Solution to Test 1 (version B)

MAT1300-3X, Summer 2015
Total = 20 marks

1. (2 marks) Find the equation of a line L that goes through point (−3, 1) and is perpendicular to the line L_1 of the equation $2x + 3y = 1$.

Solution. $2x + 3y = 1$, $y = -\frac{2}{3}x + \frac{1}{3}$. The slope of the line L_1 is $m_1 = -\frac{2}{3}$. The slope of the line L is $m = \frac{3}{2}$. The equation of L has the form $y = \frac{3}{2}x + b$. When $x = -3$, $y = 1$. $1 = -\frac{9}{2} + b$. $b = \frac{11}{2}$. The equation of L is $y = \frac{3}{2}x + \frac{11}{2}$.

2. (4 marks) Find the following limits by the limit laws:

(a) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{2x^2 - 3x - 2}$.

(b) $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$.

Solution. (a) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{2x^2 - 3x - 2} = \lim\limits_{x \to 2} \dfrac{(x+2)(x-2)}{(2x+1)(x-2)} = \lim\limits_{x \to 2} \dfrac{x+2}{2x+1} = \dfrac{4}{5}$.

(b) $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1} = \lim\limits_{x \to 1} \dfrac{\left(\sqrt{x} - 1\right)\left(\sqrt{x} + 1\right)}{(x-1)\left(\sqrt{x} + 1\right)} = \lim\limits_{x \to 1} \dfrac{x - 1}{(x-1)\left(\sqrt{x} + 1\right)} = \lim\limits_{x \to 1} \dfrac{1}{\sqrt{x} + 1} = \dfrac{1}{2}$.

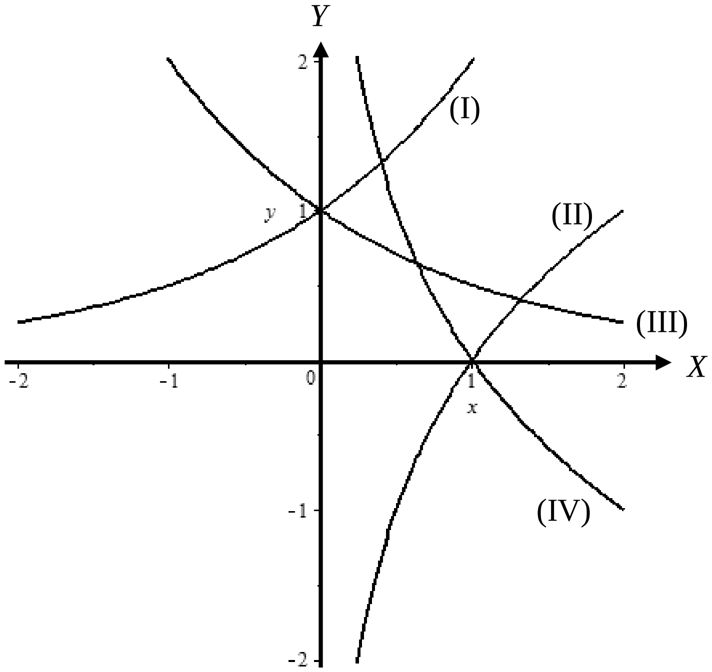
3. (3 marks) Find the vertical asymptote(s) and horizontal asymptote(s) , if any, of the graph of the function $y = \dfrac{3x^2 - x - 1}{x^2 - 1}$.

Solution. Since $x^2 - 1 = 0$ when $x = 1$ and -1, the graph of this function has two vertical asymptotes $x = 1$ and $x = -1$.

Since $\lim\limits_{x \to \infty} \dfrac{3x^2 - x - 1}{x^2 - 1} = \lim\limits_{x \to -\infty} \dfrac{3x^2 - x - 1}{x^2 - 1} = 3$, the graph of this function has a horizontal asymptote $y = 3$.

4. (2 marks) Match the following graphs with functions (a) $y = 2^x$, (b) $y = \left(\frac{1}{2}\right)^x$, (c) $y = \log_2 x$,

(d) $y = \log_{(1/2)} x$:



Answer. (I) -> (a), (II) -> (c), (III) -> (b), (IV) -> (d).

5. (3 marks) Suppose an amount \$10000 is deposit to an account with interest compounded continuously. After 5 years, the balance becomes §12000. What is the annual interest rate?

Solution. $A(t) = A(0)e^{rt}$. $e^{5r} = 1.1$. $5r = \ln 1.2$, $r = \ln 1.2 / 5$.

6. (6 marks) Find derivatives of the following functions:

(a) $y = 3x^3 + 2\sqrt{x} + 1$. (b) $y = \dfrac{x}{\ln x}$. (c) $y = \ln(e^{2x} + x)$.

Solution. (a) $y' = 9x^2 + \dfrac{1}{\sqrt{x}}$.

(b) $y' = \dfrac{\ln x - x\dfrac{1}{x}}{(\ln x)^2} = \dfrac{\ln x - 1}{(\ln x)^2}$.

(c) $y' = \dfrac{1}{e^{2x} + x}(2e^{2x} + 1)$.