x = 1$ nm and $x = 2$ nm.
 A $U(x)$ curve that satisfies all these conditions is shown below.



26. (a) From $F = -dU/dx$, we see that the sign of the force is opposite to the sign of the slope:

$$F_1: +; F_2: 0; F_3: -; F_4: 0; F_5: 0; F_6: +.$$

- (b) 1 is most positive; 3 is most negative; 2, 4, and 5 are zero.
 (c) 4 is an unstable equilibrium position;
 2 and 5 are stable equilibrium positions.



27. The potential energy is $U(x) = \alpha x^4 + \beta x^2$.

The equilibrium positions have $dU/dx = 0$:

$$dU/dx = 4\alpha x^3 + 2\beta x = 0, \text{ from which we get}$$

$$x = 0, \text{ and}$$

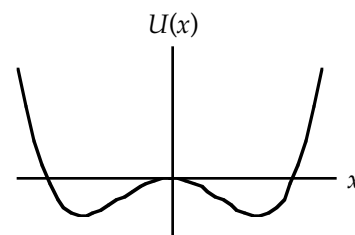
$$x = \pm (-\frac{1}{2}\beta/\alpha)^{1/2}$$

$$= \pm [-\frac{1}{2}(-3.0 \text{ J/m}^2)/(26 \text{ J/m}^4)]^{1/2}$$

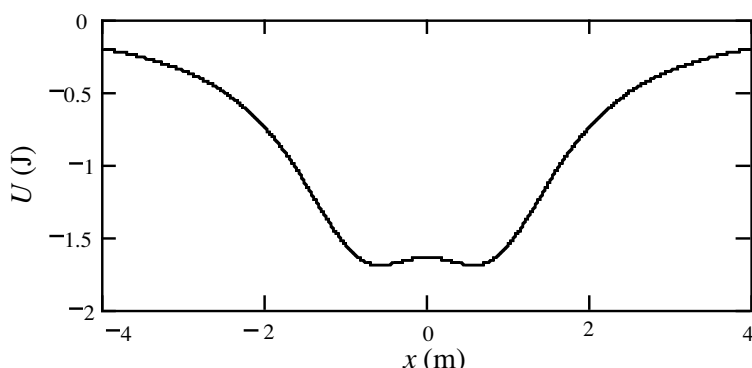
$$= \pm 0.24 \text{ m.}$$

From the sketch, we see that

$$x = 0 \text{ is unstable and } x = \pm 0.24 \text{ m is stable.}$$



28. (a)



(b) The kinetic energy of the particle at $x = -2$ m is $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.5 \text{ kg})(3.2 \text{ m/s})^2 = 10 \text{ J}$. Since from the plot above we see that $U(x)$ never dips below -2 J within the range $(-2 \text{ m}, 1.5 \text{ m})$, $E = K + U$ is never below $10 \text{ J} - 2 \text{ J} = 8 \text{ J}$; i.e., it is always positive, between $(-2 \text{ m}, 1.5 \text{ m})$. So the particle can indeed reach $x = 1.5 \text{ m}$.

29. We choose $y = 0$ at the plain. If there is no drag, energy is conserved:

$$E = \frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f;$$

$$\frac{1}{2}m(500 \text{ m/s})^2 + m(9.8 \text{ m/s}^2)(180 \text{ m}) = \frac{1}{2}mv_f^2 + m(9.8 \text{ m/s}^2)(16 \text{ m}), \text{ which gives}$$

$$v_f = \boxed{503 \text{ m/s}}.$$

30. $U = -Gm_1m_2/r$

$$= -(6.67 \times 10^{-11} \text{ m}^2/\text{kg}\cdot\text{s}) (7.35 \times 10^{22} \text{ kg})(5.98 \times 10^{24} \text{ kg}) / (3.84 \times 10^8 \text{ m})$$

$$= \boxed{-7.63 \times 10^{28} \text{ J}}.$$

31. We choose $y = 0$ at the sea. If there is no drag, energy is conserved:

$$E = \frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f;$$

$$\frac{1}{2}m(125 \text{ m/s})^2 + m(9.8 \text{ m/s}^2)(68 \text{ m}) = \frac{1}{2}mv_f^2 + m(9.8 \text{ m/s}^2)(0), \text{ which gives } v_f = \boxed{130 \text{ m/s}}.$$

When fired at an angle, the gravitational energy change is the same; the final speed will be the same. The direction of the velocity will be different.

32. Because the potential energy depends only on the distance r , the force must be a central force which depends only on r . We find the force law by differentiating:

$$F(r) = -dU/dr = -d[U_0 - (k/r^2)]/dr = -[-(-2)k/r^3] = \boxed{-2k/r^3}.$$

33. We choose $y = 0$ at the release point. If we neglect air resistance, energy is conserved:

$$E = \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2.$$

(a) The kinetic energy depends on the speed, not the direction:

$$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_h^2 + mgh, \text{ or } v_h^2 = v_0^2 - 2gh, \text{ which gives } v_h = (v_0^2 - 2gh)^{1/2}.$$

(b) For vertical motion, the speed at the highest point (where $h = H$) is zero:

$$v_H = 0 = (v_0^2 - 2gH)^{1/2}, \text{ which gives } v_0 = \boxed{(2gH)^{1/2}}.$$

(c) When the ball is thrown at an angle θ , the speed at the highest point is $v_H = v_{0x} = v_0 \cos \theta$. Here

$$v_H = v_{0x} = (v_0^2 - 2gH)^{1/2}, \text{ or } v_{0x}^2 = v_0^2 - 2gH.$$

Because $v_0^2 = v_{0x}^2 + v_{0y}^2$, we have

$$v_{0y}^2 = (v_0 \sin \theta)^2 = 2gH.$$

$$\text{Thus } v_0 = (2gH)^{1/2} / \sin 45^\circ = \boxed{2(gH)^{1/2}}.$$

34. We choose the origin at the equilibrium point.

(a) Because energies are scalars, we can add the potential energies:

$$U = U_x + U_y = \frac{1}{2}kx^2 + \frac{1}{2}ky^2 = \frac{1}{2}k(x^2 + y^2) = \boxed{\frac{1}{2}kr^2}.$$

(b) The components of the force are

$$F_x = -\partial U / \partial x = -kx \quad \text{and}$$

$$F_y = -\partial U / \partial y = -ky.$$

The direction of the force, as specified by the angle with the x -axis, is found from

$$\tan \theta = F_y / F_x = -ky / -kx = y / x, \text{ which is the angle of } \vec{r} = x\hat{i} + y\hat{j}.$$

With the negative signs for the force components, we have

$$\vec{F} = -kx\hat{i} - ky\hat{j} = -k\vec{r}.$$

(c) To have uniform circular motion with angular speed ω , we must have a centripetal force to provide the centripetal acceleration: $F_r = ma_r$. Because our net force is toward the center, if the mass is started appropriately, we can have

$$-kr = -mr\omega^2, \text{ which gives } \omega = \boxed{\sqrt{k/m}}.$$

35. With $y = 0$ at the base of the building, from energy conservation we have

$$mgh + \frac{1}{2}mv^2 = 0 + \frac{1}{2}mv_2^2.$$

If θ is above the horizontal, we get

$$v_2^2 = v^2 + 2gh = (v \sin \theta)^2 + (v \cos \theta)^2 + 2gh.$$

If θ is below the horizontal, we get

$$v_2^2 = v^2 + 2gh = (-v \sin \theta)^2 + (v \cos \theta)^2 + 2gh, \text{ which gives the same speed.}$$

The horizontal distance is $x = v_{0x}t = (v \cos \theta)t$, for either case, with t the time to reach the ground.

We find this time from

$$y = y_0 + v_{0y}t + \frac{1}{2}(-g)t^2;$$

$$0 = h \pm (v \sin \theta)t + \frac{1}{2}(-g)t^2.$$

Thus the time t depends on the sign of the angle; a knowledge of x will determine the orientation.

36. With $y = 0$ at the bottom of the hill, we call the start point A , the top of the intermediate hill point B , and the bottom point C . From energy conservation we have

$$K_A + U_A = K_B + U_B = K_C + U_C;$$

$$0 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}mv_C^2 + 0.$$

Thus $v_B^2 = 2g(h_A - h_B) = 2(9.8 \text{ m/s}^2)(43 \text{ m} - 37 \text{ m})$,

$$\text{which gives } v_B = \boxed{11 \text{ m/s}}.$$

Also $v_C^2 = 2gh_A = 2(9.8 \text{ m/s}^2)(43 \text{ m})$, which gives

$$v_C = \boxed{29 \text{ m/s}} \quad (65 \text{ mi/h}).$$

The neglect of resistive forces is not reasonable.

37. With resistive forces present, we have

$$W_f = \Delta K + \Delta U$$

$$= (\frac{1}{2}mv_C^2 - \frac{1}{2}mv_A^2) + mg(h_C - h_A)$$

$$= [\frac{1}{2}(75 \text{ kg})(23 \text{ m/s})^2 - 0] + (75 \text{ kg})(9.8 \text{ m/s}^2)(0 - 95 \text{ m})$$

$$= \boxed{-5.0 \times 10^4 \text{ J}}.$$

38. With $y = 0$ at the landing, we call the start point A, the jump point B, and the landing point C.

From energy conservation we have

$$K_A + U_A = K_B + U_B; \quad 0 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B;$$

$$m(9.8 \text{ m/s}^2)(16 \text{ m}) = \frac{1}{2}mv_B^2 + m(9.8 \text{ m/s}^2)(8 \text{ m}),$$

which gives $v_B = 12.5 \text{ m/s}$.

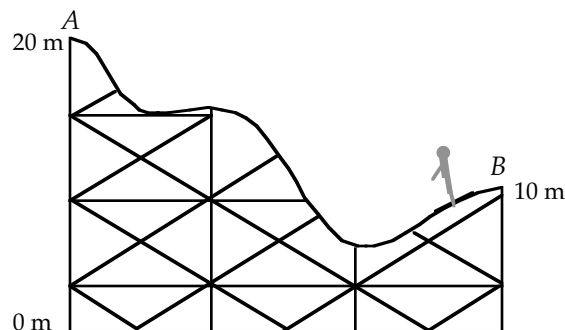
For the projectile motion from B to C, we have

$$y_C = y_B + v_{By}t - \frac{1}{2}gt^2;$$

$$0 = 8 \text{ m} + 0 - \frac{1}{2}(9.8 \text{ m/s}^2)t^2, \text{ which gives } t = 1.28 \text{ s}.$$

The horizontal distance is

$$x = v_B t = (12.5 \text{ m/s})(1.28 \text{ s}) = \boxed{16 \text{ m}}.$$



39. We choose $y = 0$ at the bottom.

- (a) For the motion from the initial point to point a, from energy conservation we have

$$K_i + U_i = K_a + U_a; \quad 0 + mgh_i = \frac{1}{2}mv_a^2 + 0;$$

$$m(9.8 \text{ m/s}^2)(0.10 \text{ m}) = 0 + \frac{1}{2}mv_a^2, \text{ which gives } v_a = \boxed{1.4 \text{ m/s}}.$$

- (b) For the motion from point a to point b, from the presence of friction we have

$$W_f = \Delta K + \Delta U; \quad -\mu_k mg \Delta x = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 + 0;$$

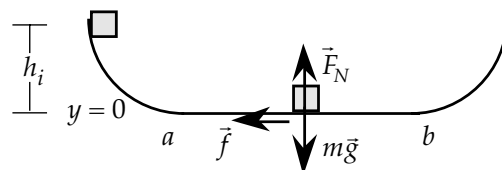
$$-(0.21)m(9.8 \text{ m/s}^2)(0.20 \text{ m}) = \frac{1}{2}mv_b^2 - \frac{1}{2}m(1.4 \text{ m/s})^2, \text{ which gives } v_b = \boxed{1.1 \text{ m/s}}.$$

- (c) For the motion from the initial point to the stopping point, we let D represent the total horizontal distance over which the friction force acts. Because the friction force always opposes the motion, we have

$$W_f = \Delta K + \Delta U; \quad -\mu_k mg D = (0 - 0) + (0 - mgh_i), \text{ which gives}$$

$$D = h_i / \mu_k = (0.10 \text{ m}) / 0.21 = 0.48 \text{ m} = 48 \text{ cm}.$$

This corresponds to $48 \text{ cm} - 2(20 \text{ cm}) = \boxed{8 \text{ cm from point a}}.$



40. With $y = 0$ at the bottom of the circle, we call the start point A, the bottom of the circle B, and the top of the circle C.

From energy conservation we have

$$K_A + U_A = K_B + U_B = K_C + U_C.$$

- (a) For the motion from A to B:

$$mgH + 0 = 0 + \frac{1}{2}mv_B^2, \text{ which gives } v_B = \boxed{\sqrt{2gH}}.$$

- (b) For the motion from A to C:

$$mgH + 0 = mg(2R) + \frac{1}{2}mv_C^2,$$

which gives $v_C = \boxed{\sqrt{2g(H - 2R)}}$

- (c) At the top of the circle we have the forces mg and N , both downward, that provide the centripetal acceleration:

$$mg + N = mv_C^2 / R, \text{ which gives}$$

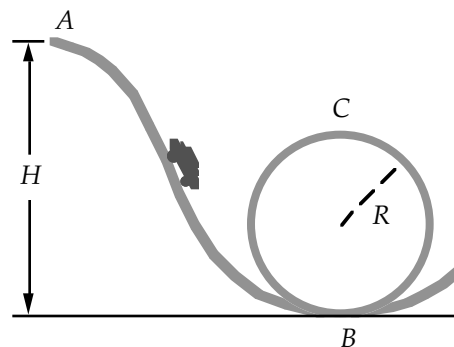
$$N = m(v_C^2 / R - g) = m[2g(H - 2R) / R - g] = \boxed{mg(2H/R - 5)}.$$

- (d) The minimum value of N is zero, since the track can only push on the car. Thus

$$2H_{\min} / R - 5 = 0, \text{ which gives } H_{\min} = \boxed{\frac{5}{2}R}.$$

The speed at C will be

$$v_{C\min} = \sqrt{2g(H_{\min} - 2R)} = \boxed{\sqrt{gR}}.$$



41. (a) We take the gravitational potential energy to be 0. The spring potential energy depends on the

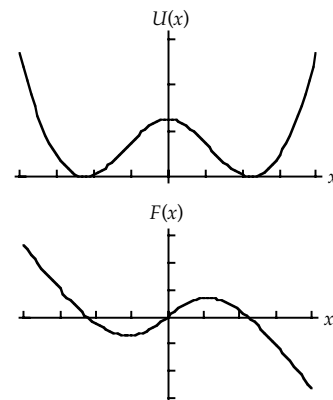
amount of compression or extension of the spring.

At the position x , the length of the spring is $\sqrt{h^2 + x^2}$. The spring potential energy is

$$U(x) = \frac{1}{2}k(\sqrt{h^2 + x^2} - L)^2.$$

- (b) We find the force produced by the spring by differentiating:

$$\begin{aligned} F &= -\frac{dU}{dx} = -\frac{1}{2}k(2)(\sqrt{h^2 + x^2} - L)\left(\frac{1}{2}\right)\frac{2x}{\sqrt{h^2 + x^2}} \\ &= -\frac{k(\sqrt{h^2 + x^2} - L)x}{\sqrt{h^2 + x^2}}. \end{aligned}$$



42. Because $U(r) = U_0[(r_0/r)^{12} - 2(r_0/r)^6]$ depends only on the separation, we find the force from

$$F_r = -dU/dr = -U_0[r_0^{12}(-12/r^{13}) - 2r_0^6(-6/r^7)] = (12U_0/r_0)[(r_0/r)^{13} - (r_0/r)^7].$$

The force will be zero when $(r_0/r)^{13} = (r_0/r)^7$, which gives $(r_0/r)^6 = 1$, or $r = \boxed{r_0}$.

At this separation, the potential energy is $U = U_0(1 - 2) = \boxed{-U_0}$.

43. (a) Because the potential energy $U(r) = -GMm/r$ depends only on the radius, the force will be radial and we find it from

$$F_r = -dU/dr = -[-GMm(-1/r^2)] = -GMm/r^2, \text{ so the force is } \boxed{GMm/r^2 \text{ toward } M}.$$

- (b) The force provides the centripetal acceleration:

$$F_r = ma_r; \quad GMm/r^2 = mv^2/r, \text{ which gives } v^2 = GM/r.$$

The kinetic energy is $K = \frac{1}{2}mv^2 = \boxed{GMm/2r}$.

- (c) The total energy is $E = K + U = GMm/2r - GMm/r = \boxed{-GMm/2r}$.

44. From the potential energy $U(x, y) = a_1x^2 + a_2xy + a_3y^2$, we find the force components:

$$F_x = -\partial U/\partial x = -2a_1x - a_2y \quad \text{and} \quad F_y = -\partial U/\partial y = -a_2x - 2a_3y. \text{ Thus}$$

$$\vec{F} = \boxed{[(-6x + 14y)\hat{i} + (14x - 3y)\hat{j}] \text{ N}}, \text{ with } x \text{ and } y \text{ in meters.}$$

45. With $y = 0$ at the landing, we call the start point A, the take-off point B, and the landing point C.

From energy conservation we have

$$K_A + U_A = K_B + U_B; \quad 0 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B;$$

$$mg(H + D) = \frac{1}{2}mv_B^2 + mgD, \text{ which gives } v_B = \sqrt{2gH}.$$

For the projectile motion from B to C, we have

$$y_C = y_B + v_{By}t + \frac{1}{2}(-g)t^2;$$

$$0 = D + v_B \sin \theta t - \frac{1}{2}gt^2 = D + \sqrt{2gH} \sin \theta t - \frac{1}{2}gt^2; \text{ and}$$

$$x_C = v_B \cos \theta t = \sqrt{2gH} \cos \theta t.$$

If we eliminate t between these two equations, we get a quadratic equation for x_C :

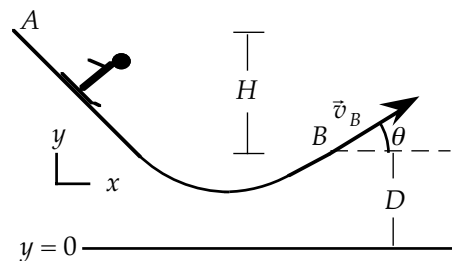
$$x_C^2/4H \cos^2 \theta - x_C \tan \theta - D = 0, \text{ which has the solution}$$

$$x_C = 2H \cos \theta [\sin \theta \pm \sqrt{\sin^2 \theta + (D/H)}].$$

To find the angle that maximizes x_C , we set $dx_C/d\theta = 0$:

$$\begin{aligned} dx_C/d\theta &= 2H[-\sin \theta [\sin \theta \pm \sqrt{\sin^2 \theta + (D/H)}] + \cos \theta [\cos \theta \pm (\sin \theta \cos \theta)/\sqrt{\sin^2 \theta + (D/H)}]] = 0, \text{ or} \\ \sin^4 \theta - \cos^4 \theta &= -(D/H) \sin^2 \theta. \end{aligned}$$

If we let $z = \sin^2 \theta$, we obtain $z^2 - (1 - z)^2 = -(D/H)z$, which has the solution $z = \sin^2 \theta = 1/(2 + D/H)$.



46. The work done by the nonconservative forces changes the energy of the system, with $K_i = K_f = 0$:

$$W_{nc} = \Delta(K + U) = U_f - U_i = mg(h_f - h_i) = (10 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)(0.90 \text{ m} - 1.00 \text{ m}) = \boxed{-9.8 \times 10^{-3} \text{ J}}.$$

47. The work done by the nonconservative drag forces changes the energy of the system:

$$W_{nc} = \Delta(K + U) = (K_f + U_f) - (K_i + U_i) = (\tfrac{1}{2}mv_f^2 + 0) - (0 + mgh_i) \\ = \tfrac{1}{2}(75 \text{ kg})(5.0 \text{ m/s})^2 - (75 \text{ kg})(9.8 \text{ m/s}^2)(85 \text{ m}) = \boxed{-6.2 \times 10^4 \text{ J}}.$$

48. (a) The work done by the nonconservative friction force on the horizontal track of length L , where the normal force is mg , changes the energy of the system, with $K_i = K_f = 0$:

$$W_{\text{friction}} = \Delta(K + U); \quad -\mu_k mgL = mg(h_f - h_i).$$

After cancelling mg , we have

$$-0.18L = 0.55 \text{ m} - 1.3 \text{ m}, \text{ which gives } L = \boxed{4.2 \text{ m}}.$$

- (b) When the object slides back, it will come to rest on the horizontal track, so we have

$$W_{\text{friction}} = \Delta(K + U); \quad -\mu_k mgx = mg(h_f - h_i);$$

$$-0.18x = 0.0 \text{ m} - 0.55 \text{ m}, \text{ which gives } x = \boxed{3.1 \text{ m}}.$$

49. We assume one 75-W bulb in each room that is kept lit for 2 hours in the morning and 5 hours in the evening each day:

$$\text{Cost} = 3(75 \text{ W})(2 \text{ h} + 5 \text{ h})(1 \text{ kW} / 1000 \text{ W})(12 \text{ cents} / \text{kWh}) \approx \boxed{20 \text{ cents}}.$$

50. The mechanical energy of the rock is conserved, so

$$E_i = K_i + U_i = \tfrac{1}{2}mv_i^2 + mgh = \tfrac{1}{2}mv_f^2, \text{ or}$$

$$v_f = (v_i^2 + 2gh)^{1/2}.$$

The difference in final speeds in question is then

$$\Delta v_f = (v_i^2 + 2gh)^{1/2} - (2gh)^{1/2} \\ = [(2 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(20 \text{ m})]^{1/2} - [2(9.8 \text{ m/s}^2)(20 \text{ m})]^{1/2} \\ = \boxed{0.1 \text{ m/s}}.$$

51. The change in potential energy is $mg \Delta y = (60 \text{ kg})(9.8 \text{ m/s}^2)(1.0 \text{ m}) \approx \boxed{5.9 \times 10^2 \text{ J}}.$

52. We choose $y = 0$ at the water surface. If there is no drag, energy is conserved:

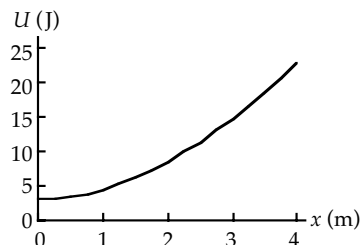
$$E = \tfrac{1}{2}mv_i^2 + mgy_i = \tfrac{1}{2}mv_f^2 + mgy_f;$$

$$\tfrac{1}{2}m(3.2 \text{ m/s})^2 + m(9.8 \text{ m/s}^2)(6.0 \text{ m}) = \tfrac{1}{2}mv_f^2 + m(9.8 \text{ m/s}^2)(0), \text{ which gives } v_f = \boxed{11.3 \text{ m/s down}}.$$

53. (a) We find the potential energy for the force $F(x) = -ax + bx^2$ from

$$U(x) = U(0) - \int_0^x (-ax' + bx'^2) dx' = U(0) + \frac{ax^2}{2} - \frac{bx^3}{3} \\ = 3 + \frac{3x^2}{2} - \frac{0.2x^3}{3} \text{ J, with } x \text{ in m.}$$

(b)



54. We choose $y = 0$ at the initial position, with up positive, and let $y_0 = 0.050$ m and y_1 be magnitudes as indicated on the diagram. From the stretch to the new equilibrium position, we find the spring constant:

$$mg = ky_0, \text{ or } k = mg/y_0.$$

For the motion from the release point to the initial point, energy

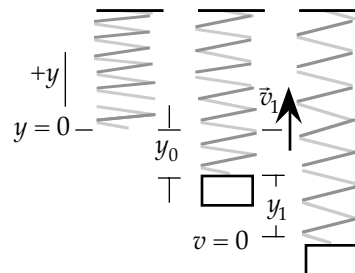
conservation $(K + U_g + U_{\text{spring}})_i = (K + U_g + U_{\text{spring}})_f$, gives

$$0 - mg(y_0 + y_1) + \frac{1}{2}k(y_0 + y_1)^2 = \frac{1}{2}mv_1^2 + 0 + 0;$$

$$\frac{1}{2}(mg/y_0)(y_0 + y_1)^2 - mg(y_0 + y_1) = \frac{1}{2}mv_1^2;$$

$$\frac{1}{2}[(9.8 \text{ m/s}^2)/(0.050 \text{ m}))(0.050 \text{ m} + y_1)^2 - (9.8 \text{ m/s}^2)(0.050 \text{ m} + y_1) - \frac{1}{2}(2.1 \text{ m/s})^2 = 0.$$

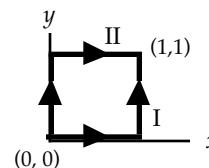
Solving this quadratic equation for y_1 , we get $y_1 = 0.16$ m = **16 cm**.



55. (a) Because $F(x) = +ax + bx^3 + cx^4$ is a one-dimensional force that depends only on position, it is **conservative**.
- (b) To test $\vec{F} = Ax^2\hat{i} + Bxy\hat{j}$, we find the work for a displacement from $(0, 0)$ to $(1, 1)$ for the two paths indicated in the diagram:

$$\begin{aligned} W_I &= \int_{0,0}^{1,0} F_x dx + \int_{1,0}^{1,1} F_y dy = \int_{0,0}^{1,0} Ax^2 dx + \int_{1,0}^{1,1} Bxy dy \\ &= \left. \frac{1}{3}Ax^3 \right|_{0,0}^{1,0} + \left. \frac{1}{2}Bxy^2 \right|_{1,0}^{1,1} = \left(\frac{A}{3} - 0 \right) + \left(\frac{B}{2} - 0 \right) = \frac{A}{3} + \frac{B}{2}. \end{aligned}$$

$$\begin{aligned} W_{II} &= \int_{0,0}^{0,1} F_y dy + \int_{0,1}^{1,1} F_x dx = \int_{0,0}^{0,1} Bxy dy + \int_{0,1}^{1,1} Ax^2 dx \\ &= \left. \frac{1}{2}Bxy^2 \right|_{0,0}^{0,1} + \left. \frac{1}{3}Ax^3 \right|_{0,1}^{1,1} = (0 - 0) + \left(\frac{A}{3} - 0 \right) = \frac{A}{3}. \end{aligned}$$



Because the work depends on the path, the force is **not conservative**.

56. The initial mechanical energy of the ball at the top of its path is mgh_0 , where h_0 is the initial height from which it is dropped. After the 1st bounce its mechanical energy becomes mgh_1 , where $h_1 = 95\%h_0$; so the energy dissipated at the first bounce is

$$\Delta E_1 = mgh_0 - mgh_1 = (1 - 95\%) mgh_0 = (1 - 95\%)(0.050 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m}) = \mathbf{0.025 \text{ J}}.$$

Similarly, the energy dissipated at the second bounce is

$$\Delta E_2 = mgh_1 - mgh_2 = (1 - 95\%) mgh_1 = (1 - 95\%) mg(95\%h_0) = 95\%\Delta E_1 = \mathbf{0.023 \text{ J}}; \text{ while that at the}$$

third one is

$$\Delta E_3 = mgh_2 - mgh_3 = (1 - 95\%) mgh_2 = (1 - 95\%) mg(95\%h_1) = 95\%\Delta E_2 = \mathbf{0.022 \text{ J}}; \text{ etc. In general, at the}$$

n -th bounce

$$\Delta E_n = 95\%\Delta E_{n-1} = (95\%)^2 \Delta E_{n-2} = \dots = (95\%)^{n-1} \Delta E_1 = \mathbf{0.95^{n-1} (0.025 \text{ J})}.$$

57. From $(0, 0)$ to $(1, 0)$ we have $y = 0$, so $dy = 0$ and $F_x = 2Ax^2y = 0$ and the integral is

$$\int F_x dx + \int F_y dy = 0.$$

From $(1, 0)$ to $(1, 1)$ we have $dx = 0$ and $x = 1$, so

$$\int F_x dx + \int F_y dy = \int F_y dy = \int A(1)y^2 dy = \frac{1}{3}A.$$

From $(1, 1)$ to $(0, 1)$ we have $dy = 0$ and $y = 1$, so

$$\int F_x dx + \int F_y dy = \int F_x dx = \int 2Ax^2(1) dx = -\frac{2}{3}A.$$

From $(0, 1)$ to $(0, 0)$ we have $dx = 0$ and $x = 0$, so

$$\int F_x dx + \int F_y dy = 0.$$

Overall, for the entire loop

$$W = \int F_x dx + \int F_y dy = 0 + \frac{1}{3}A - \frac{2}{3}A + 0 = -\frac{1}{3}A \neq 0.$$

Thus the force is not conservative, so we **cannot** define a potential energy associated with it.

58. We must find the work by integrating the force $F = -0.9x - 1.4x^3$:

(a)

$$W_I = \int_0^3 (-0.9x - 1.4x^3) dx$$

$$= \left[-0.9 \frac{x^2}{2} - 1.4 \frac{x^3}{3} \right]_0^3 = -17 \text{ J.}$$

(b)

$$W_{II} = \int_0^{-2} (-0.9x - 1.4x^3) dx + \int_{-2}^7 (-0.9x - 1.4x^3) dx + \int_7^3 (-0.9x - 1.4x^3) dx$$

$$= \left[-0.9 \frac{x^2}{2} - 1.4 \frac{x^3}{3} \right]_0^{-2} + \left[-0.9 \frac{x^2}{2} - 1.4 \frac{x^3}{3} \right]_{-2}^7 + \left[-0.9 \frac{x^2}{2} - 1.4 \frac{x^3}{3} \right]_7^3$$

$$= -1.8 \text{ J} - 180.3 \text{ J} + 165.5 \text{ J} = -17 \text{ J} = W_I.$$

Thus we see that the work done is the same for both paths, as it should be for a conservative force.

59. We can use any path to find the work done by the conservative force in moving the particle between two points. The work done while traversing a path in reverse is the negative of the work done for the original path:

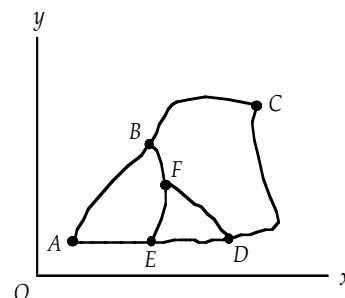
$$W_{CA} = -W_{ABC} = \boxed{-2 \text{ J}}.$$

$$W_{AE} = W_{ABC} + W_{CB} + W_{BE} = W_{ABC} - W_{BC} - W_{EB}$$

$$= +2 \text{ J} - 1 \text{ J} - 3 \text{ J} = \boxed{-2 \text{ J}}.$$

$$W_{DC} = W_{DF} + W_{FE} + W_{EB} + W_{BC}$$

$$= W_{DF} - W_{EF} + W_{EB} + W_{BC} = -1 \text{ J} - 1 \text{ J} + 3 \text{ J} + 1 \text{ J} = \boxed{+2 \text{ J}}.$$



60. We choose $y = 0$ at the bottom of the loop.

With no friction, energy is conserved.

The initial (and constant) energy is

$$E = E_a = mgy_a + \frac{1}{2}mv_a^2$$

$$= (0.050 \text{ kg})(9.8 \text{ m/s}^2)(10 \times 10^{-2} \text{ m}) + 0 = 4.9 \times 10^{-2} \text{ J.}$$

(a) $E_b = 0 + \frac{1}{2}mv_b^2$

$$= \frac{1}{2}(0.050 \text{ kg})v_b^2 = 4.9 \times 10^{-2} \text{ J, which gives}$$

$$v_b = \boxed{1.4 \text{ m/s}}.$$

$$E_c = mgy_c + \frac{1}{2}mv_c^2$$

$$= (0.050 \text{ kg})(9.8 \text{ m/s}^2)(8 \times 10^{-2} \text{ m}) + \frac{1}{2}(0.050 \text{ kg})v_c^2$$

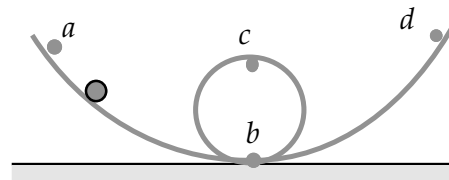
$$= 4.9 \times 10^{-2} \text{ J, which gives } v_c = \boxed{0.63 \text{ m/s}}.$$

$$E_d = mgy_d + \frac{1}{2}mv_d^2$$

$$= (0.050 \text{ kg})(9.8 \text{ m/s}^2)(12 \times 10^{-2} \text{ m}) + \frac{1}{2}(0.050 \text{ kg})v_d^2 = 4.9 \times 10^{-2} \text{ J, which gives } v_d^2 < 0.$$

Thus v_d is not possible and the particle never reaches point d .

(b) At the highest point the particle has no kinetic energy, so it must have the same potential energy as the initial point: $y_{\max} = y_a = \boxed{10 \text{ cm}}.$



61. We choose $y = 0$ at the bottom of the loop.

With no friction, energy is conserved.

The initial (and constant) energy is

$$E = \frac{1}{2}mv_1^2 + mgh_1 = 0 + mgh_1 = mgh_1.$$

We find the speed at a height h from

$$\frac{1}{2}mv^2 + mgh = mgh_1; \quad v = \sqrt{2g(h_1 - h)}.$$

- (a) The skier starts from rest: $v_1 = 0$.

$$\begin{aligned} v_2 &= \sqrt{2g(h_1 - h_2)} \\ &= \sqrt{2(9.8 \text{ m/s}^2)(18 \text{ m} - 15 \text{ m})} = \boxed{7.7 \text{ m/s}}. \end{aligned}$$

$$\begin{aligned} v_3 &= \sqrt{2g(h_1 - h_3)} \\ &= \sqrt{2(9.8 \text{ m/s}^2)(18 \text{ m} - 7 \text{ m})} = \boxed{14.7 \text{ m/s}}. \end{aligned}$$

- (b) At x_3 gravity is down and the normal force must be up.

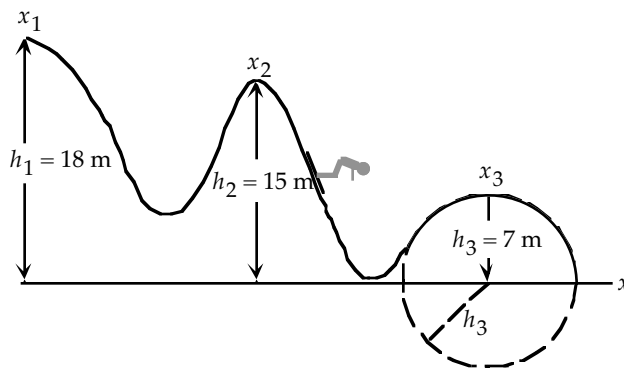
These two provide the centripetal acceleration: $mg - F_N = mv_3^2/R$, so the normal force is

$$\begin{aligned} F_N &= m(g - v_3^2/R) \\ &= m[g - 2g(h_1 - h_3)/h_3] \\ &= mg(3 - 2h_1/h_3) = mg[3 - 2(18 \text{ m})/7 \text{ m}], \text{ which is negative.} \end{aligned}$$

The normal force cannot be negative (it can not pull on the skier), so **the skier leaves the surface**.

We set F_N to its minimum value (zero) to find the maximum value of h_1 at which the skier stays on:

$$F_N = 0; \quad 3 - 2h_1/h_3 = 0, \text{ which gives } h_1 = 3h_3/2 = 3(7 \text{ m})/2 = 10.5 \text{ m}.$$



62. As the mass falls, its kinetic energy increases due to the decrease in potential energy. The kinetic energy then decreases as the resistance force does (negative) work. If we take the entire motion of the mass and neglect the small change in distance as the pile sinks, after n drops we have

$$W_F = \Delta K + \Delta U;$$

$$-F_R d = 0 - 0 + (0 - mgh);$$

$$-(2.5 \times 10^6 \text{ N})(5 \text{ m}) = -n(1300 \text{ kg})(9.8 \text{ m/s}^2)(6 \text{ m}), \text{ which gives } n = \boxed{164}.$$

Energy conservation can be used for the gravity force but not for the resistance force.

63. We choose $y = 0$ at the release position. Because the tension does no work, we can apply conservation of energy.

- (a) From the release point to the lowest position, we have

$$K_i + U_i = K_f + U_f;$$

$$0 + 0 = \frac{1}{2}mv_a^2 + mg(-L), \text{ which gives}$$

$$v_a^2 = 2gL = 2(9.8 \text{ m/s}^2)(1.0 \text{ m}) = \boxed{44 \text{ m/s}}.$$

- (b) From the release point to an angle θ with the vertical, we have

$$K_i + U_i = K_f + U_f;$$

$$0 + 0 = \frac{1}{2}mv_b^2 + mg(-L \cos \theta), \text{ which gives}$$

$$v_a^2 = 2gL \cos \theta = 2(9.8 \text{ m/s}^2)(1.0 \text{ m}) \cos 45^\circ = \boxed{3.7 \text{ m/s}}.$$

- (c) The tension and the appropriate component of the weight must provide the centripetal acceleration. At an angle θ with the vertical, we have

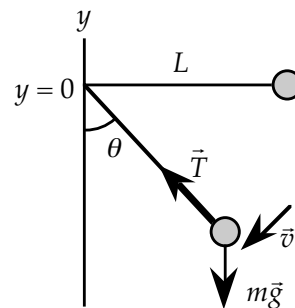
$$T - mg \cos \theta = mv^2/L.$$

Using the result from part (b), we get

$$T - mg \cos \theta = 2mg \cos \theta, \text{ or } T = 3mg \cos \theta.$$

At the bottom of the swing, $\theta = 0^\circ$, so $T_a = 3(0.20 \text{ kg})(9.8 \text{ m/s}^2) \cos 0^\circ = \boxed{5.9 \text{ N}}.$

When $\theta = 45^\circ$, we get $T_b = 3(0.20 \text{ kg})(9.8 \text{ m/s}^2) \cos 45^\circ = \boxed{4.2 \text{ N}}.$



64. We choose $y = 0$ at the lowest position. At the top of the swing the tension and the weight must provide the centripetal acceleration;

$$T + mg = mv^2/h.$$

The tension must pull on the mass, so we have

$$T = m(v^2/h - g) \geq 0, \text{ or } v^2 \geq gh.$$

Because the tension does no work, we can apply conservation of energy from the release point to the position of the mass directly above the nail:

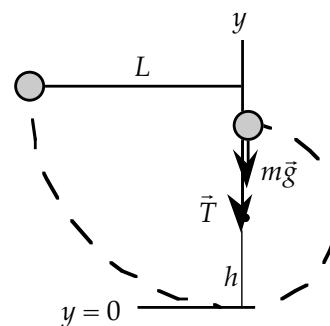
$$K_i + U_i = K_f + U_f;$$

$$0 + mgL = \frac{1}{2}mv^2 + mg(2h), \text{ or } v^2 = 2g(L - 2h).$$

From the condition on the tension, we have

$$2g(L - 2h) \geq gh, \text{ which gives}$$

$$h \leq 2L/5 = 0.40 \text{ m}.$$



65. (a) We take the proton to remain stationary. From the discussion in Section 7-3, this central force (a function of r only) is a conservative force.
 (b) Because the force is attractive, its direction is opposite to that of \vec{r} . We find the potential energy:

$$U(r) = U(s) - \int_s^r F(r') dr' = U(s) - \int_s^r -\frac{C}{r'^2} dr' = U(s) - \frac{C}{r} + \frac{C}{s}.$$

For such a force we take $s = \infty$ and $U(\infty) = 0$. The potential energy is then

$$U(r) = -C/r.$$

- (c) From energy conservation we have

$$K_i + U_i = K_f + U_f;$$

$$0 + 0 = \frac{1}{2}(9.1 \times 10^{-31})v^2 + [-(2.3 \times 10^{-28} \text{ kg} \cdot \text{m}^3/\text{s}^2)/(1.2 \times 10^{-12} \text{ m})], \text{ which gives}$$

$$v = 2.1 \times 10^7 \text{ m/s}.$$

66. For the projectile motion we choose the origin at the bat and up as positive.

- (a) For the vertical component, with the ball hitting the ground, we have

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2; \quad 0 = 0 + v_0 \sin \theta t + \frac{1}{2}(-g)t^2, \text{ which gives}$$

$$t = 0 \text{ and } t = 2v_0 \sin \theta / g.$$

For the horizontal component, we have

$$x = v_0 \cos \theta t = 2(v_0^2 \sin \theta \cos \theta) / g;$$

$$130 \text{ m} = 2(v_0^2 \sin 46^\circ \cos 46^\circ) / (9.8 \text{ m/s}^2), \text{ which gives}$$

$$v_0 = 36 \text{ m/s}.$$

- (b) At the maximum height, the speed is $v_0 \cos \theta$. From energy conservation, we have

$$K_1 + U_1 = K_2 + U_2;$$

$$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}m(v_0 \cos \theta)^2 + mgH;$$

$$H = \frac{1}{2}v_0^2(1 - \cos^2 \theta) / g = \frac{1}{2}(35.7 \text{ m/s})^2(1 - \cos^2 46^\circ) / (9.8 \text{ m/s}^2) = 34 \text{ m}.$$

- (c) At half the maximum height, we have

$$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_3^2 + mg\frac{1}{2}H;$$

$$v_3^2 = v_0^2 - gH = (35.7 \text{ m/s})^2 - (9.8 \text{ m/s}^2)(33.6 \text{ m}), \text{ which gives}$$

$$v_3 = 31 \text{ m/s}.$$

- (e) Because the kinetic energy must be the same at the same height, we have

$$v_4 = 31 \text{ m/s}.$$

67. (a) $W_{\text{spring}} = -\Delta U_{\text{spring}}$
 $= -[0 - \frac{1}{2}k(\Delta x)^2] = \frac{1}{2}(2500 \text{ N/m})(3 \times 10^{-2} \text{ m})^2 = \boxed{+1.13 \text{ J}}.$

(b) $W_f = -\mu_k(mg \cos \theta) \Delta x$
 $= -0.1(3.0 \text{ kg})(9.8 \text{ m/s}^2)(\cos 20^\circ)(3 \times 10^{-2} \text{ m}) = \boxed{-0.083 \text{ J}}.$

(c) $W_g = -\Delta U_g = -(mg \sin \theta) \Delta x$
 $= -(3.0 \text{ kg})(9.8 \text{ m/s}^2)(\sin 20^\circ)(3 \times 10^{-2} \text{ m}) = \boxed{-0.30 \text{ J}}.$

(d) From the work-energy theorem, we have

$$W_{\text{net}} = W_{\text{spring}} + W_f + W_g = \Delta K;$$

$$1.13 \text{ J} - 0.083 \text{ J} - 0.30 \text{ J} = \frac{1}{2}(3.0 \text{ kg})v^2 - 0, \text{ which gives } v = \boxed{0.70 \text{ m/s}}.$$

(e) We apply the work-energy theorem from the time the block leaves the spring until the block stops:

$$W_{\text{net}} = W_{\text{spring}} + W_f + W_g = \Delta K;$$

$$0 - \mu_k(mg \cos \theta)d - (mg \sin \theta)d = 0 - \frac{1}{2}mv^2, \text{ which gives}$$

$$d = v^2 / [2g(\mu_k \cos \theta + \sin \theta)] = (0.70 \text{ m/s})^2 / [2(9.8 \text{ m/s}^2)(0.1 \cos 20^\circ + \sin 20^\circ)] = 0.058 \text{ m}.$$

The total distance will be $0.058 \text{ m} + 0.030 \text{ m} = 0.088 \text{ m} = \boxed{8.8 \text{ cm}}.$

(f) The work-energy theorem applied from the equilibrium point to the rest point becomes

$$W_{\text{net}} = W_{\text{spring}} + W_f + W_g = \Delta K;$$

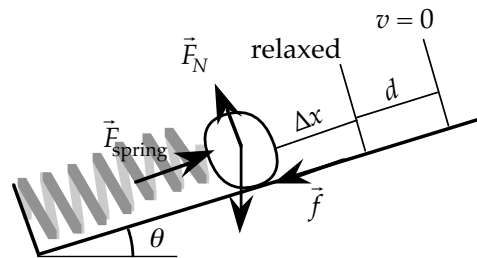
$$\frac{1}{2}k d^2 - 0 - \mu_k(mg \cos \theta)d - (mg \sin \theta)d = 0 - \frac{1}{2}mv^2,$$

This is a quadratic equation for d :

$$\frac{1}{2}(2500 \text{ N/m})d^2 - (0.1 \cos 20^\circ + \sin 20^\circ)(3.0 \text{ kg})(9.8 \text{ m/s}^2)d = \frac{1}{2}(3.0 \text{ kg})(0.70 \text{ m/s})^2.$$

We select the positive solution, which is $d = 0.020 \text{ m}.$

The total distance will be $0.020 \text{ m} + 0.030 \text{ m} = 0.050 \text{ m} = \boxed{5.0 \text{ cm}}.$



68. Because the tension in the rope is perpendicular to the motion, it does no work.

(a) From the work-energy theorem we have

$$W_{\text{net}} = W_f = \Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2;$$

$$W_f = \frac{1}{2}(1.8 \text{ kg})(2.1 \text{ m/s})^2 - \frac{1}{2}(1.8 \text{ kg})(3.5 \text{ m/s})^2 = \boxed{-7.1 \text{ J}}.$$

(b) Friction opposes the tangential motion.

$$W_f = -\mu_k mgs = -\mu_k mg(2\pi r);$$

$$-7.1 \text{ J} = -\mu_k(1.8 \text{ kg})(9.8 \text{ m/s}^2)[2\pi(0.31 \text{ m})], \text{ which gives } \mu_k = \boxed{0.21}.$$

(c) If we take the reference level at the table top, $U = 0$ at all times, since the table is horizontal.

(d) Because the work done by friction does not depend on the speed, $W_f = -7.1 \text{ J}$ for each revolution.

If the block stops after n revolutions, we have

$$W_{\text{net}} = \Delta K;$$

$$(-7.1 \text{ J})n = 0 - \frac{1}{2}(1.8 \text{ kg})(3.5 \text{ m/s})^2, \text{ which gives } n = \boxed{1.5 \text{ revolutions}}.$$

69. Because the length of the rope is constant, when m_1 moves

down Δy_1 , the two segments above m_1 both increase by Δy_1 , and

the segment above m_2 must decrease by twice that amount:

$$\Delta y_2 = -2\Delta y_1 \quad (- \text{ indicates the opposite direction}).$$

If we differentiate with respect to time, we have $v_2 = -2v_1$.

With $y = 0$ at the ground, from energy conservation we have

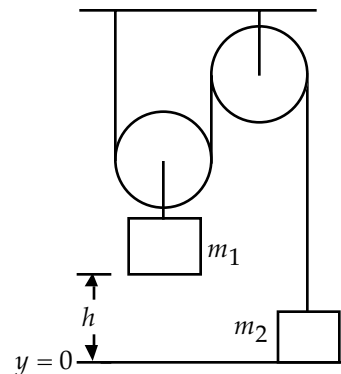
$$K_i + U_i = K_f + U_f; \quad 0 + m_1 g h_1 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + m_2 g h_2;$$

$$m_1 g h = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 (-2v_1)^2 + m_2 g(2h), \quad \text{or}$$

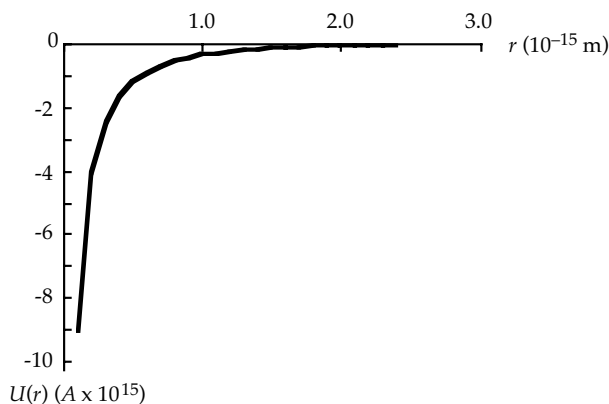
$$v_1^2 = 2gh(m_1 - 2m_2) / (m_1 + 4m_2)$$

$$= 2(9.8 \text{ m/s}^2)(0.8 \text{ m})[(5.0 \text{ kg}) - 2(2.0 \text{ kg})] / [5.0 \text{ kg} + 4(2.0 \text{ kg})],$$

which gives $v_1 = \boxed{1.1 \text{ m/s}}.$



70. (a)



- (b) Normally we would find r at which $U = -Ae^{-kr}/r$ is minimum by setting $dU/dr = 0$:
 $dU/dr = +Ae^{-kr}/r^2 + Ake^{-kr}/r = Ae^{-kr}(1 + kr)/r^2 = 0$,
 which gives $r = \infty$, for which U is maximum (zero). The difficulty is that U is not defined for $r < 0$.
 From the plot we see that U is minimum ($-\infty$) at $r = 0$.
- (c) $F(r) = -dU/dr = -Ae^{-kr}[(1 + kr)/r^2]$, which is attractive.
- (d) $|F(0.1)| = Ae^{-0.1}(1 + 0.1)/(0.1 \times 10^{-15} \text{ m})^2 = (1.0 \times 10^{32}) \text{ A}$.
 $|F(10)| = Ae^{-10}(1 + 10)/(10 \times 10^{-15} \text{ m})^2 = (5.0 \times 10^{24}) \text{ A}$.

71. For the potential energy $U(x_1 - x_2)$, the force on the particle at x_1 is determined by the spatial variation of the potential energy, keeping x_2 fixed. We differentiate, using the chain rule:

$$F_1 = -\frac{\partial U}{\partial x_1} = -\frac{\partial U}{\partial(x_1 - x_2)} \frac{\partial(x_1 - x_2)}{\partial x_1} = -\frac{\partial U}{\partial(x_1 - x_2)} (+1) = -\frac{\partial U}{\partial(x_1 - x_2)}.$$

The force on the particle at x_2 is determined by the spatial variation of the potential energy, keeping x_1 fixed:

$$F_2 = -\frac{\partial U}{\partial x_2} = -\frac{\partial U}{\partial(x_1 - x_2)} \frac{\partial(x_1 - x_2)}{\partial x_2} = -\frac{\partial U}{\partial(x_1 - x_2)} (-1) = +\frac{\partial U}{\partial(x_1 - x_2)}.$$

We see that $F_1 = -F_2$, so that Newton's third law is satisfied.

72. Since the gravitational energy is proportional to the height and equal to the total energy when the projectile reaches the top of its path, the projectile must have 90% of the initial energy left as it reaches the top, i.e., it has lost 10% of its initial energy. So

$$W_f = \Delta E = (90\% E_i - E_i) = -10\% E_i \\ = -10\% \left(\frac{1}{2} m v_0^2 \right) = -10\% \left[\frac{1}{2} (5.0 \text{ kg}) (30 \text{ m/s})^2 \right] = \boxed{-0.23 \text{ kJ}}.$$

73. (a) For the given force, $F(r) = -kr$, we find the potential energy:

$$U = \int kr \, dr = \frac{1}{2} kr^2, \text{ with } U = 0 \text{ at } r = 0. \text{ The total energy of the object is then}$$

$$E = K + U = \boxed{\frac{1}{2} m v^2 + \frac{1}{2} k r^2}.$$

(b) The object is in circular motion of radius R , so $F_r = mv^2/R = kR$, and the velocity must have a constant magnitude of

$$v = \boxed{R(k/m)^{1/2}}.$$

(c) Write $v^2 = v_r^2 + (\omega r)^2$, where $v_r = dr/dt$ is the radial velocity. Since $r = R = \text{constant}$, $v_r = 0$; and so $v = \omega r$. Plug this into the expression for E : $E = \frac{1}{2} m (\omega r)^2 + \frac{1}{2} k r^2$. Thus

$$\omega = \boxed{(2E/mr^2 - k/m)^{1/2}}.$$

74. The linear mass density of the chain is $\mu = m/L = 3.5 \text{ kg/m}$. We choose $y = 0$ at the table top and up positive. As indicated on the diagram, the differential mass $dm = \mu dy$ has a potential energy of

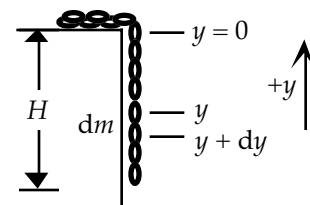
$$dU = yg dm = yg\mu dy. \quad (\text{Note that since } y < 0, U < 0.)$$

When dm is raised to the table, where $U = 0$, the change in potential energy is $-yg\mu dy$ (which is positive). For the total change in potential energy when all of the elements are raised to the tabletop, we can sum the potential energy changes by integrating:

$$\begin{aligned} W = \Delta U &= - \int_H^0 yg\mu dy = -\frac{1}{2}g\mu y^2 \Big|_H^0 = \frac{1}{2}g\mu H^2 \\ &= \frac{1}{2}(9.8 \text{ m/s}^2)(3.5 \text{ kg/m})(0.6 \text{ m})^2 = \boxed{6.2 \text{ J}}. \end{aligned}$$

Note that this is the result we find for the work required to raise the center of mass of the piece that is hanging to the tabletop:

$$W = mg \Delta h = (3.5 \text{ kg/m})(0.6 \text{ m})(9.8 \text{ m/s}^2)(0.30 \text{ m}) = \boxed{6.2 \text{ J}}.$$



75. We choose the coordinate system shown in the diagram, with $y = 0$ at the lowest point. Because the tension does no work from the release point to the point where the string is cut, we can use conservation of energy:

$$K_i + U_i = K_f + U_f;$$

$$0 + mgL = \frac{1}{2}mv_0^2 + mgL(1 - \cos \alpha), \text{ which gives}$$

$$v_0^2 = 2gL \cos \alpha.$$

When the string is cut, the ball becomes a projectile with initial speed v_0 at an angle α with the horizontal.

For the projectile motion, we have

$$x = x_0 + v_0 (\cos \alpha) t \quad \text{and} \quad y = y_0 + v_0 (\sin \alpha) t + \frac{1}{2}(-g)t^2.$$

When the ball hits the floor, $y = 0$:

$$0 = L(1 - \cos \alpha) + v_0 (\sin \alpha) t - \frac{1}{2}gt^2,$$

from which we can find the time to hit the floor:

$$t = (v_0/g) \sin \alpha + \{[(v_0/g) \sin \alpha]^2 + (2L/g)(1 - \cos \alpha)\}^{1/2}.$$

We take the positive value from the solution to the quadratic equation for t .

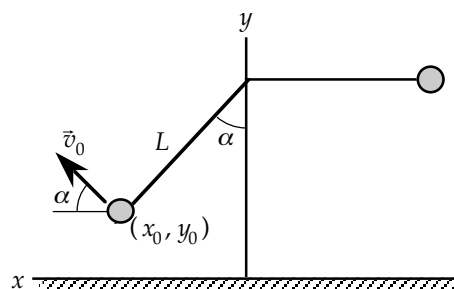
The horizontal distance traveled in this time is

$$x = L \sin \alpha + v_0 (\cos \alpha) t$$

$$= L \sin \alpha + (v_0 \cos \alpha)(v_0/g) \sin \alpha + (v_0 \cos \alpha) \{[(v_0/g) \sin \alpha]^2 + (2L/g)(1 - \cos \alpha)\}^{1/2}.$$

When we use $v_0^2 = 2gL \cos \alpha$ and $\sin^2 \alpha = 1 - \cos^2 \alpha$ and do some algebra, we get

$$x = \boxed{L[\sin \alpha + 2 \sin \alpha \cos^2 \alpha + 2(\cos^3 \alpha - \cos^6 \alpha)^{1/2}]}. \quad \text{}$$



76. We consider a small mass m on the surface of the water.

In the rotating reference frame in which the water is at rest, there are two forces acting on m :

gravity: mg , which is conservative with $U_g = mgy$;

centrifugal: $mr\omega^2$, which is conservative, since it depends only on position.

We find the centrifugal potential energy from

$$U_c = - \int F_c dr = - \int mr\omega^2 dr = -\frac{1}{2}mr^2\omega^2 + U(0).$$

At the surface, the total potential energy per unit mass is constant:

$$U/m = U_g/m + U_c/m = gy - \frac{1}{2}r^2\omega^2 = C_1, \text{ from which we get}$$

$$y = \frac{1}{2}\omega^2 r^2/g + C.$$

The constant C is the depth of the water at the center of the bucket ($r = 0$):

$$\boxed{y = (\frac{1}{2}\omega^2/g)r^2 + y_0}.$$

