

CHAPTER 5 Applications of Newton's Laws

Answers to Understanding the Concepts Questions

1. The tightrope walker has an upward force acting on him or her that must balance the force of gravity. This force is a contact force from the wire that must come from the tension in the wire. If the wire were perfectly horizontal, the tension would have only a horizontal component. In order for the tension to have a vertical component, the wire must be aligned with a nonzero vertical component.
2. The engine forces the tires to turn, creating a tendency for the tire to move backwards at the contact point between the tire and the ground. This causes the road to exert a forward friction (against the tendency of motion of the tire) on the tire. The role of the muscles is similar. They create a backward-moving tendency for the foot, which in turn receives a forward friction from the road.
3. If the string is itself massless, then the tension must equal the weight of the object suspended from it. Only if the string has a mass significant compared to the mass of the object will the tension vary along the string. The tension in the string at a particular point must match the weight of the object plus the weight of all the string below that point. In that case the tension except at the very top of the string is greater than the weight of the object.
4. Besides the obvious relevance of the power of the engine, other factors include the retarding force of the wind, as well as the conditions of the road and the tires, which determine the coefficient of static friction between the tire and the road. In fact, regardless of the power of the engine, the force of acceleration exerted on the vehicle by the road cannot exceed its maximum value, which is proportional to the coefficient of static friction.
5. When a rope has mass, then the tension above a certain point must be large enough to move not only the mass attached to the end, but the mass of the rope below that point as well. The rope must be strong enough to support this additional tension. Friction in a pulley produces much the same effect. The tension must increase to the point where not only the weight of an object being accelerated against gravity can be overcome, but the force due to friction as well — remember, the friction acts against the motion. A properly lubricated axle can help reduce friction a great deal when a pulley is in use.
6. Putting a heavy load on the driving wheels increases the normal force exerted by the road on the wheels. This in turn increases the maximum available value of the static friction force exerted by the road on the driving wheels, reducing the chance for the vehicle to skid.
7. A cyclist making a turn must make use of a centripetal force, one that is perpendicular to his direction of motion. The only external force able to do this on a flat road is the force of friction between tire and road, and in leaning into the curve the cyclist is attempting to increase this force by trying to put the bike into a position where it would “slide out” if there were no friction. If the curve is banked, then the cyclist can make use of the normal force that keeps the bicycle from moving into the road surface. By tilting the bicycle so that it is perpendicular to the road, the cyclist avoids introducing a sideways friction force which in this case would be undesirable.
8. In the case of an uneven distribution, the wheel that carries the lightest load has the least amount of available static friction from the road, making it the most vulnerable to skidding. To ensure that this wheel does not skid, the maximum possible acceleration of the vehicle has to be reduced. If the vehicle has rear-wheel drive, then only the load on the rear wheels can be counted when deciding the

maximum available static friction from the road. For example, if the weight is still evenly distributed on all four wheels, then the combined normal force on the two rear (driving) wheels is only half of the weight of the vehicle, and the corresponding maximum static friction available from the road is also only half as much. This cuts the maximum possible acceleration of the vehicle to half as much as the case of four-wheel drive.

9. For a massless rope, the tension in the rope that balances the weight of a suspended object is the same everywhere. That is because the rope itself, and hence any part of it, has no weight that itself might need balancing. In this case, the spring scale would reveal the same tension everywhere. On the other hand, if the rope is not massless, then the tension in the rope at a particular point must match the weight of the object plus the weight of all the rope below that point. Thus the tension would be larger and larger as we followed the rope up away from the object, and this is what the inserted spring would reveal.
10. No. The scale reading is equal in magnitude to the combined weight of the bowl, water, and ice. As ice melts its weight stays the same, so the reading should not change.
11. A centerboard has a single purpose: to keep the boat from moving sideways through the water when the wind blows at right angles to the boat's direction. It does this by experiencing a drag force from the water that is largest when the boat moves sideways through the water. A keel plays this role plus one more: it is a very massive object and an enormous force is required to lift it. Thus the keel also tends to keep the boat bottom down. The keel is important when the wind forces are so large that the boat may be overturned, and this is typically associated with larger boats carrying sails of comparatively large area. On the other hand, a keel is so massive that it has a significant effect on the acceleration of the boat. Thus a centerboard is adequate when overturning is not a problem and is advantageous when rapid accelerations are desired.
12. The mass of the lineman is now 149.5 kg, which is slightly more than 4 times the mass of the woman. So, to lift him up the woman needs to pull the rope with slightly more than her own body weight. In return, the rope would lift her off the ground. So, as she starts from rest and pulls the rope with a force that is just able to lift the lineman up, she experiences an upward acceleration, and gets lifted off the ground. Once she is airborne, the pulley system loses balance and the weight of the lineman, being slightly greater than 4 times her weight, causes the lineman to accelerate (slowly) downward while she is accelerated upward at 4 times the rate of the lineman. Once the lineman touches the ground again the ground gives him an additional supporting force, which restores the balance of the pulley system. The lineman stays on the ground while she remains suspended on the rope.
13. Yes. In the free-body diagram for the lineman, there are 4 segments of the rope that pull the lineman up, which is why a force that equals only $1/4$ of his weight would be enough to lift him up. Note that each movable pulley adds 2 segments of the rope. So, to lift the lineman with a force that equals $1/100$ of his weight we would need 100 segments of the rope, which would require 50 movable pulleys. In reality, a large number of pulleys in the system would significantly increased the frictional resistance of the system; and the person who lifts the lineman would have to pull the rope for over 100 meters just to lift him up by 1 meter.
14. The mass of the rope is negligible if the tension in the rope is considerably greater than the weight of the rope.
15. Drag forces on boats are a type of friction that depends on the surface area of the boat in contact with the water. The more people in a scull, the lower she sits in the water. This increases the area of surface in contact with the water, and as a consequence the drag increases as well.
16. Consider, for example, the setup depicted in Figure 5-34. For simplicity we may take $\theta = 0$ (so the inclined plane becomes a horizontal tabletop and the mass m hangs over the edge of the table). It is clear that the smallest weight of the hanging mass that can initiate motion must be equal to the

maximum static friction between the mass M and the tabletop. By varying the mass M and measuring the corresponding minimum mass m that causes M to slide, he can deduce the relationship between the contact force F_N (which is equal to Mg) and the maximum value of the static friction. He can also vary the contact area between M and the tabletop and check if that changes the required minimum mass m that causes sliding. If not, then static friction does not depend on the contact area.

17. All of the statements are true. Any net force, friction or not, will cause an object to accelerate. Frictions can also be used to increase the speed of an object. Static friction exerted by the road on the tires, for example, is responsible for the increase in speed of a car when its accelerator pedal is pushed. As an example of kinetic friction causing the speed of an object to increase, consider a box that rests on a stationary cart. If the cart is suddenly pulled forward with sufficient acceleration, the box will start to slide forward as well; although its speed is not as fast as that of the cart, it does increase its speed from zero, due to the kinetic friction it receives from the cart.
18. Not only can there be a normal force on the car, but we strongly recommend that you only take rides in roller coasters where there is such a force at the top of a loop. If there were not, gravity would be the only force acting, and if this were true even slightly away from the top of the loop the car would not execute the circular motion that corresponds to contact with the track! Note that the force required for circular motion at the top of the loop is a downward force, and this can come both from gravity and the normal force.
19. The rider is moving in a vertical circular path, which requires a centripetal force pointing radially inward. At the top of the path the centripetal force is vertically downward, which happens to be in the same direction as the weight of the rider. So at this point the rider's weight is (mostly) being used as the centripetal force. He or she is in fact accelerating downward at nearly the acceleration of gravity. This is not unlike a free fall. The coins would not fall out, as everything (the rider and the coins) is falling at the same rate.
20. No. Since the elevator is moving with a constant velocity its acceleration is zero, and so must be the net force exerted on the box. So the normal force exerted on it must still be equal in magnitude to its weight.
21. Yes. The block is now accelerating upward, so the net force must also be upward, meaning that the magnitude of the upward normal force exerted on it by the elevator floor must exceed that of its downward weight.
22. Any circular motion requires a centripetal force, and the boat following a great circle is no exception. For the boat, circular motion along the great circle requires a force directed to Earth's center. The net effect of gravity and the normal (upward) force on the boat, plus the net vertical component of wind forces, must combine to give a net acceleration v^2/R , where v is the speed of the boat and R is Earth's radius. Since this is a very small number, the excess of gravity over the other forces must be very tiny.
23. Neglecting air friction, the bob is subject to two forces: its weight, down; and the tension in the string. When the string is vertical the bob passes through the lowest point of its circular motion, at which point T has to exceed mg so as to provide the centripetal force. At any other point, the tension is not vertical and cannot completely cancel with the weight of the bob. So the net force on the bob is never zero. As the bob reaches the right turning point its speed is zero. It is subject to the tension in the string, which has both upward and leftward components, and its weight, down. The net force exerted on it has both a leftward and downward component, causing it to move both downward and leftward from the turning point.
24. A centripetal force is needed to keep you moving in a circle along with the car. Your seat belt or a contact force from the car door and seat provides this force. When it is not present, you would move in a straight line; that is, you would be thrown from the car.

25. No. If the plane of the circle were to be perfectly horizontal, then the tension in the rope would also be horizontal, and there would be no force to balance the downward force of gravity on the book, which would not be able to stop falling vertically and staying in the same horizontal plane.
26. Newton's third law states that the forces on you and your partner are equal and opposite. (We are here ignoring the vertical forces due to gravity and the normal force from the ice; these cancel in any case for each partner, and all the motion discussed here is horizontal.) If you have equal masses, then the result, according to Newton's second law, is that you will experience an equal and opposite acceleration. If partner B is half as massive as partner A, then partner B will experience an acceleration of twice the magnitude as that of partner A.
27. The coefficient of static friction between the ground and the shoes would be low, and so would be the maximum available static friction from the ground that can cause you to accelerate. If you try just a little too hard, your feet would slip on ice.
28. Tension arises as an object is being stretched, be it a string or a stick. If you pull the mass in a direction along the orientation of the stick, then the force in the stick is indeed tension, just as in a string. But unlike a soft string, a stick can also be pulled sideways, in which case the force exerted on the mass attached at the end of the stick is no longer parallel to the stick. It can also be used to push the mass instead of pulling, and the force in the stick is now due to its compression, rather than stretch. These forces are not tension and do not occur in a string.
29. The net force on Tarzan is such that he moves in a circle. The tension in the vine is the only force that can act to move Tarzan in this way. The tension at the bottom of the swing must also compensate for the entire force of gravity on Tarzan, and moreover, the speed is greatest at the bottom of the swing, because the movement has a tangential component due to gravity that increases Tarzan's speed to the bottom. Thus the tension must be greatest at the bottom, and this is the point of greatest danger, meaning the point where the tension is most likely to exceed the vine's breaking strength.
30. As we explained in Question 19, the weight of the person is largely being used as centripetal force, so there is very little supporting force from the seat (which would otherwise be required to balance the weight), making her feel "empty", as if she is just about to lose contact with the seat and "fly off".
31. We can think about this most easily by first visualizing the situation from the outside. The die slides smoothly downward in a straight line, because without friction there is no force that will start the die moving around the central axis. Let's say that in looking straight down we see the die sliding along the x -axis. As it does so an observer turning with the bowl who starts by looking outward from the vertical axis along the x -axis will have rotated away from the x -axis, say towards the y -axis, if the rotation seen from above is counterclockwise. That observer will see the die curving away from his or her left hand as it descends the bowl's wall. Such curving motion suggests a force acting, a phenomenon associated with the fact that the rotating observer is in a noninertial frame.
32. In the vertical direction she is subject to her weight, down; and the supporting force from the ground, up. In the horizontal direction, as she pushes the ground backward with her back foot the ground exerts a forward static friction force on her, while she is also subject to a backward air friction.
33. The first term is proportional to v while the second one is proportional to v^2 ; so at low speeds the first term dominates (as v is small and v^2 is even smaller), while at high speeds the second term dominates (as v is large but v^2 is even larger). At a certain speed, v_1 , the two terms become the same: $bv_1 = cv_1^2$, or $v_1 = b/c$. When $v \ll v_1$ it's the low-speed regime, where the first term dominates; and when $v \gg v_1$ it's the high-speed regime, where the second one dominates.
34. Whenever you make contact with the cushion there is always an upward normal force acting on you. The magnitude of this normal force depends on your acceleration. As you sink into the seat cushion with

a certain initial downward velocity you must soon be brought to rest, so there is a net upward force on you (causing an upward acceleration), so the normal force exerted by the seat cushion exceeds your weight. As you settle into the seat and stops sinking the net force on you becomes zero, and the normal force from the cushion is now equal to your weight (assuming, for simplicity, that your feet do not touch the ground, as that would introduce another normal force from the floor).

Solutions to Problems

1. For the one-dimensional motion with constant acceleration, we can write
 $v^2 = v_0^2 + 2a(x - x_0)$; $0 = (500 \text{ m/s})^2 + 2a [7 \text{ cm} (1 \text{ m}/100 \text{ cm}) - 0]$, which gives

$$a = -1.8 \times 10^6 \text{ m/s}^2.$$

If we select the bullet as the object, we apply Newton's second law to find the magnitude of F_{av} :

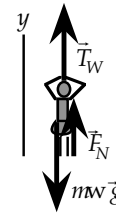
$$\Sigma F = ma; F_{av} = [8 \text{ g}/(10^3 \text{ g/kg})](1.8 \times 10^6 \text{ m/s}^2) = \boxed{1.4 \times 10^4 \text{ N}}.$$

2. The forces on the woman are her weight, the tension in the rope and the normal force from the scale, as shown in the diagram.

We can write

$$\Sigma \vec{F} = m\vec{a}; T\hat{j} + F_N\hat{j} - m_W g\hat{j} = m_W(0), \text{ so}$$

$$F_N = m_W g - T = (50 \text{ kg})(9.8 \text{ m/s}^2) - 365 \text{ N} = \boxed{123 \text{ N} (27 \text{ lb})}.$$



3. The two forces are the weight (down) and the buoyancy (up). With up as positive we write

$$\Sigma F_y = ma_y; F_B - m_T g = m(0), \text{ which gives}$$

$$F_B = m_T g = (100 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{980 \text{ N}}.$$

4. For the horizontal motion, assuming constant acceleration,
 $v = v_0 + at$; $(210 \text{ km/h})(10^3 \text{ m/km})/(3600 \text{ s/h}) = 0 + a(7.3 \text{ s})$, which gives
 $a = 8.0 \text{ m/s}^2$.

To get the net force on the woman that produces this acceleration,

$$\Sigma F = ma = (61 \text{ kg})(8.0 \text{ m/s}^2) = \boxed{4.9 \times 10^2 \text{ N}} \text{ forward.}$$

This net horizontal force is mostly from the seat back, with some from the steering wheel and a negative force from the accelerator pedal. (In the vertical direction there is the weight balanced by the normal force from the seat.)

5. For the equation of motion of the puck we can write
 $\Sigma \vec{F} = m\vec{a}$; $4.0\hat{i} \text{ N} = (0.10 \text{ kg})\vec{a}$, which gives $\vec{a} = 40\hat{i} \text{ m/s}^2$.

Thus v_y does not change.

For a speed of $v = 6 \text{ m/s}$, we can find v_x :

$$v^2 = v_x^2 + v_y^2; (6.0 \text{ m/s})^2 = v_x^2 + (3.0 \text{ m/s})^2, \text{ which gives } v_x = 5.2 \text{ m/s.}$$

For the constant acceleration in the x -direction we can write

$$v_x = v_{0x} + a_x t; 5.2 \text{ m/s} = 1.4 \text{ m/s} + (40 \text{ m/s}^2)t, \text{ which gives } t = \boxed{0.095 \text{ s}}.$$

6. The tension in the rope is created by the contact between the rope and the man's hands. The man will pull down on the rope, and the rope will pull up on the man. As shown, we take the positive direction down, since this is the direction of the acceleration.

(a) For the man we can write

$$\Sigma F_y = ma_y; mg - T = ma.$$

From this we see that there is a minimum acceleration that produces the maximum tension:

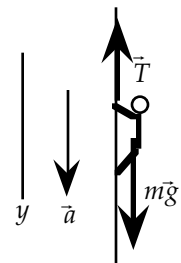
$$(80 \text{ kg})(9.8 \text{ m/s}^2) - 600 \text{ N} = (80 \text{ kg})a_{\min}, \text{ which gives } a_{\min} = \boxed{2.3 \text{ m/s}^2}.$$

(b) For this constant acceleration, we can write

$$y = y_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(2.3 \text{ m/s}^2)(5.0 \text{ s})^2 = \boxed{29 \text{ m}}.$$

To find the velocity we use

$$v = v_0 + at = 0 + (2.3 \text{ m/s}^2)(5.0 \text{ s}) = \boxed{12 \text{ m/s}} \text{ down.}$$



7. The mass of the bottom section of the uniform rod, with d the distance from the top, is $m' = (M/L)(L - d)$.

For the y -direction we can write: $\sum F = m'a$; $T_d - m'g = 0$, so

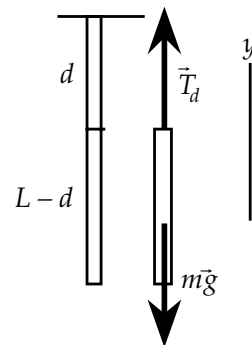
$T_d = (M/L)(L - d)g$; thus T_d is maximum at the top and zero at the bottom.

- (a) When $d = 2.0$ m,

$$T_2 = [(5.6 \text{ kg}) / (3.5 \text{ m})](3.5 \text{ m} - 2.0 \text{ m})(9.8 \text{ m/s}^2) = \boxed{24 \text{ N}}.$$

- (b) When $d = 3.0$ m

$$T_3 = [(5.6 \text{ kg}) / (3.5 \text{ m})](3.5 \text{ m} - 3.0 \text{ m})(9.8 \text{ m/s}^2) = \boxed{7.8 \text{ N}}.$$



8. For the stationary brick, with no acceleration, we can write

$$\sum F_x = ma_x:$$

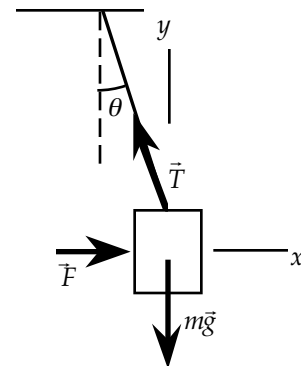
$$P - T \sin \theta = 0; \quad 12 \text{ N} - T \sin 25^\circ = 0, \text{ which gives}$$

$$T = 28 \text{ N}.$$

$$\sum F_y = ma_y:$$

$$T \cos \theta - mg = 0; \quad (28 \text{ N}) \cos 25^\circ - m(9.8 \text{ m/s}^2) = 0, \text{ which gives}$$

$$m = \boxed{2.6 \text{ kg}}.$$



9. (a) In equilibrium, there is no acceleration of the block.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the block:

$$y\text{-component: } F_N - F \sin \theta - mg \cos \theta = 0,$$

$$x\text{-component: } F \cos \theta - mg \sin \theta = 0,$$

$$F \cos 15^\circ - (25 \text{ kg})(9.8 \text{ m/s}^2) \sin 15^\circ = 0, \text{ which gives}$$

$$F = \boxed{66 \text{ N}}.$$

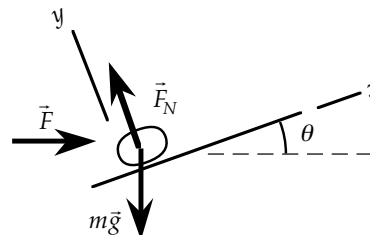
- (b) With a larger force, the block will accelerate up the plane:

$$x\text{-component: } F \cos \theta - mg \sin \theta = ma,$$

$$3(66 \text{ N}) \cos 15^\circ - (25 \text{ kg})(9.8 \text{ m/s}^2) \sin 15^\circ = (25 \text{ kg})a,$$

which gives

$$a = \boxed{5.1 \text{ m/s}^2 \text{ up}}.$$



10. (a) If we choose the two blocks as the system,

we can write for $\sum \vec{F} = m\vec{a}$:

$$x\text{-component: } F = (m_1 + m_2)a, \text{ which gives}$$

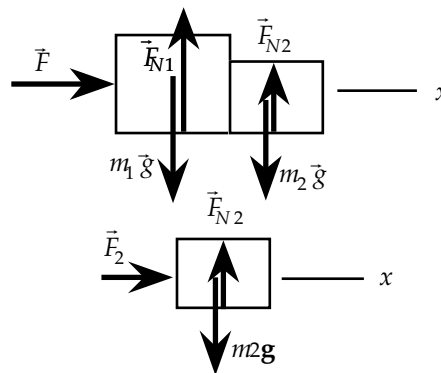
$$a = \boxed{F / (m_1 + m_2)}.$$

- (b) For the block on the right as the system,

we can write for $\sum \vec{F} = m\vec{a}$:

$$x\text{-component: } F_2 = m_2 a = \boxed{m_2 F / (m_1 + m_2)}.$$

Note that an equal and opposite force acts on m_1 .



11. (a) We can find the acceleration from the motion data:

$$v^2 = v_0^2 + 2a(x - x_0);$$

$$[(55 \text{ mi/h})(1.6 \times 10^3 \text{ m/mi}) / (3.6 \times 10^3 \text{ s/h})]^2 = 0 + 2a(120 \text{ m} - 0), \text{ which gives}$$

$$a = \boxed{2.5 \text{ m/s}^2 \text{ forward}}.$$

- (b) With the pulled automobile as the system, the tension in the tow rope must provide its acceleration. Thus $\Sigma F_x = ma_x$;

$$T = m_A a = (1400 \text{ kg})(2.5 \text{ m/s}^2) = \boxed{3.5 \times 10^3 \text{ N forward}}.$$

12. Forces are drawn for each of the blocks for the situation when m_1 leaves the floor (no normal force). Because the string doesn't stretch, the tension is the same at each end of the string, and the accelerations of the blocks have the same magnitude. Note that we take the positive direction in the direction of the acceleration for each block.

- (a) We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for each block:

$$y\text{-component (block 1): } T - m_1 g = m_1 a.$$

$$y\text{-component (block 2): } m_2 g - T = m_2 a.$$

By adding the equations, we find the acceleration:

$$\begin{aligned} a &= (m_2 - m_1)g / (m_1 + m_2) \\ &= (1.700 \text{ kg} - 1.650 \text{ kg})(9.8 \text{ m/s}^2) / (1.70 \text{ kg} + 1.65 \text{ kg}) \\ &= \boxed{0.15 \text{ m/s}^2} \text{ for both blocks.} \end{aligned}$$

- (b) For the motion of block 2:

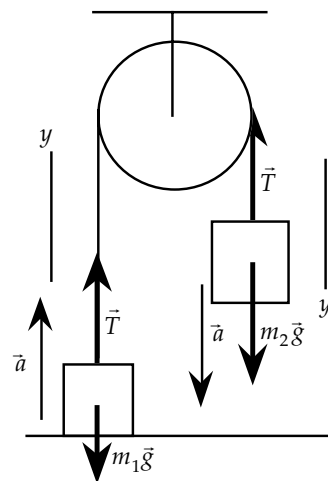
$$v_2^2 = v_{02}^2 + 2a(y_2 - y_{02}) = 0 + 2(0.15 \text{ m/s}^2)(2.15 \text{ m} - 0),$$

$$\text{which gives } v_2 = \boxed{0.80 \text{ m/s}}.$$

(Note: block 1 has the same speed. Once block 2 hits the floor, $T \rightarrow 0$ and the motions of the two blocks will differ.)

- (c) To find the time to reach the floor, for block 2:

$$v_2 = v_{02} + at; \quad 0.803 \text{ m/s} = 0 + (0.15 \text{ m/s}^2)t, \text{ which gives } t = \boxed{5.4 \text{ s}}.$$



13. For the largest value of M , the block is on the verge of slipping down the plane, so the static friction force will be up the plane and maximum, $f_s = f_{s,\max} = \mu_s F_N$.

From the force diagram, with the block M as the system, we can write $\Sigma \vec{F} = M\vec{a}$:

$$x\text{-component: } T - M_{\max} g \sin \theta + f_{s,\max} = 0;$$

$$y\text{-component: } F_N - M_{\max} g \cos \theta = 0; \text{ or } F_N = M_{\max} g \cos \theta.$$

From the force diagram, with the block m as the system,

we can write $\Sigma \vec{F} = m\vec{a}$:

$$y\text{-component: } T - mg = 0; \text{ or } T = mg.$$

The x -equation becomes

$$mg - M_{\max} g \sin \theta + \mu_s M_{\max} g \cos \theta = 0, \text{ or } M_{\max} (\sin \theta - \mu_s \cos \theta) = m;$$

$$M_{\max} (\sin 30^\circ - 0.20 \cos 30^\circ) = 3.0 \text{ kg}, \text{ which gives } M_{\max} = \boxed{9.2 \text{ kg}}.$$

For the smallest value of M , the block is on the verge of slipping up the plane, so the static friction force will be down the plane and maximum, $f_s = f_{s,\max} = \mu_s F_N$. The only change will be in the x -equation. From

the force diagram, with the block M as the system, we can write $\Sigma \vec{F} = M\vec{a}$:

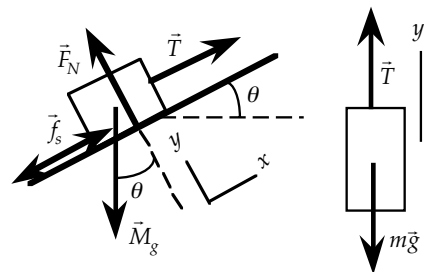
$$x\text{-component: } T - M_{\min} g \sin \theta - f_{s,\max} = 0; \quad mg - M_{\min} g \sin \theta - \mu_s M_{\min} g \cos \theta = 0, \text{ or}$$

$$M_{\min} (\sin 30^\circ + 0.20 \cos 30^\circ) = 3.0 \text{ kg}, \text{ which gives } M_{\min} = \boxed{4.5 \text{ kg}}.$$

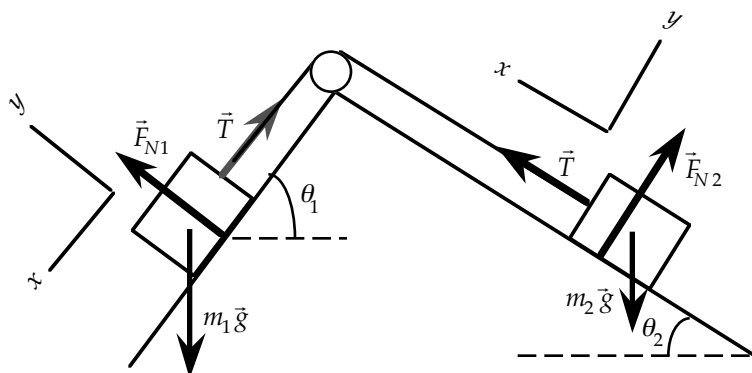
If $M = 6 \text{ kg}$, the block will remain at rest. We assume that the static friction force will be up the plane.

The x -equation becomes $mg - Mg \sin \theta + f_s = 0$;

$$(3.0 \text{ kg})(9.8 \text{ m/s}^2) - (6.0 \text{ kg})(9.8 \text{ m/s}^2) \sin 30^\circ + f_s = 0, \text{ which gives } f_s = \boxed{0}.$$



14.



Forces are drawn for each of the blocks. Because the string doesn't stretch, the tension is the same at each end of the string. Note that we use two different coordinate systems.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for each block:

For block 1:

$$x\text{-component: } m_1 g \sin \theta_1 - T = 0.$$

For block 2:

$$x\text{-component: } T - m_2 g \sin \theta_2 = 0.$$

By adding the equations, we have $m_1 g \sin \theta_1 = m_2 g \sin \theta_2$;

$$(1.50 \text{ kg}) \sin 62^\circ = (2.50 \text{ kg}) \sin \theta_2, \text{ which gives } \theta_2 = \boxed{32^\circ}.$$

15. Let the tension in the string be T . For the woman (1) we have

$$T - m_1 g = m_1 a_1, \text{ and for the lineman (2)}$$

$$m_2 g - 4T = m_2 a_2. \text{ Also}$$

$$a_1 = 4a_2.$$

Solve these equations to obtain

$$\begin{aligned} a_1 &= (m_2 - 4m_1)g / (4m_1 + m_2/4) \\ &= [149.5 \text{ kg} - 4(37.5 \text{ kg})](9.8 \text{ m/s}^2) / [4(37.3 \text{ kg}) + (149.5 \text{ kg})/4] \\ &= \boxed{9.8 \text{ mm/s}^2} \text{ and} \\ a_2 &= \frac{1}{4}a_1 = \boxed{2.5 \text{ mm/s}^2}. \end{aligned}$$

16. The mass of the element of rope is

$$\Delta m = \lambda \Delta h.$$

If we write $\sum \vec{F} = m\vec{a}$ for the element, with up positive, we get

$$T_{\text{up}} - T_{\text{down}} - (\Delta m)g = 0 \quad \text{or}$$

$$T(h + \Delta h) - T(h) = (\lambda \Delta h)g.$$

We divide by Δh and take the limit as $\Delta h \rightarrow 0$:

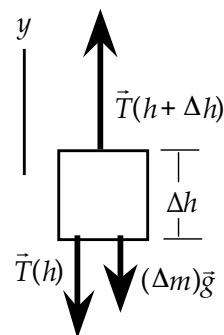
$$\lim_{\Delta h \rightarrow 0} \frac{T(h + \Delta h) - T(h)}{\Delta h} = \frac{dT(h)}{dh} = \lambda g, \quad \text{or } dT = \lambda g dh.$$

This can be integrated to give

$$T(h) = \lambda gh + \text{a constant.}$$

Because $T = 0$ at $h = 0$, the constant = 0 and

$$T(h) = \boxed{\lambda g h}.$$



17. The force diagrams for each of the masses and the movable pulley are shown. Note that we take down as positive and the indicated accelerations are relative to the fixed pulley. A downward acceleration of m_1 means an upward acceleration of the movable pulley. If we call a_r the acceleration of m_2 with respect to the movable pulley, then

$$a_2 = a_r - a_1 \quad \text{and} \quad a_3 = -a_r - a_1,$$

because the acceleration of m_3 with respect to the pulley must be the negative of m_2 's acceleration with respect to the pulley.

If the mass of the pulley is negligible, we write $\sum F_y = ma_y$:

$$2T_2 - T_1 = (0)(-a_1), \text{ so } 2T_2 = T_1.$$

For each of the masses, for $\sum F_y = ma_y$ we get

$$\text{mass } m_1: \quad m_1 g - T_1 = m_1 a_1,$$

$$\text{mass } m_2: \quad m_2 g - T_2 = m_2 a_2 = m_2 (a_r - a_1),$$

$$\text{mass } m_3: \quad m_3 g - T_2 = m_3 a_3 = m_3 (-a_r - a_1).$$

We have four equations for the four unknowns:

$$T_1, T_2, a_r, \text{ and } a_1.$$

After some careful algebra, we get

$$a_1 = [(m_1 m_2 + m_1 m_3 - 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g;$$

$$a_r = [2m_1(m_2 - m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g;$$

$$T_1 = [8m_1 m_2 m_3 / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g; \text{ and}$$

$$T_2 = [4m_1 m_2 m_3 / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g.$$

We can now find the other accelerations:

$$a_2 = [(m_1 m_2 - 3m_1 m_3 + 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g;$$

$$a_3 = [(-3m_1 m_2 + m_1 m_3 + 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g.$$

If $m_2 = m_3 \neq m_1$:

$$a_r = 0;$$

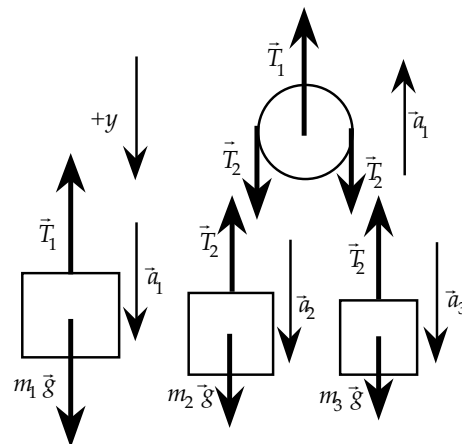
$$a_1 = [(m_1 - 2m_2) / (m_1 + 2m_2)]g$$

$$T_1 = [4m_1 m_2 / (m_1 + 2m_2)]g;$$

$$T_2 = [2m_1 m_2 / (m_1 + 2m_2)]g;$$

$$a_2 = a_3 = -a_1 = [(2m_2 - m_1) / (m_1 + 2m_2)]g.$$

Thus m_2 and m_3 have the same acceleration as the pulley. Note that neither tension is equal to an mg !



18. From the results for Problem 17, we have for the common denominator

$$(m_1 m_2 + m_1 m_3 + 4m_2 m_3) = (2.00)(1.20) + (2.00)(0.800) + 4(1.20)(0.800) = 7.84.$$

Thus for the accelerations (with down positive) and tensions we get

$$(a) \quad a_1 = [(m_1 m_2 + m_1 m_3 - 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g \\ = \{[(2.00)(1.20) + (2.00)(0.800) - 4(1.20)(0.800)] / 7.84\}(9.8 \text{ m/s}^2) = \boxed{+0.20 \text{ m/s}^2};$$

$$a_r = [2m_1(m_2 - m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g \\ = \{[2(2.00)(1.20 - 0.800)] / 7.84\}(9.8 \text{ m/s}^2) = \boxed{-0.78 \text{ m/s}^2};$$

$$a_2 = [(m_1 m_2 - 3m_1 m_3 + 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g \\ = \{[(2.00)(1.20) - 3(2.00)(0.800) + 4(1.20)(0.800)] / 7.84\}(9.8 \text{ m/s}^2) = \boxed{1.80 \text{ m/s}^2};$$

$$a_3 = [(-3m_1 m_2 + m_1 m_3 + 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g \\ = \{[-3(2.00)(1.20) + (2.00)(0.800) + 4(1.20)(0.800)] / 7.84\}(9.8 \text{ m/s}^2) = \boxed{-2.20 \text{ m/s}^2}.$$

$$(b) \quad T_1 = [8m_1 m_2 m_3 / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g = [8(2.00)(1.20)(0.800) / 7.84](9.8 \text{ m/s}^2) = \boxed{19.2 \text{ N}};$$

$$T_2 = \frac{1}{2}T_1 = \boxed{9.6 \text{ N}}.$$

19. There are only two tensions. We will label the tension in the string connecting m_1 and m_3 as T_1 and the tension in the string supporting m_2 as T_2 . The forces on each mass and the central pulley are shown. We choose up positive and assume that each acceleration is up.

If the mass of the pulley is negligible, we can write

$$\sum F_y = ma_y; \quad 2T_1 - T_2 = 0, \text{ which gives } T_2 = 2T_1.$$

For each mass we can write $\sum F_y = ma_y$:

$$\text{mass } m_1: \quad T_1 - m_1g = m_1a_1;$$

$$\text{mass } m_2: \quad 2T_1 - m_2g = m_2a_2;$$

$$\text{mass } m_3: \quad T_1 - m_3g = m_3a_3.$$

It appears that we have only three equations for four unknowns: T_1 , a_1 , a_2 and a_3 ; however, the accelerations are related because the length of the string is constant. If we call the various segments L_1 , L_2 , and L_3 , we have

$$L_1 + 2L_2 + L_3 = \text{a constant. By differentiating this twice, we get } a_1 + 2a_2 + a_3 = 0.$$

The three force equations can be written as

$$T_1/m_1 - g = a_1;$$

$$4T_1/m_2 - 2g = 2a_2;$$

$$T_1/m_3 - g = a_3.$$

From the relation between the accelerations, the sum of these three equations must be 0, from which we get

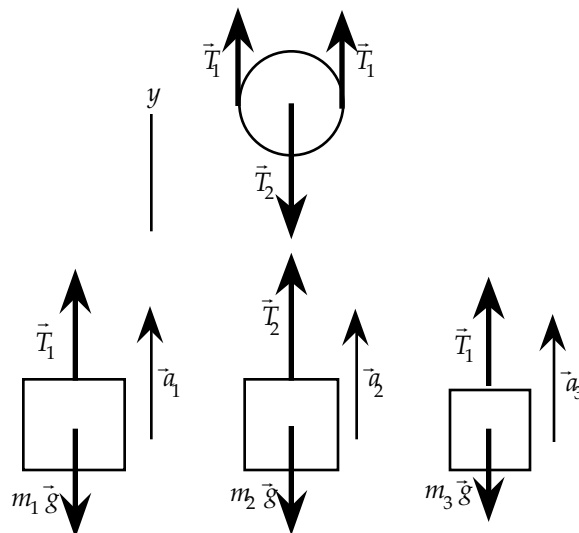
$$\begin{aligned} T_1 &= 4g(m_1m_2m_3)/(m_2m_3 + 4m_1m_3 + m_1m_2) \\ &= 4(9.8 \text{ m/s}^2)(4.00 \text{ kg})(10.00 \text{ kg})(6.00 \text{ kg}) / [(10.00 \text{ kg})(6.00 \text{ kg}) + 4(4.00 \text{ kg})(6.00 \text{ kg}) + (4.00 \text{ kg})(10.00 \text{ kg})] \\ &= \boxed{48.0 \text{ N}} \quad \text{and thus} \quad T_2 = \boxed{96.0 \text{ N}}. \end{aligned}$$

When we substitute the value of T_1 into the force equations, we get

$$a_1 = T_1/m_1 - g = \boxed{2.2 \text{ m/s}^2 \text{ (up)}};$$

$$a_2 = 2T_1/m_2 - g = \boxed{-0.2 \text{ m/s}^2 \text{ (down)}};$$

$$a_3 = -(a_1 + 2a_2) = \boxed{-1.8 \text{ m/s}^2 \text{ (down)}}.$$



20. The acceleration can be found from the boy's one-dimensional motion:

$$v = v_0 + at; \quad 0 = 5.2 \text{ m/s} + a(4.7 \text{ s}), \text{ which gives}$$

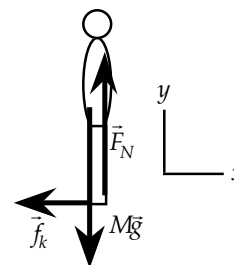
$$a = -1.1 \text{ m/s}^2.$$

Using the force diagram for the boy, we can write $\sum \vec{F} = m\vec{a}$:

$$x\text{-component: } -\mu_k F_N = Ma;$$

$$y\text{-component: } F_N - Mg = 0.$$

$$\text{Thus } F_N = Mg \text{ and } \mu_k = -a/g = -(-1.1 \text{ m/s}^2)/(9.8 \text{ m/s}^2) = \boxed{0.11}.$$



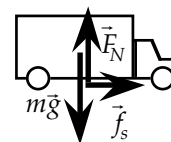
21. We assume that all tires are the same, so we can use one normal force.

The static friction force provides the acceleration. We can write $\sum \vec{F} = m\vec{a}$:

$$x\text{-component (horizontal): } f_s, \text{ max} = \mu_s F_N = ma;$$

$$y\text{-component (vertical): } F_N - mg = 0.$$

$$\text{Thus } F_N = mg \text{ and } \mu_s = a/g = (1.2 \text{ m/s}^2)/(9.8 \text{ m/s}^2) = \boxed{0.12}.$$



22. If the crate is about to slide, the static friction force on the crate will be maximum, $f_2 = \mu_{s2}F_{N2}$. The static friction force on the worker must not exceed its maximum value, $f_1 \leq \mu_{s1}F_{N1}$.

Using the force diagram for the crate, we can write $\Sigma \vec{F} = m\vec{a}$:

$$y\text{-component: } F_{N2} - m_2g = 0, \text{ or } F_{N2} = m_2g.$$

Using the force diagram for the worker, we can write $\Sigma \vec{F} = m\vec{a}$:

$$y\text{-component: } F_{N1} - m_1g = 0, \text{ or } F_{N1} = m_1g.$$

For the worker and crate, we can write $\Sigma \vec{F} = m\vec{a}$:

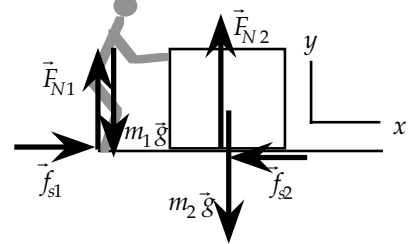
$$x\text{-component: } f_1 - f_2 = 0, \text{ or}$$

$$f_1 = f_2 = \mu_{s2}F_{N2} = (0.43)(140 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{5.9 \times 10^2 \text{ N}}.$$

We must see if this exceeds the maximum friction force on the worker:

$$f_{1,\text{max}} = \mu_{s1}F_{N1} = (0.81)(80 \text{ kg})(9.8 \text{ m/s}^2) = 6.4 \times 10^2 \text{ N}.$$

Because this is greater than f_1 , our result is valid.



23. We assume that the static friction force is the maximum, without skidding. The acceleration can be found from the car's one-dimensional motion:

$$v = v_0 + at; \quad 0 = 25 \text{ m/s} + a(4.2 \text{ s}), \text{ which gives}$$

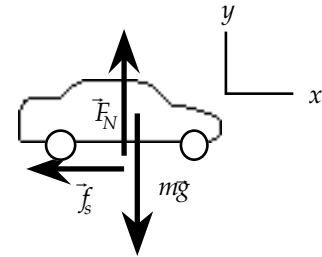
$$a = -5.6 \text{ m/s}^2.$$

We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for the car:

$$x\text{-component: } -f_s = ma$$

$$y\text{-component: } F_N - mg = 0.$$

Thus $F_N = Mg$ and $\mu_s = -a/g = -(-5.6 \text{ m/s}^2)/(9.8 \text{ m/s}^2) = \boxed{0.61}$.



24. (a) With the block as the system, there is no acceleration.

Using the force diagram, we can write $\Sigma \vec{F} = m\vec{a}$:

$$x\text{-component: } T_1 - mg \sin \theta - f_{k1} = 0;$$

$$y\text{-component: } F_{N1} - mg \cos \theta = 0.$$

$$\text{Thus } T_1 = mg \sin \theta + \mu_k mg \cos \theta$$

$$= mg(\sin \theta + \mu_k \cos \theta)$$

$$= (25 \text{ kg})(9.8 \text{ m/s}^2)(\sin 25^\circ + 0.4 \cos 25^\circ) = \boxed{1.9 \times 10^2 \text{ N}}.$$

- (b) We have a new force diagram, so $\Sigma \vec{F} = m\vec{a}$ becomes

$$x\text{-component: } T_2 \cos(\theta - \phi) - mg \sin \theta - \mu_k F_{N2} = 0;$$

$$y\text{-component: } T_2 \sin(\theta - \phi) + F_{N2} - mg \cos \theta = 0.$$

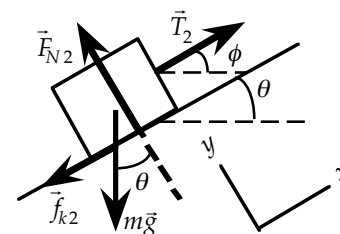
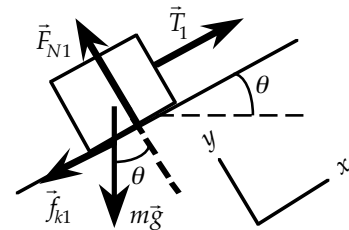
If we substitute the data, we get

$$T_2 \cos(40^\circ - 25^\circ) - (25 \text{ kg})(9.8 \text{ m/s}^2) \sin 25^\circ - (0.4)F_{N2} = 0;$$

$$T_2 \sin(40^\circ - 25^\circ) + F_{N2} - (25 \text{ kg})(9.8 \text{ m/s}^2) \cos 25^\circ = 0.$$

When these two equations are solved for the two unknowns, we get

$$F_{N2} = 1.8 \times 10^2 \text{ N} \quad \text{and} \quad T_2 = \boxed{1.8 \times 10^2 \text{ N}}.$$



25. With a minimum force, the acceleration of the crate will be zero.

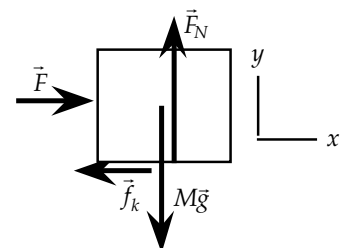
Using the force diagram for the crate, we can write $\Sigma \vec{F} = m\vec{a}$:

$$x\text{-component: } F_{\min} - f_k = 0;$$

$$y\text{-component: } F_N - Mg = 0.$$

Thus $F_N = Mg$ and

$$F_{\min} = f_k = \mu_k F_N = \mu_k Mg = (0.63)(58 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{3.6 \times 10^2 \text{ N}}.$$



26. The kinetic friction force will be down the roof to oppose the motion.

(a) From the force diagram for the package; $\sum \vec{F} = m\vec{a}$:

$$x\text{-component: } F - Mg \sin \theta - f_k = Ma$$

$$y\text{-component: } F_N - Mg \cos \theta = 0.$$

Thus $F_N = Mg \cos \theta$ and

$$\begin{aligned} F &= Mg \sin \theta + \mu_k F_N + Ma \\ &= (15 \text{ kg})[(9.8 \text{ m/s}^2) \sin 27^\circ + \\ &\quad (0.55)(9.8 \text{ m/s}^2) \cos 27^\circ + (0.15 \text{ m/s}^2)] \\ &= \boxed{1.4 \times 10^2 \text{ N}}. \end{aligned}$$

(b) We will assume that the package remains stationary on the roof.

Static friction will be up,

to oppose the impending motion down the roof. We can find its

maximum possible value from

part (a): $F_N = Mg \cos \theta$, so

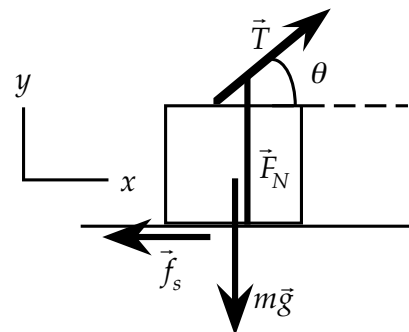
$$f_{s,\max} = \mu_s F_N = (0.58)(15 \text{ kg})(9.8 \text{ m/s}^2) \cos 27^\circ = 76 \text{ N}.$$

Because we have assumed no acceleration, we can find the required friction force from

$$x\text{-component: } f_s - Mg \sin \theta = 0;$$

$$f_s = Mg \sin \theta = (15 \text{ kg})(9.8 \text{ m/s}^2) \sin 27^\circ = 67 \text{ N}.$$

Since $f_s < f_{s,\max}$ the package will stay on the roof.



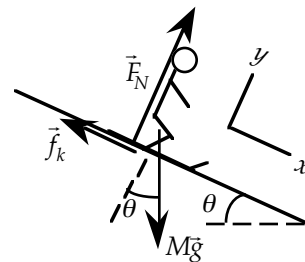
27. We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the skier, with $\vec{a} = 0$:

$$x\text{-component: } Mg \sin \theta - f_k = 0;$$

$$y\text{-component: } F_N - Mg \cos \theta = 0.$$

Thus $Mg \sin \theta = f_k = \mu_k F_N = \mu_k Mg \cos \theta$, which gives

$$\mu_k = (\sin \theta) / (\cos \theta) = \tan \theta = \tan 22^\circ = \boxed{0.40}.$$



28. Static friction must be backward, to oppose the impending slide along the truck bed and provide the deceleration of the crate.

We can find the acceleration from the motion of the truck:

$$v^2 = v_0^2 + 2a(x - x_0);$$

$$0 = (26.7 \text{ m/s})^2 + 2a(140 \text{ m} - 0), \text{ which gives}$$

$$a = -2.6 \text{ m/s}^2.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the crate:

$$x\text{-component: } -f_s = ma;$$

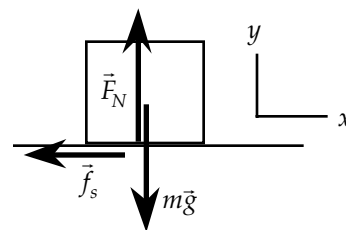
$$y\text{-component: } F_N - Mg = 0.$$

Thus $f_s = -ma = -(250 \text{ kg})(-2.6 \text{ m/s}^2) = 650 \text{ N}$ and

$$F_N = Mg = (250 \text{ kg})(9.8 \text{ m/s}^2) = 2450 \text{ N}.$$

Because $f_s \leq f_{s,\max} = \mu_s F_N$, the minimum μ required is

$$\mu_{s,\min} = f_s / F_N = (650 \text{ N}) / (2450 \text{ N}) = \boxed{0.27}.$$



29. For the shortest distance, maximum acceleration is required, thus maximum static friction:

$$f_s = f_{s,\max} = \mu_s F_N.$$

(a) On a horizontal road, $F_N = Mg$, and the static friction force provides the acceleration: $f_s = Ma$.

$$\text{Thus } a = \mu_s Mg / M = \mu_s g = (0.7)(9.8 \text{ m/s}^2) = 6.9 \text{ m/s}^2.$$

For the motion we can write

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$[(96 \text{ km/h}) / (3600 \text{ s/h})]^2 = 0 + 2(6.9 \text{ m/s}^2)(x - 0), \text{ which gives } x = \boxed{52 \text{ m}} \quad (171 \text{ ft})$$

(b) To find the time we use $v = v_0 + at$;

We have ignored rolling friction and drag; actual distance and time will be longer.

30. Until the box moves, friction is static, opposing the impending motion.

$$(96 \text{ km/h}) / (3600 \text{ s/h}) = 0 + (6.9 \text{ m/s}^2)t, \text{ which gives } t = \boxed{3.9 \text{ s}}.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the mass:

$$x\text{-component: } T \cos \theta - f_s = 0;$$

$$y\text{-component: } T \sin \theta + F_N - mg = 0.$$

When the box is on the verge of moving, the friction force is maximum, so we have

$$f_s = f_{s,\max} = \mu_s F_N.$$

When this is put into the x -component equation, we can solve for T to get

$$\begin{aligned} T &= \mu_s mg / (\cos \theta + \mu_s \sin \theta) \\ &= 0.40(2.0 \text{ kg})(9.8 \text{ m/s}^2) / (\cos 50^\circ + 0.40 \sin 50^\circ) = \boxed{8.3 \text{ N}}. \end{aligned}$$

31. (a) For the motion of the box we can write

$$x = x_0 + v_0 t + \frac{1}{2} a t^2;$$

$$0.8 \text{ m} = 0 + 0 + \frac{1}{2}(0.5 \text{ m/s}^2)t^2, \text{ which gives}$$

$$t = \boxed{1.8 \text{ s}}.$$

- (b) We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the box:

$$x\text{-component: } mg \sin \theta - f_k = ma, \text{ which gives}$$

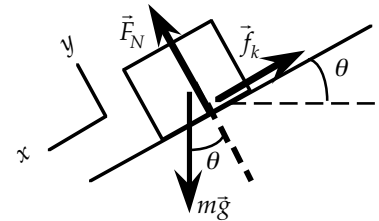
$$\begin{aligned} f_k &= mg \sin \theta - ma \\ &= (0.500 \text{ kg})(9.8 \text{ m/s}^2) \sin 35^\circ - (0.500 \text{ kg})(0.5 \text{ m/s}^2) \\ &= \boxed{2.6 \text{ N}}, \text{ up the incline.} \end{aligned}$$

- (c) From the force diagram for the box, $\sum \vec{F} = m\vec{a}$:

$$y\text{-component: } F_N - Mg \cos \theta = 0, \text{ which gives}$$

$$F_N = Mg \cos \theta = (0.500 \text{ kg})(9.8 \text{ m/s}^2) \cos 35^\circ = 4.0 \text{ N}.$$

$$\text{Thus } \mu_k = f_k / F_N = 2.6 \text{ N} / 4.0 \text{ N} = \boxed{0.65}.$$



32. The two blocks must have the same acceleration.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the top block:

$$x\text{-component: } F - f = m_1 a;$$

$$y\text{-component: } F_{N1} - m_1 g = 0.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the bottom block:

$$x\text{-component: } f = m_2 a;$$

$$y\text{-component: } F_{N2} - F_{N1} - m_2 g = 0.$$

From the y -equations, we get

$$F_{N1} = m_1 g \quad \text{and} \quad F_{N2} = (m_1 + m_2)g.$$

If we eliminate a from the two x -equations, we get:

$$F = (m_1 + m_2)f / m_2.$$

This means that F will be F_{\max} when $f = f_{\max} = \mu_s F_{N1} = \mu_s m_1 g$.

$$\text{Thus } F_{\max} = (0.7 \text{ kg} + 0.9 \text{ kg})[0.45(0.7 \text{ kg})(9.8 \text{ m/s}^2)] / (0.9 \text{ kg}) = \boxed{5.5 \text{ N}}.$$

If the force F is applied to the lower block, the x -equations will change:

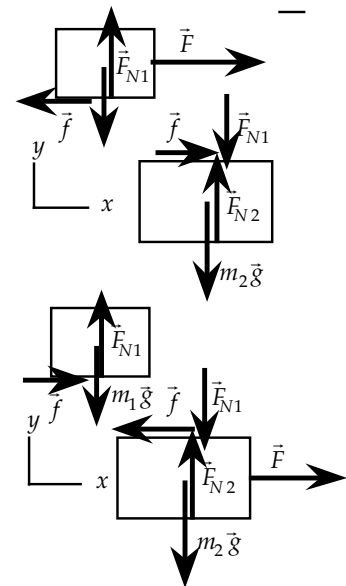
$$\text{top block: } f = m_1 a;$$

$$\text{bottom block: } F - f = m_2 a.$$

If we eliminate a from these equations, we get:

$$F = (m_1 + m_2)f / m_1, \text{ so}$$

$$F_{\max} = (0.7 \text{ kg} + 0.9 \text{ kg})[0.45(0.7 \text{ kg})(9.8 \text{ m/s}^2)] / (0.7 \text{ kg}) = \boxed{7.1 \text{ N}}.$$



33. Until the box moves, friction is static, opposing the impending motion.

(a) We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the box:

$$x\text{-component: } F \cos \theta - f_s = 0;$$

$$y\text{-component: } F \sin \theta + F_N - mg = 0.$$

When the box is on the verge of moving, we have

$$f_s = f_{s,\max} = \mu_s F_N.$$

When this is put into the x -component equation, we can solve for F to get

$$\begin{aligned} F &= \mu_s mg / (\cos \theta + \mu_s \sin \theta) \\ &= 0.75(50 \text{ kg})(9.8 \text{ m/s}^2) / (\cos \theta + 0.75 \sin \theta) \\ &= \boxed{370 / (\cos \theta + 0.75 \sin \theta) \text{ N}}. \end{aligned}$$

(b) To find the angle at which F is a minimum, we set $dF/d\theta = 0$:

$$dF/d\theta = \mu_s mg (-1)(-\sin \theta + \mu_s \cos \theta) / (\cos \theta + \mu_s \sin \theta)^2 = 0.$$

This is true when $\sin \theta_{\min} = \mu_s \cos \theta_{\min}$, or

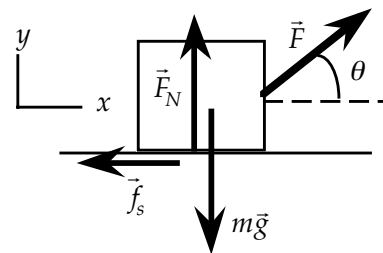
$$\tan \theta_{\min} = \mu_s = 0.75; \text{ which gives}$$

$$\theta_{\min} = \boxed{37^\circ}.$$

The force F is then

$$\begin{aligned} F_{\min} &= \mu_s mg / [\cos(\tan^{-1} \mu_s) + \mu_s \sin(\tan^{-1} \mu_s)] \\ &= (370 \text{ N}) / [0.80 + 0.75(0.60)] \\ &= \boxed{2.9 \times 10^2 \text{ N}}. \end{aligned}$$

A minimum value occurs because at small angles the normal force, F_N , and thus the friction force, f_s , are large, which requires a large force F . At angles near 90° , F_N is small, creating a small friction force; however, the small horizontal component of F requires a large magnitude of F . There is an angle where the decrease in F_N is balanced by the decrease in horizontal component, and thus a minimum value of F .



34. (a) We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the snow while it is on the roof:

$$x\text{-component: } mg \sin \theta - \mu_k F_N = ma;$$

$$y\text{-component: } F_N - mg \cos \theta = 0.$$

Thus $a = g(\sin \theta - \mu_k \cos \theta)$

$$= (9.8 \text{ m/s}^2)[\sin 40^\circ - (0.1) \cos 40^\circ] = 5.55 \text{ m/s}^2.$$

For the motion on the roof, we have

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) = 0 + 2(5.55 \text{ m/s}^2)(8 \text{ m}),$$

which gives

$$v_1 = \boxed{9.4 \text{ m/s}}.$$

- (b) The motion when the snow leaves the roof is projectile motion, with an initial velocity of $v_1 = 9.4 \text{ m/s}$ at 40° below the horizontal.

If we use the new coordinate system shown, we can write

$$y = y_0 + (v_1 \sin 40^\circ)t + \frac{1}{2}(-g)t^2;$$

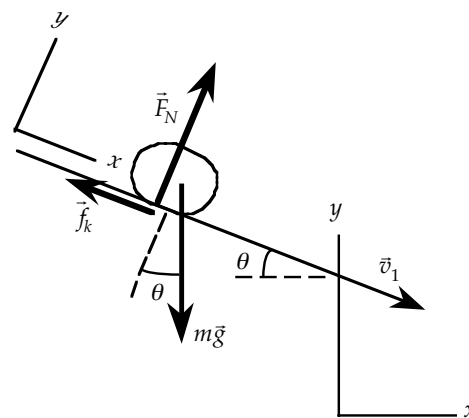
$$0 = 6 \text{ m} - [(9.4 \text{ m/s}) \sin 40^\circ]t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2, \text{ which}$$

gives

$$t = 0.65 \text{ s (keeping the positive value).}$$

For the horizontal motion, we can write

$$\begin{aligned} x &= (v_1 \cos 40^\circ)t = (9.4 \text{ m/s})(\cos 40^\circ)(0.65 \text{ s}) \\ &= \boxed{4.7 \text{ m}}. \end{aligned}$$



35. Because the professor does not slip, from the force diagram we have

$$\begin{aligned}\sum \vec{F} &= \vec{F} + \vec{F}_N + m\vec{g} + \vec{f} = 0; \\ -F \cos \theta \hat{i} - F \sin \theta \hat{j} + F_N \hat{j} - mg \hat{j} + f \hat{i} &= 0.\end{aligned}$$

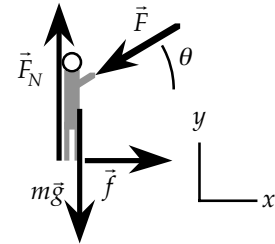
From the two components, we get

$$f = F \cos \theta \quad \text{and} \quad F_N = mg + F \sin \theta.$$

Slipping will not occur if $f \leq f_{s,\max} = \mu_s F_N$.

For a fixed value of f , there will be a minimum μ_s when $f = \mu_{s,\min} F_N$:

$$\mu_{s,\min} = f/F_N = F \cos \theta / (mg + F \sin \theta) = (50 \text{ N}) \cos 30^\circ / [(55 \text{ kg})(9.8 \text{ m/s}^2) + (50 \text{ N}) \sin 30^\circ] = \boxed{0.077}.$$



36. (a) We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the top mass:

$$x\text{-component: } 0 = m_1 a_1;$$

$$y\text{-component: } F_{N1} - m_1 g = 0.$$

We write $\sum \vec{F} = m\vec{a}$ from the diagram for the bottom mass:

$$x\text{-component: } F = m_2 a_2;$$

$$y\text{-component: } F_{N2} - F_{N1} - m_2 g = 0.$$

Thus we have $F_{N1} = m_1 g$; $F_{N2} = F_{N1} + m_2 g$; and

$$\boxed{a_1 = 0; \quad a_2 = F/m_2}.$$

- (b) We write $\sum \vec{F} = m\vec{a}$ for the combined masses:

$$x\text{-component: } F = (m_1 + m_2)a;$$

$$\text{Thus } a = \boxed{a_1 = a_2 = F/(m_1 + m_2)}.$$

- (c) Assuming the top mass is sliding to the left, we have

$$\vec{F}_{N1} = m_1 g \hat{j} \quad (\text{up}) \quad \text{and} \quad \vec{f}_1 = \mu_k m_1 g \hat{i} \quad (\text{right}), \text{ so}$$

$$\boxed{\vec{F}(\text{lower on upper}) = (\mu_k m_1 g) \hat{i} + (m_1 g) \hat{j}}.$$

- (d) The y -component equations are the same as in part (a).

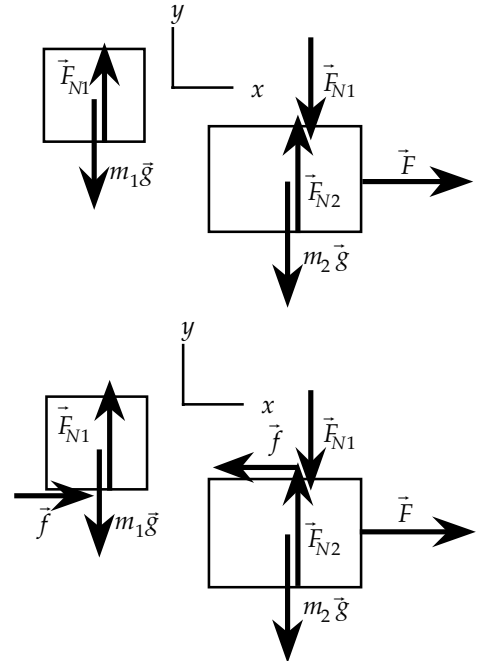
We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the top mass:

$$x\text{-component: } \mu_k m_1 g = m_1 a_1.$$

We write $\sum \vec{F} = m\vec{a}$ from the diagram for the bottom mass:

$$x\text{-component: } F - \mu_k m_1 g = m_2 a_2.$$

$$\text{Thus } \boxed{a_1 = \mu_k g} \quad \text{and} \quad \boxed{a_2 = (F - \mu_k m_1 g)/m_2}.$$



37. We convert the speed: $(65 \text{ mi/h})(1.61 \times 10^3 \text{ m/mi})/(3.6 \times 10^3 \text{ s/h}) = 29 \text{ m/s}$.

The drag force is

$$F_D = \frac{1}{2} \rho A C_D v^2 = \frac{1}{2} (1.25 \text{ kg/m}^3) (4 \text{ m}^2) (0.45) (29 \text{ m/s})^2 = \boxed{9.5 \times 10^2 \text{ N}}.$$

38. At terminal velocity, $a = 0$ and the weight is balanced by the drag force:

$$F_d = mg = kv^2, \text{ so}$$

$$k = mg/v^2 = 116 \text{ kg}(9.8 \text{ m/s}^2)/(4.9 \text{ m/s})^2 = 47 \text{ N} \cdot \text{s}^2/\text{m}^2 = \boxed{47 \text{ kg/m}}.$$

39. Because $F_D = \frac{1}{2} \rho A C_D v^2$, at terminal speed we have $mg = \frac{1}{2} \rho A C_D v^2$, so

$$A = 2mg/\rho C_D v^2 = 2(0.50 \text{ kg})(9.8 \text{ m/s}^2)/(1.25 \text{ kg/m}^3)(0.4)(18 \text{ m/s})^2 = 0.060 \text{ m}^2 = \boxed{600 \text{ cm}^2}.$$

40. The maximum acceleration is achieved when there is no drag (low speed).

$$\text{Thus } F_{\text{engine}} = ma = (800 \text{ kg})(4.8 \text{ m/s}^2) = 3.8 \times 10^3 \text{ N}.$$

At a top speed of $v_{\max} = 90 \text{ m/s}$, the acceleration is zero and

$$F_D = \frac{1}{2} \rho A C_D v^2 = F_{\text{engine}}, \text{ so}$$

$$C_D = 2F_{\text{engine}}/\rho A v^2 = 2(3.8 \times 10^3 \text{ N})/(1.25 \text{ kg/m}^3)(1.8 \text{ m}^2)(90 \text{ m/s})^2 = \boxed{0.42}.$$

41. At constant speed,
- $a = 0$
- .

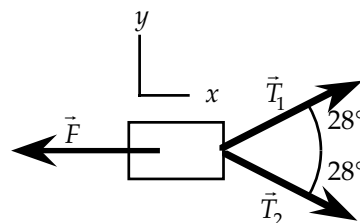
We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the barge:

$$x\text{-component: } T_1 \cos 32^\circ + T_2 \cos 32^\circ - F = 0;$$

$$y\text{-component: } T_1 \sin 32^\circ - T_2 \sin 32^\circ = 0.$$

$$\text{Thus } T_1 = T_2 \quad \text{and} \quad 2(74 \text{ N})(\cos 32^\circ) = (220 \text{ N} \cdot \text{s/m})v,$$

$$\text{which gives } v = \boxed{0.57 \text{ m/s}}.$$



42. (a) Because
- $F_D = \frac{1}{2}\rho AC_D v^2$
- at terminal speed we have
- $mg = \frac{1}{2}\rho AC_D v^2$
- , so

$$\rho_s \frac{4}{3}\pi r_1^3 g = \frac{1}{2}\rho(4\pi r_1^2)C_D v_1^2 \quad \text{for the first sphere and}$$

$$\rho_s \frac{4}{3}\pi r_2^3 g = \frac{1}{2}\rho(4\pi r_2^2)C_D v_2^2 \quad \text{for the second sphere.}$$

Thus, by dividing we get

$$(v_2^2/v_1^2)(r_2^2/r_1^2) = r_2^3/r_1^3, \quad \text{or} \quad (v_2/v_1)^2 = r_2/r_1 = 2, \text{ so}$$

$$\boxed{v_2 = \sqrt{2} v_1}.$$

- (b) For a radius of
- $r_z = z r_1$
- , we get

$$(v_z/v_1)^2 = r_z/r_1 = z, \text{ so } \boxed{v_z = \sqrt{z} v_1}.$$

43. Because
- $F_D = \frac{1}{2}\rho AC_D v^2$
- , at terminal speed we have
- $mg = \frac{1}{2}\rho AC_D v^2$
- . Because the mass of the skydiver does not change, and the maximum area corresponds to the minimum speed, we can write

$$mg = \frac{1}{2}\rho A_{\max} C_{D\min} v_{\min}^2 = \frac{1}{2}\rho A_{\min} C_{D\max} v_{\max}^2. \text{ Thus}$$

$$C_{D\max}/C_{D\min} = (A_{\max}/A_{\min})(v_{\min}/v_{\max})^2 = (1.5)[(40 \text{ m/s})/(60 \text{ m/s})]^2 = \boxed{0.67}.$$

44. Because
- $F_D = \frac{1}{2}\rho AC_D v^2$
- , at terminal speed we have
- $mg = \frac{1}{2}\rho AC_D v^2$
- , so

$$C_D = 2mg/\rho A v^2$$

$$= 2(90 \text{ kg})(9.8 \text{ m/s}^2)/(1.25 \text{ kg/m}^3)(30 \text{ m}^2)(6 \text{ m/s})^2 = \boxed{1.3}.$$

45. Similar to the previous problem, set
- $mg = \frac{1}{2}\rho AC_D v^2$
- , which gives
- $v = \text{constant} \times m^{1/2}$
- . With
- $m_1 = 90 \text{ kg}$
- ,
- $v_1 = 6 \text{ m/s}$
- , and
- $m_2 = 60 \text{ kg}$
- , the terminal speed
- v_2
- of the 60-kg person must satisfy

$$v_1/v_2 = (m_1/m_2)^{1/2}, \text{ which gives}$$

$$v_2 = (m_2/m_1)^{1/2} v_1 = (60 \text{ kg}/90 \text{ kg})^{1/2} (6 \text{ m/s}) = \boxed{5 \text{ m/s}}.$$

46. From
- $F_d = -Av$
- , we find the dimensions of
- A
- :

$$[A] = [F]/[v] = [MLT^{-2}]/[LT^{-1}] = [MT^{-1}].$$

We assume that the time will be a function of A , g and m , so we write $t = m^\alpha g^\beta A^\gamma$.

In terms of dimensions, this gives

$$[t] = [m]^\alpha [g]^\beta [A]^\gamma, \quad \text{or} \quad [T] = [M]^\alpha [LT^{-2}]^\beta [MT^{-1}]^\gamma.$$

Equating the exponents of the various dimensions, we get $1 = -2\beta - \gamma$, $0 = \alpha + \gamma$, and $0 = \beta$.Thus $\beta = 0$, $\gamma = -1$, and $\alpha = +1$, so $\boxed{t \approx m/A}.$

47. The centripetal acceleration is

$$a_r = v^2/r = (200 \text{ m/s})^2/(30 \times 10^3 \text{ m}) = \boxed{1.3 \text{ m/s}^2 \text{ toward the center}}.$$

48. Because the rock has a centripetal acceleration provided by the tension, we can write:

$$F_r = T = ma_r = mv^2/r; \text{ thus the speed will be maximum when the tension is maximum:}$$

$$26 \text{ N} = (0.220 \text{ kg})v_{\max}^2/(0.35 \text{ m}), \text{ which gives } v_{\max} = \boxed{6.4 \text{ m/s}}.$$

49. Because the man has a centripetal acceleration, we can write:

$$F_r = ma_r = mr\omega^2$$

$$= (65 \text{ kg})(5.3 \text{ m})[(6 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min})]^2 = \boxed{1.4 \times 10^2 \text{ N toward the center}}.$$

50. The airplane has a centripetal acceleration:

$$a_r = v^2/r; \quad 0.30(9.8 \text{ m/s}^2) = [(650 \times 10^3 \text{ m/h})/(3600 \text{ s/h})]^2/r, \text{ which gives } r = 1.1 \times 10^4 \text{ m} = \boxed{11 \text{ km}}.$$

51. If the automobile does not skid, the friction is static, with $f_s \leq \mu_s F_N$. At high speed f_s will be down the incline. Note that we take a coordinate system with the x -axis in the direction of the centripetal acceleration. We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for the auto:

$$x\text{-component: } F_N \sin \theta + f_s \cos \theta = ma = mv^2/R;$$

$$y\text{-component: } F_N \cos \theta - f_s \sin \theta - mg = 0.$$

The speed is maximum when $f_s = f_{s,\max} = \mu_s N$.

From the y -equation we get

$$F_N \cos \theta - \mu_s F_N \sin \theta = mg, \text{ or } F_N = mg/(\cos \theta - \mu_s \sin \theta).$$

From the x -equation we get

$$v_{\max}^2/R = g(\sin \theta + \mu_s \cos \theta)/(\cos \theta - \mu_s \sin \theta).$$

$$v_{\max}^2 = (150 \text{ m})(9.8 \text{ m/s}^2)(\sin 18^\circ + 0.3 \cos 18^\circ)/(\cos 18^\circ - 0.3 \sin 18^\circ), \text{ which gives } v_{\max} = \boxed{32 \text{ m/s}}.$$

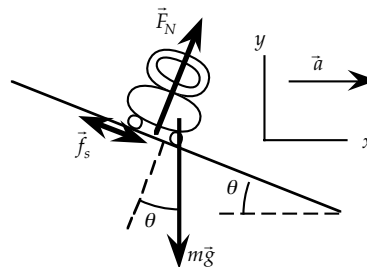
At low speed, the automobile will tend to slide down the incline, so f_s will be up the incline.

The speed is minimum when $f_s = f_{s,\max} = \mu_s F_N$.

If we change the sign of f_s in the equations, we get

$$F_N = mg/(\cos \theta + \mu_s \sin \theta) \text{ and } v_{\min}^2/R = mg(\sin \theta - \mu_s \cos \theta)/(\cos \theta + \mu_s \sin \theta). \text{ Thus}$$

$$v_{\min}^2 = (150 \text{ m})(9.8 \text{ m/s}^2)(\sin 18^\circ - 0.3 \cos 18^\circ)/(\cos 18^\circ + 0.3 \sin 18^\circ), \text{ which gives } v_{\min} = \boxed{5.8 \text{ m/s}}.$$



52. In the noninertial frame of the passenger there is a fictitious force $m\vec{r}\omega^2$ away from the center, and the passenger has no acceleration. Thus the force from the shoes will be along the body direction and balances a horizontal force of $m\vec{r}\omega^2$ and a vertical force of mg .

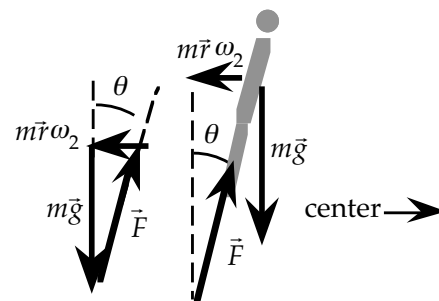
If θ is the angle from the vertical, we have

$$\tan \theta = m\vec{r}\omega^2/mg = r\omega^2/g, \text{ with } \omega = (1 \text{ rev})(2\pi \text{ rad/rev})/10 \text{ s}.$$

$$(a) \tan \theta_1 = (1 \text{ m})(\frac{1}{5}\pi \text{ rad/s})^2/(9.8 \text{ m/s}^2) = 0.040, \quad \theta_1 = \boxed{2.3^\circ}.$$

$$(b) \tan \theta_3 = (3 \text{ m})(\frac{1}{5}\pi \text{ rad/s})^2/(9.8 \text{ m/s}^2) = 0.121, \quad \theta_3 = \boxed{6.9^\circ}.$$

$$(c) \tan \theta_5 = (5 \text{ m})(\frac{1}{5}\pi \text{ rad/s})^2/(9.8 \text{ m/s}^2) = 0.201, \quad \theta_5 = \boxed{11.4^\circ}.$$



53. For the mass in circular motion $F = mR\omega^2$, where $F = (4.2 \text{ N/m})x$ and $R = 0.6 \text{ m} + x$. So $(4.2 \text{ N/m})x = m(0.6 \text{ m} + x)\omega^2$, and

$$x = (0.6 \text{ m})m\omega^2/(4.2 \text{ N/m} - m\omega^2)$$

$$= (0.6 \text{ m})(0.400 \text{ kg})(1.2 \text{ rad/s})^2/[4.2 \text{ N/m} - (0.400 \text{ kg})(1.2 \text{ rad/s})^2] = \boxed{0.095 \text{ m}} = 9.5 \text{ cm}.$$

54. $a = v^2/R$, so $v_1 = (a_1 R_1)^{1/2}$ for the first curve and $v_2 = (a_2 R_2)^{1/2}$ for the second one. The ratio is
- $$v_2/v_1 = (a_2 R_2/a_1 R_1)^{1/2} = \{(2.20 \text{ m/s}^2)(\frac{1}{2}R_1)/[(0.93 \text{ m/s}^2)R_1]\}^{1/2} = \boxed{1.1}.$$

55. While the stone does not slide on the turntable, the static friction force provides the centripetal acceleration:

$$f_s = ma_r = mR\omega^2.$$

Thus a larger friction force is needed at larger R . At the critical distance the friction force is maximum

$$f_s = f_{s,\max} = \mu_s mg, \text{ so we have}$$

$$\mu_s mg = mR\omega^2, \text{ or}$$

$$\mu_s = R\omega^2/g = (0.21 \text{ m})[(33 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min})]^2/(9.8 \text{ m/s}^2) = \boxed{0.26}.$$

56. The angular speed of the Ferris wheel is $\omega = 2\pi/75 \text{ rad/s}$.

At the top and bottom the forces are vertical, with the weight down and the normal force up.

- (a) At the bottom, the centripetal acceleration is up, so we take up as positive and from $\sum \vec{F} = m\vec{a}$

$$F_{NB} - mg = mR\omega^2, \text{ so}$$

$$F_{NB} = m(g + R\omega^2) = (60 \text{ kg})[9.8 \text{ m/s}^2 + (30 \text{ m})(2\pi/75 \text{ rad/s})^2] = \boxed{6.0 \times 10^2 \text{ N}}.$$

- (b) At the top, the centripetal acceleration is down, we take down as positive, and from $\sum \vec{F} = m\vec{a}$ we get

$$mg - F_{NT} = mR\omega^2, \text{ so}$$

$$F_{NT} = m(g - R\omega^2) = (60 \text{ kg})[9.8 \text{ m/s}^2 - (30 \text{ m})(2\pi/75 \text{ rad/s})^2] = \boxed{5.8 \times 10^2 \text{ N}}.$$

57. Because the acceleration of the Moon is centripetal, $a = v^2/R = R\omega^2$, we have

$$R = a/\omega^2 = (0.0027 \text{ m/s}^2)/[2\pi \text{ rad}/(28 \text{ d})(24 \text{ h/d})(3600 \text{ s/h})]^2 = 4.0 \times 10^8 \text{ m. Then}$$

$$v = \sqrt{aR} = \sqrt{(0.0027 \text{ m/s}^2)(4.0 \times 10^8 \text{ m})} = \boxed{1.0 \times 10^3 \text{ m/s tangent to the orbit}}.$$

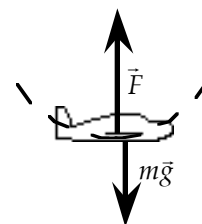
58. At the bottom of the dive the centripetal acceleration is up and has magnitude

$$a = v^2/R$$

$$= [(1500 \text{ km/h})(10^3 \text{ m/km})/(3.6 \times 10^3 \text{ s/h})]^2/(1.75 \times 10^3 \text{ m})$$

$$= 99.2 \text{ m/s}^2 = 10.1g.$$

Thus from $\sum \vec{F} = m\vec{a}$ we get $F - mg = ma$, which gives $F = m(a + g) = m(11.1g)$. So the pilot feels $\boxed{11.1g}$. The maneuver is **unsafe**, since this exceeds $11g$.



59. We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the hanging mass, with down positive:

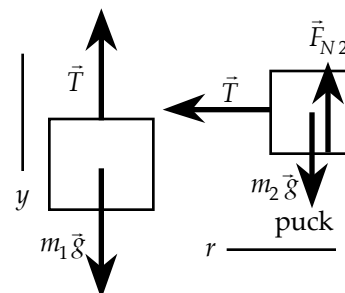
$$m_1g - T = m_1a_1 = 0; \text{ which gives}$$

$$T = m_1g = (1.00 \text{ kg})(9.8 \text{ m/s}^2) = 9.80 \text{ N.}$$

For the rotating puck, the tension provides the centripetal acceleration, $\sum F_r = ma_r$:

$$T = m_2v^2/r; \quad 9.80 \text{ N} = (0.400 \text{ kg})v^2/(0.80 \text{ m}), \text{ which gives}$$

$$v = \boxed{4.43 \text{ m/s}}.$$



60. (a) For the rotating mass, the tension provides the centripetal acceleration, $\sum F_r = ma_r$:

$$T = \boxed{mv^2/r}.$$

- (b) From the speed of the first mass, v , we find the angular speed: $\omega = v/r$.

The speed of the second mass is then

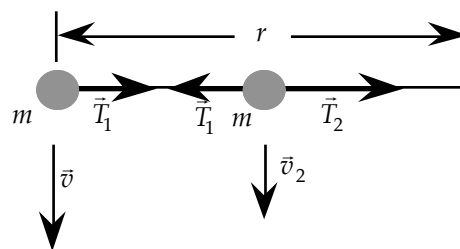
$$v_2 = (\frac{1}{2}r)\omega = (\frac{1}{2}r)(v/r) = \frac{1}{2}v.$$

For $\sum F_r = ma_r$ we have

$$\text{mass 1: } T_1 = \boxed{mv^2/r};$$

$$\text{mass 2: } T_2 - T_1 = mv_2^2/\frac{1}{2}r = mv^2/\frac{1}{2}r.$$

When we use the value for T_1 we get $T_2 = \boxed{\frac{3}{2}mv^2/r}.$



61. (b) The mass moves in a circle of radius $L \sin \theta$ and has a centripetal acceleration. We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the mass:

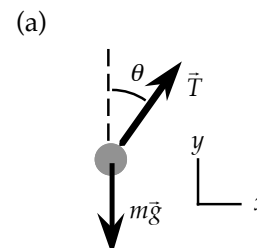
$$x\text{-component: } T \sin \theta = mv^2/r;$$

$$y\text{-component: } T \cos \theta - mg = 0.$$

Combining these, we get

$$v^2 = rg \tan \theta$$

$$= (0.5 \text{ m})(\sin 10^\circ)(9.8 \text{ m/s}^2)(\tan 10^\circ), \text{ which gives } v = \boxed{0.4 \text{ m/s}}$$



62. (a) **False**. A force can be applied in the direction of motion (along the unit vector) to cause the particle's speed to increase, or in the opposite direction to cause the speed to decrease.
- (b) **True**. The object is essentially in a fixed direction, so the direction of its velocity cannot change. If the magnitude of the velocity does not change either, then the velocity is unchanged and the acceleration is therefore zero, and so is the net force on the object.
- (c) **False**. We can only say that the net force is zero in the direction perpendicular to \hat{r} , but that does not mean there cannot be any forces in that direction, as long as they cancel with each other.

63. The bob is subject to three forces: T and mg . The net force is horizontal and points towards the center of its circular path. We write $\Sigma = m\vec{a}$ for the bob:

$$x\text{-component: } T \sin \theta = ma_x = mR\omega^2 = m(L \sin \theta) \omega^2;$$

$$y\text{-component: } T \cos \theta - mg = ma_y = 0.$$

Solve for L , the length of the string:

$$L = g / \omega^2 \cos \theta = (9.8 \text{ m/s}^2) / [(\frac{1}{2}\pi \text{ rad/s})^2 \cos 30^\circ] = \boxed{4.5 \text{ m}}.$$

64. The forces on the mass are mg and the normal force.

To find the direction of the normal force, we can find the slope of the bowl from $h = br^2$:

$$\tan \theta = dh/dr = 2br.$$

We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for the mass:

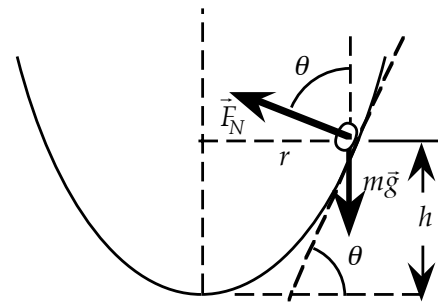
$$\text{radial-component: } F_N \sin \theta = mv^2/r;$$

$$\text{vertical-component: } F_N \cos \theta - mg = 0.$$

Combining these, we get

$$\tan \theta = v^2/gr = 2br. \text{ Thus}$$

$$r^2 = \frac{1}{2}v^2/bg \text{ and } h = br^2 = \boxed{\frac{1}{2}v^2/g}.$$



65. We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for the block:

$$x\text{-component: } F_N \cos \theta = mr\omega^2;$$

$$y\text{-component: } F_N \sin \theta - mg = 0.$$

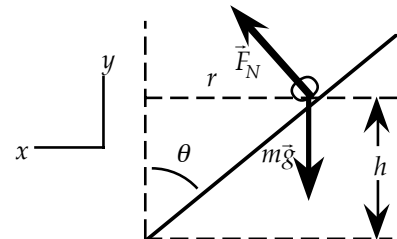
Combining these we get

$$r = g / (\omega^2 \tan \theta). \text{ Then}$$

$$h = r / \tan \theta = g / (\omega^2 \tan^2 \theta)$$

$$= (9.8 \text{ m/s}^2) / [(3.8 \text{ rad/s})^2 \tan^2 44^\circ]$$

$$= \boxed{0.72 \text{ m}}.$$



66. (a) We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for the mass:

$$x\text{-component: } T_1 \cos 30^\circ - T_2 \cos 30^\circ = mr\omega^2;$$

$$y\text{-component: } T_1 \sin 30^\circ - T_2 \sin 30^\circ - mg = 0.$$

From the y -equation we get:

$$(150 \text{ N}) \sin 30^\circ - T_2 \sin 30^\circ - (5.0 \text{ kg})(9.8 \text{ m/s}^2) = 0,$$

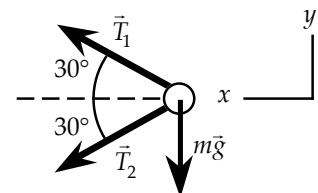
$$\text{which gives } T_2 = \boxed{52 \text{ N}}.$$

- (b) From the x -equation we get

$$(150 \text{ N}) \cos 30^\circ + (52 \text{ N}) \cos 30^\circ = (5.0 \text{ kg})(1.0 \text{ m}) \cos 30^\circ \omega^2;$$

$$\text{which gives } \omega = 6.36 \text{ rad/s}.$$

$$\text{The time for one circuit is then } t = 2\pi / \omega = \boxed{0.99 \text{ s}}.$$



67. If the automobile does not skid, the friction is static and down the incline, with $f_s \leq \mu_s F_N$. Note that we take a coordinate system with the x -axis in the direction of the centripetal acceleration.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the auto:

$$x\text{-component: } F_N \sin \theta + f_s \cos \theta = ma = mv^2/R;$$

$$y\text{-component: } F_N \cos \theta - f_s \sin \theta - mg = 0.$$

The speed is maximum when $f_s = f_{s,\max} = \mu_s F_N$.

From the y -equation we get

$$F_N \cos \theta - \mu_s F_N \sin \theta = mg, \text{ or } F_N = mg/(\cos \theta - \mu_s \sin \theta).$$

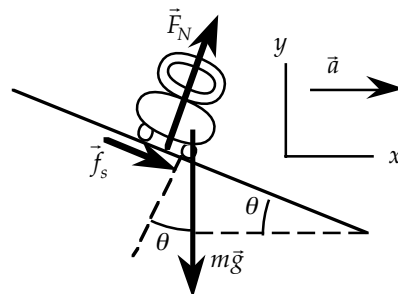
From the x -equation we get

$$v_{\max}^2/R = g(\sin \theta + \mu_s \cos \theta)/(\cos \theta - \mu_s \sin \theta).$$

$$v_{\max}^2 = [(9.8 \text{ m/s}^2)(\sin 5.1^\circ + 0.8 \cos 5.1^\circ)/(\cos 5.1^\circ - 0.8 \sin 5.1^\circ)](320 \text{ m}), \text{ which gives}$$

$$v_{\max} = \boxed{55 \text{ m/s}} \quad (\approx 120 \text{ mi/h}).$$

There are many slower speeds, because friction can be less than $\mu_s F_N$.



68. The speed is $(80 \text{ km/h})(10^3 \text{ m/km})/(3.6 \times 10^3 \text{ s/h}) = 22.2 \text{ m/s}$.

Because the tangential acceleration is small, we will assume that the tangential speed is approximately constant.

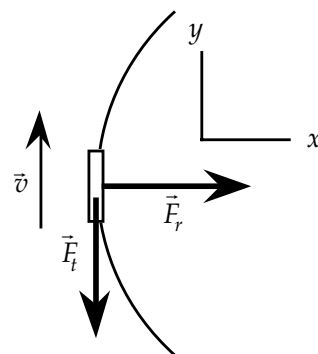
The train will have both a radial and a tangential acceleration:

$$F_r = mv^2/r = (1.5 \times 10^5 \text{ kg})(22.2 \text{ m/s})^2/(2 \times 10^3 \text{ m}) = 3.7 \times 10^4 \text{ N};$$

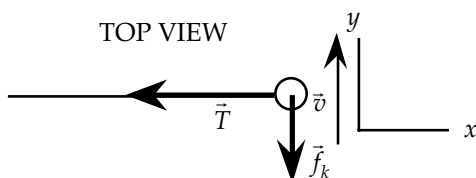
$$F_t = m(dv/dt) = (1.5 \times 10^5 \text{ kg})(-0.2 \text{ m/s}^2) = -3.0 \times 10^4 \text{ N}.$$

From the coordinate system on the diagram, the net force is

$$\vec{F}_{\text{net}} = \boxed{(3.7 \times 10^4 \hat{i} - 3.0 \times 10^4 \hat{j}) \text{ N}} \text{ and } |\vec{F}_{\text{net}}| = \boxed{4.8 \times 10^4 \text{ N}}.$$



- 69.



We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the puck:

$$x\text{-component: } -T = -mv^2/R;$$

$$y\text{-component: } -f_k = m(dv/dt);$$

$$z\text{-component: } F_N - mg = 0.$$

- (a) At $t = 0$ s:

$$T = mv_0^2/R = (0.1 \text{ kg})(8.0 \text{ m/s})^2/(0.3 \text{ m}) = \boxed{21 \text{ N}}.$$

- (b) Because $F_N = mg$, $f_k = \mu_k F_N = \mu_k mg$.

From the y -equation we can find the tangential acceleration:

$$-0.25(9.8 \text{ m/s}^2) = dv/dt, \text{ which gives } dv/dt = a_t = -2.45 \text{ m/s}^2.$$

For the tangential motion of one revolution we can write

$$v^2 = v_0^2 + 2a_t(C - 0) = (8.0 \text{ m/s})^2 + 2(-2.45 \text{ m/s}^2)[2\pi(0.3 \text{ m}) - 0], \text{ which gives the speed after}$$

one revolution: $v = 7.4 \text{ m/s}$. The tension after one revolution is

$$T = mv^2/R = (0.1 \text{ kg})(7.4 \text{ m/s})^2/(0.3 \text{ m}) = \boxed{18 \text{ N}}.$$

70. The coordinate system and angles are shown in the diagram. The two forces, gravity and tension, cause the circular motion of the pendulum of radius $r = R \sin \theta$.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the pendulum:

$$x\text{-component: } T \sin \phi - mg \sin \theta = -mr\omega^2;$$

$$y\text{-component: } T \cos \phi - mg \cos \theta = 0;$$

If we use $r = R \sin \theta$ and eliminate T , we find

$$\tan \phi = (g \sin \theta - R \sin \theta \omega^2) / (g \cos \theta), \text{ or}$$

$$\phi = \tan^{-1}[(1 - R\omega^2/g) \tan \theta].$$

The angle from the radial direction is

$$\alpha = \theta - \phi = \theta - \tan^{-1}[(1 - R\omega^2/g) \tan \theta].$$

Because $R\omega^2 = (6.4 \times 10^6 \text{ m})[2\pi/(24 \text{ h})(3600 \text{ s/h})]^2 = 0.034 \text{ m/s}^2$,

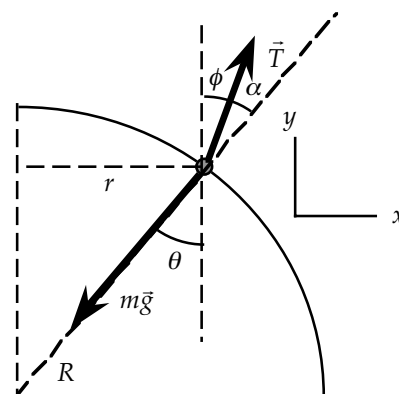
the angle α (and $\tan \alpha$) will be small. Then

$$\tan \alpha = \tan(\theta - \phi) = (\tan \theta - \tan \phi) / (1 + \tan \theta \tan \phi)$$

$$= [\tan \theta - (1 - R\omega^2/g) \tan \theta] / (1 + \tan \theta \tan \phi)$$

$$\approx (R\omega^2/g) \tan \theta / (1 + \tan^2 \theta) = (R\omega^2/g) \sin \theta \cos \theta.$$

Thus $\alpha \approx \tan \alpha \approx \boxed{(R\omega^2/g) \sin \theta \cos \theta}$.



71. Because gravity provides the centripetal acceleration, for $\sum F_r = ma_r$ we have

$$mg' = mv^2/r.$$

If we call the first orbit 1 and the orbit after one year 2, with $g' = 0.95g$ we can write

$$v_1 = (0.95gr_1)^{1/2} \text{ and } v_2 = (0.95gr_2)^{1/2}.$$

With h the height above the surface of the earth and R the radius of Earth, $r = R + h$, and $h \ll R$, so we can write

$$v_2/v_1 = (r_2/r_1)^{1/2} = [(R + h_2)/(R + h_1)]^{1/2}$$

$$\approx [(1 + h_2/R)(1 - h_1/R)]^{1/2}$$

$$\approx [1 + (h_2 - h_1)/R]^{1/2}$$

$$\approx 1 + \frac{1}{2}(h_2 - h_1)/R.$$

With the given data, we get

$$v_2/v_1 = 1 + (-5 \text{ km})/[2(6.37 \times 10^3 \text{ km})] = 1 - 3.92 \times 10^{-4}.$$

The change in the tangential speed is

$$v_2 - v_1 = v_1(1 - 3.92 \times 10^{-4} - 1) = 3.92 \times 10^{-4} v_1.$$

Because this change in tangential speed is caused by the drag force, for $\sum F_t = ma_t$ we write

$$\frac{1}{2}\rho AC_D v_1^2 = m(v_2 - v_1)/T, \text{ which gives}$$

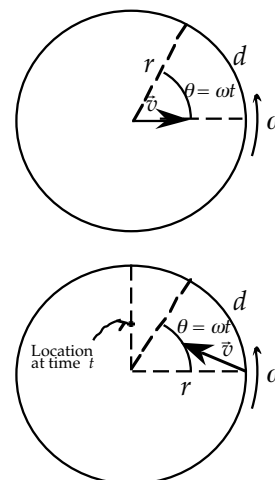
$$\rho = 2m(v_2 - v_1)/(TAC_D v_1^2)$$

$$= 2m(3.92 \times 10^{-4})/[TAC_D(0.95gr_1)^{1/2}]$$

$$= 2(3000 \text{ kg})(3.92 \times 10^{-4})/\{(365 \text{ d/yr})(24 \text{ h/d})(3600 \text{ s/d})(6 \text{ m}^2)(1.0)[0.95(9.8 \text{ m/s}^2)(6.55 \times 10^6 \text{ m})^{1/2}]\}$$

$$= \boxed{1.6 \times 10^{-12} \text{ kg/m}^3}.$$

72. (a) In the time $t = r/v$ that it takes the ball to reach the rim of the platform, it will have rotated an angle $\theta = \omega t$. A point on the rim will have moved a distance $d = r\theta = r\omega t = v\omega t^2$.
- (b) To the observer on the platform, the ball will have moved opposite to the direction of rotation a distance d in a time t . The apparent acceleration can be found from $d = v_{0\perp}t + \frac{1}{2}at^2$; $a = 2d/t^2 = 2(v\omega t^2)/t^2 = 2v\omega$, perpendicular to v .
- (c) In the inertial frame, the ball thrown from the rim also has the initial tangential velocity of the rim, $v = r\omega$. In this frame, as the ball moves in a straight line, it will not go to the center of the platform but will go to the right of the direction of throw. To an observer on the platform, the ball will have an acceleration to the right of the radial line perpendicular to the motion.



73. For a cable of length L , the radius of the circle is L . The tension in the cable provides the centripetal acceleration. For $\Sigma F_r = ma_r$ we have

$$T = mr\omega^2 = mL\omega^2 \\ = (5.0 \text{ kg})(1.2 \text{ m})[(1 \text{ rev})(2\pi \text{ rad/rev})/(1.4 \text{ s})]^2 = \boxed{1.2 \times 10^2 \text{ N}}.$$

74. Until the die begins to slide, there will be a static friction force from the turntable that provides the centripetal acceleration. The die will begin to slide when this force reaches its maximum limit:

$$f_s = f_{s,\max} = \mu_s F_N.$$

We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for the die:

$$x\text{-component: } f_s = mR\omega^2;$$

$$y\text{-component: } F_N - mg = 0.$$

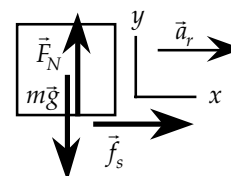
Thus a larger R requires a larger f_s .

R will be maximum when f_s is maximum:

$$f_s = \mu_s F_N = \mu_s mg.$$

Thus $\mu_s mg = mR_{\max}\omega^2$, or $\mu_s = R\omega^2/g$;

$$\mu_s = (4 \text{ in})(2.54 \text{ cm/in})(10^{-2} \text{ m/cm})[(45 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min})]^2/(9.8 \text{ m/s}^2) = \boxed{0.23}.$$



75. Because there is no friction, we need to consider horizontal forces only.

We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for the system of the three masses:

$$x\text{-component: } F = (m_1 + m_2 + m_3)a;$$

$$1.5 \text{ N} = (0.3 \text{ kg} + 0.4 \text{ kg} + 0.2 \text{ kg})a, \text{ which gives}$$

$$a = 1.7 \text{ m/s}^2.$$

- (a) We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for m_1 :

$$x\text{-component: } T_1 = m_1 a;$$

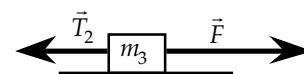
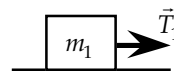
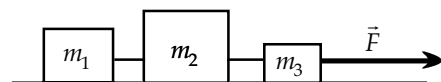
$$T_1 = (0.3 \text{ kg})(1.7 \text{ m/s}^2) = \boxed{0.5 \text{ N}}.$$

- (b) We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for m_3 :

$$x\text{-component: } F - T_2 = m_3 a;$$

$$1.5 \text{ N} - T_2 = (0.2 \text{ kg})(1.7 \text{ m/s}^2), \text{ which gives}$$

$$T_2 = \boxed{1.2 \text{ N}}.$$



76. From the dimensions of the triangle, we find that

$$L_1^2 + L_2^2 = D^2,$$

which means that the triangle is a right triangle:

$$\cos \theta_1 = L_1/D = 2.96 \text{ m}/4.96 \text{ m} = 0.60, \text{ so } \theta_1 = 53^\circ;$$

$$\cos \theta_2 = L_2/D = 3.99 \text{ m}/4.96 \text{ m} = 0.80, \text{ so } \theta_2 = 37^\circ.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the mass:

$$x\text{-component: } -T_1 \cos \theta_1 + T_2 \cos \theta_2 = 0;$$

$$y\text{-component: } T_1 \sin \theta_1 + T_2 \sin \theta_2 - mg = 0.$$

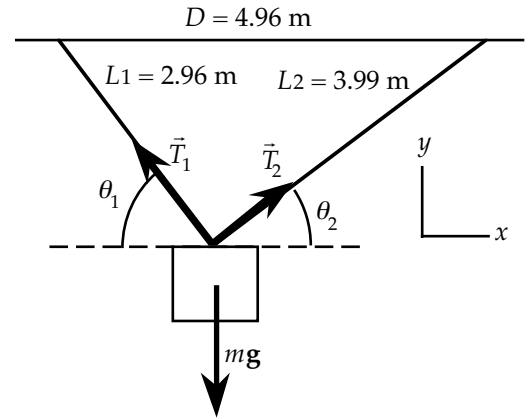
With the given data, these equations become

$$0.60T_1 - 0.80T_2 = 0;$$

$$0.80T_1 + 0.60T_2 = (3.88 \text{ kg})(9.8 \text{ m/s}^2).$$

Solving these two equations simultaneously, we get

$$T_1 = \boxed{30.4 \text{ N}} \quad \text{and} \quad T_2 = \boxed{22.8 \text{ N}}.$$



77. (a) Because the length of the rope is constant, when m_1 moves up Δx_1 , the segment above m_1 decreases by Δx_1 , and each segment above m_2 must increase by one-half that amount:

$$\boxed{\Delta x_2 = -\frac{1}{2}\Delta x_1} \quad (- \text{ indicates the opposite direction}).$$

- (b) If we differentiate with respect to time twice:

$$v_2 = -\frac{1}{2}v_1;$$

$$a_2 = -\frac{1}{2}a_1.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for m_1 :

$$x\text{-component: } T - m_1g = m_1a_1.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for m_2 :

$$x\text{-component: } 2T - m_2g = m_2a_2 = -\frac{1}{2}m_2a_1.$$

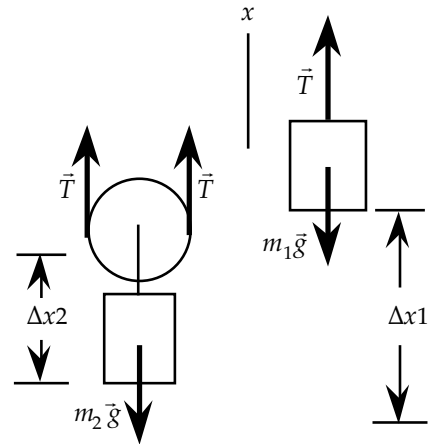
By eliminating T between these equations, we get

$$\begin{aligned} a_1 &= -2g[(2m_1 - m_2)/(4m_1 + m_2)] \\ &= -2(9.8 \text{ m/s}^2)[2(1.2 \text{ kg}) - 1.8 \text{ kg}]/[4(1.2 \text{ kg}) + 1.8 \text{ kg}] \\ &= \boxed{-1.78 \text{ m/s}^2 \text{ (down)}}; \end{aligned}$$

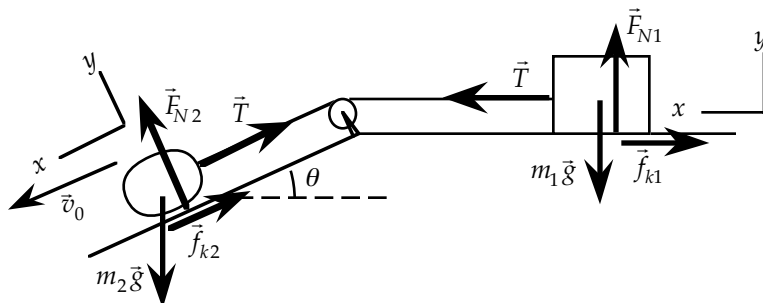
$$a_2 = -\frac{1}{2}a_1 = \boxed{+0.89 \text{ m/s}^2 \text{ (up)}}.$$

- (c) For the tension we have

$$T = m_1(g + a_1) = (1.2 \text{ kg})(9.8 \text{ m/s}^2 - 1.78 \text{ m/s}^2) = \boxed{9.6 \text{ N}}.$$



78. The two masses will have the same acceleration and the same tension. The kinetic friction forces will oppose the motion.
(a)



- (b) We write $\sum \vec{F} = m\vec{a}$ from the force diagram for m_1 :

$$x\text{-component: } T - \mu_k F_{N1} = m_1 a;$$

$$y\text{-component: } F_{N1} - m_1 g = 0.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for m_2 :

$$x\text{-component: } m_2 g \sin \theta - T - \mu_k F_{N2} = m_2 a;$$

$$y\text{-component: } F_{N2} - m_2 g \cos \theta = 0.$$

By eliminating F_{N1} and F_{N2} from these equations, we get

$$m_2 g \sin \theta - T - \mu_k m_2 g \cos \theta = m_2 a;$$

$$T - \mu_k m_1 g = m_1 a.$$

By eliminating T from these equations, we get

$$a = [m_2 (\sin \theta - \mu_k \cos \theta) - \mu_k m_1] g / (m_1 + m_2);$$

$$= [(1.1 \text{ kg})(\sin 25^\circ - 0.25 \cos 25^\circ) - (0.25)(0.8 \text{ kg})](9.8 \text{ m/s}^2) / (0.8 \text{ kg} + 1.1 \text{ kg}) = \boxed{0.08 \text{ m/s}^2}.$$

For the motion we get

$$v = v_0 + at = 1.2 \text{ m/s} + (0.08 \text{ m/s}^2)t \quad \text{and}$$

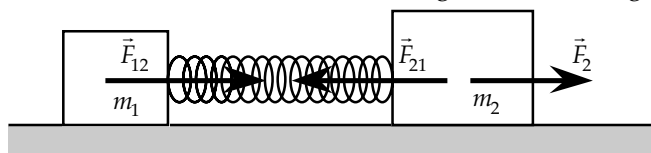
$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + (1.2 \text{ m/s})t + (0.04 \text{ m/s}^2)t^2.$$

79. The net force exerted on mass-1 (5 kg) is

$$F_{12} = m_1 a_1, \text{ and that on mass-2 (10 kg) is}$$

$$F_2 - F_{21} = m_2 a_2. \text{ Here } F_{12} = F_{21} \text{ (action and reaction). Solve for } a_2:$$

$$a_2 = (F_2 - m_1 a_1) / m_2 = [12 \text{ N} - (5 \text{ kg})(3 \text{ m/s}^2)] / 10 \text{ kg} = \boxed{-0.3 \text{ m/s}^2}, \text{ toward mass-1.}$$



80. (a) **No**. The most likely point where the string will break is at the bottom of the circular track, not at the top. The least likely point for the string to break is at the top of the track, where the tension in the string is the lowest in magnitude.

- (b) At the bottom of the track $T - mg = ma = mv^2/R$. Set $T = 40 \text{ N}$ to obtain

$$v = (TR/m - gR)^{1/2} = [(40 \text{ N})(1.0 \text{ m}) / (0.150 \text{ kg}) - (9.8 \text{ m/s}^2)(1.0 \text{ m})]^{1/2} = \boxed{16 \text{ m/s}}.$$

81. The force that prevents slipping is an upward friction force. The normal force provides the centripetal acceleration.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the person:

$$x\text{-component: } F_N = mR\omega^2;$$

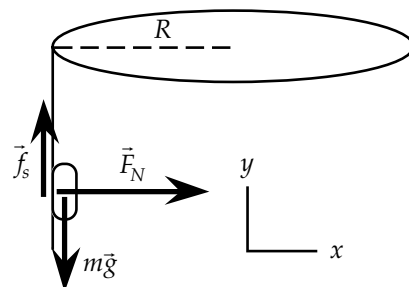
$$y\text{-component: } f_s - mg = 0.$$

At the critical condition where slipping begins,

$$f_s = f_{s,\max} = \mu_s F_N.$$

Thus we have $\mu_s(mR\omega^2) = mg$, which gives

$$\omega = \boxed{(g/\mu_s R)^{1/2}}.$$



82. The force that prevents falling is an upward friction force. The normal force provides the centripetal acceleration. If we use the force diagram from Problem 81, for $\sum \vec{F} = m\vec{a}$, we get

$$x\text{-component: } F_N = mv^2/R;$$

$$y\text{-component: } f_s - mg = 0.$$

At the critical condition where falling begins, $f_s = f_{s,\max} = \mu_s F_N$.

Thus we have $\mu_s(mv^2/R) = mg$, which gives

$$v^2 = gR/\mu_s = (9.8 \text{ m/s}^2)(8 \text{ m})/0.9, \text{ thus } v = \boxed{9.3 \text{ m/s}} \quad (\approx 21 \text{ mi/h}).$$

83. From Problem 61 we have

$$\tan \theta = v^2/rg = r\omega^2/g = (\ell_4^1 \sin \theta)\omega^2/g, \text{ which gives } \omega = \boxed{[g/(\ell \cos \theta)]^{1/2}}.$$

84. The forces are shown in the diagrams. At constant speed, $\vec{a} = 0$.

(a) We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the tractor:

$$x\text{-component: } F - T = 0;$$

$$y\text{-component: } F_{N1} - m_1g = 0.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the sled:

$$x\text{-component: } T - \mu_k N_2 = 0;$$

$$y\text{-component: } F_{N2} - m_2g = 0.$$

When these equations are combined, we get

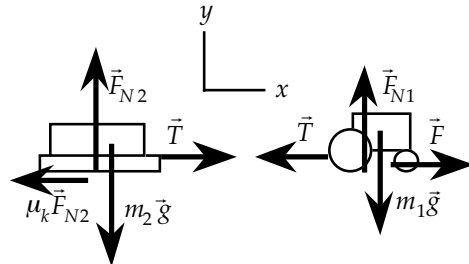
$$F = T = \mu_k m_2g = 0.68(1450 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{9.7 \times 10^3 \text{ N}}$$

forward.

(b) From part (a), $T = F = \boxed{9.7 \times 10^3 \text{ N}}$.

(c) Once stopped, the sled will start moving again when the tractor force overcomes the maximum static friction force:

$$F = \mu_s m_2g = 0.70(1450 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{9.9 \times 10^3 \text{ N}} \text{ forward.}$$



85. The static friction force provides the centripetal acceleration.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the bicycle:

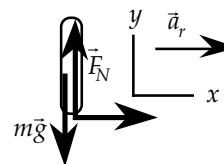
$$x\text{-component: } f_s = mv^2/R;$$

$$y\text{-component: } F_N - mg = 0.$$

The shortest turn (smallest R) requires $f_{s,\max} = \mu_s F_N$.

Thus $R_{\min} = mv^2/f_{s,\max} = mv^2/\mu_s mg$

$$= v^2/\mu_s g = (10 \text{ m/s})^2/0.4(9.8 \text{ m/s}^2) = \boxed{26 \text{ m}}.$$



86. The speed of 60 mi/h = (60 mi/h)(1.6 × 10³ m/mi)/(3.6 × 10³ s/h) = 26.7 m/s. We assume that each car will stop in the shortest time, which means maximum static friction force: $f_s = \mu_s F_N$.

Each car will have the same force diagram, except for different magnitudes.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for each car:

Car 1: x -component: $-f_{s1} = \mu_s F_{N1} = ma_1$;

y -component: $F_{N1} - m_1 g = 0$.

Car 2: x -component: $-f_{s2} = \mu_s F_{N2} = ma_2$;

y -component: $F_{N2} - m_2 g = 0$.

From these we can get the two accelerations:

$$a_1 = -\mu_1 g = -(0.8)(9.8 \text{ m/s}^2) = -7.8 \text{ m/s}^2;$$

$$a_2 = -\mu_2 g = -(0.7)(9.8 \text{ m/s}^2) = -6.9 \text{ m/s}^2.$$

For the one-dimensional motion we take the origin at the point where the driver of the second car observes the first car braking.

During the reaction time, the second car will move at constant speed a distance

$$x_r = v_0 t_r = (26.7 \text{ m/s})(0.8 \text{ s}) = 21.4 \text{ m}.$$

We find the distance the second car moves during the acceleration, before it stops:

$$v^2 = v_0^2 + 2a_2 x_2; \quad 0 = (26.7 \text{ m/s})^2 + 2(-6.9 \text{ m/s}^2)x_2, \text{ which gives}$$

$$x_2 = 52.0 \text{ m}.$$

The total distance car 2 travels is $x_r + x_2 = 21.4 \text{ m} + 52.0 \text{ m} = 73.4 \text{ m}$.

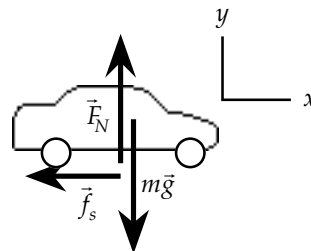
We find the distance the first car moves during the acceleration, before it stops:

$$v^2 = v_0^2 + 2a_1 x_1; \quad 0 = (26.7 \text{ m/s})^2 + 2(-7.8 \text{ m/s}^2)x_1, \text{ which gives}$$

$$x_1 = 45.6 \text{ m}.$$

Thus, to avoid a collision, the second car must be behind the first car by at least

$$D = x_r + x_2 - x_1 = 21.4 \text{ m} + 52.0 \text{ m} - 45.6 \text{ m} = \boxed{28 \text{ m}} \quad (\approx 91 \text{ ft})$$



87. At the top of the loop, both the normal force and the weight are downward.

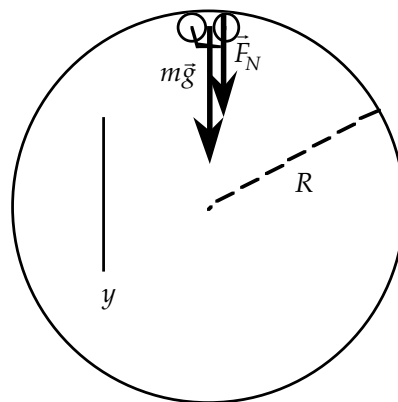
We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the motorcycle:

$$y\text{-component: } F_N + mg = mv^2/R.$$

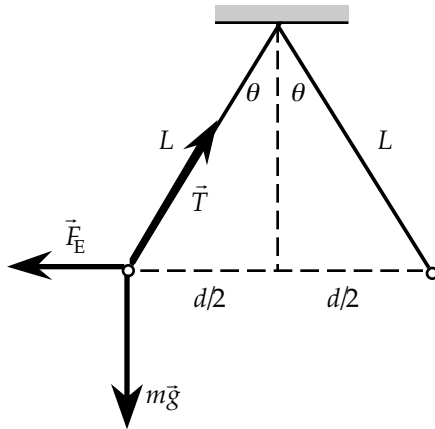
The speed v will be minimum when the normal force is minimum. The normal force can only push away from the ramp, that is, with our coordinate system it must be positive, so $F_{N\min} = 0$.

Thus we have $v_{\min}^2 = gR$, or

$$\begin{aligned} v_{\min} &= (gR)^{1/2} \\ &= [(9.8 \text{ m/s}^2)(12 \text{ m})]^{1/2} \\ &= \boxed{11 \text{ m/s}} \quad (\approx 24 \text{ mi/h}). \end{aligned}$$



88.



Three forces exert on each ball: its weight (mg), the tension in the string (T), and the electric force (F_E) due to the electric field of the other ball. For the ball to be in mechanical equilibrium the net force exerted on it must vanish:

$$\sum F_x = T \sin \theta - kq^2/d^2 = 0 \quad (x\text{-component});$$

$$\sum F_y = T \cos \theta - mg = 0 \quad (y\text{-component}),$$

where $k = 9 \times 10^9$ and $\theta = 30^\circ$. Also,

$$\frac{1}{2}d/L = \sin \theta. \text{ Solve for } q:$$

$$q = 2L \sin \theta (mg \tan \theta / k)^{1/2}$$

$$= 2(0.60)(\sin 30^\circ)[(0.0005)(9.8)(\tan 30^\circ)/(9 \times 10^9)] = \boxed{3 \times 10^{-7} \text{ C}}.$$

89. Because the belt is moving at constant velocity, it is an inertial frame. In a coordinate system attached to the belt, the block has an initial velocity of 3.0 m/s in the $-x$ -direction. The kinetic friction force will provide an acceleration which will slow the block until it comes to a full stop. Once the block comes to rest, the static friction force will be 0.

We find the acceleration from $\sum \vec{F} = m\vec{a}$:

$$x\text{-component: } f_k = \mu_k F_N = ma;$$

$$y\text{-component: } F_N - mg = 0, \text{ so we have}$$

$$a = \mu_k g = (0.20)(9.8 \text{ m/s}^2) = 1.96 \text{ m/s}^2.$$

We find the sliding distance from

$$v^2 = v_0^2 + 2a(x - x_0);$$

$$0 = (-3.0 \text{ m/s})^2 + 2(1.96 \text{ m/s}^2)x, \text{ which gives}$$

$$x = -2.3 \text{ m}.$$

The block slides to the left and leaves a black mark $\boxed{2.3 \text{ m}}$ long.

