Solution to Test 1 (version A)

MAT1300-3X, Summer 2015
Total = 20 marks

1. (2 marks) Find the equation of a line L that goes through point (3, −1) and is perpendicular to the line L_1 of the equation $3x - 2y = 1$.

Solution. $3x - 2y = 1$, $y = \frac{3}{2}x - \frac{1}{2}$. The slope of the line L_1 is $m_1 = \frac{3}{2}$. The slope of the line L is $m = -\frac{2}{3}$. The equation of L has the form $y = -\frac{2}{3}x + b$. When $x = 3$, $y = -1$. $-1 = -2 + b$. $b = 1$. The equation of L is $y = -\frac{2}{3}x + 1$.

2. (4 marks) Find the following limits by the limit laws:

(a) $\lim_{x\to 2} \frac{2x^2 - 3x - 2}{x^2 - 4}$.

(b) $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$.

Solution. (a) $\lim_{x\to 2} \frac{2x^2 - 3x - 2}{x^2 - 4} = \lim_{x\to 2} \frac{(2x+1)(x-2)}{(x+2)(x-2)} = \lim_{x\to 2} \frac{2x+1}{x+2} = \frac{5}{4}$.

(b) $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x\to 1} \frac{(x-1)\left(\sqrt{x}+1\right)}{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)} = \lim_{x\to 1} \frac{(x-1)\left(\sqrt{x}+1\right)}{x-1} = \lim_{x\to 1}(\sqrt{x}+1) = 2$.

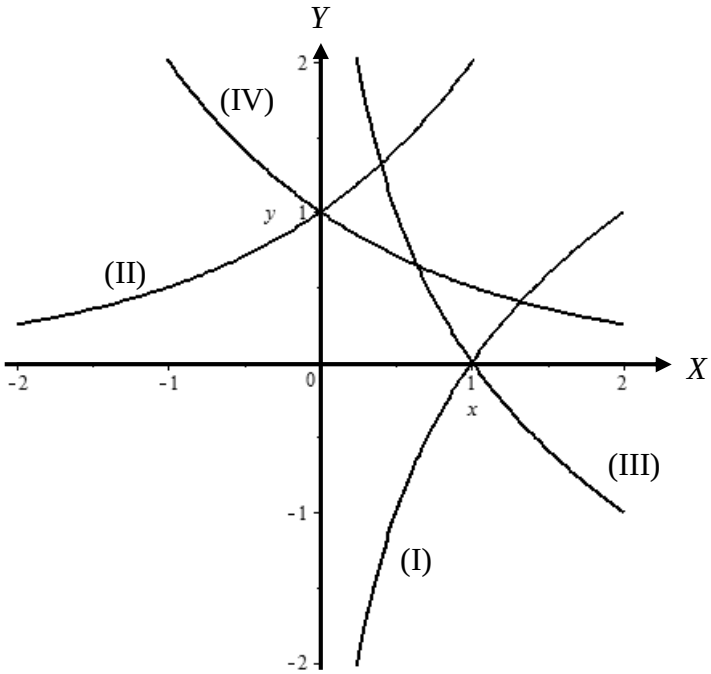
3. (3 marks) Find the vertical asymptote(s) and horizontal asymptote(s) , if any, of the graph of the function $y = \frac{2x^2 - 3x - 1}{x^2 - 4}$.

Solution. Since $x^2 - 4 = 0$ when $x = 2$ and -2, the graph of this function has two vertical asymptotes $x = 2$ and $x = -2$.

Since $\lim_{x\to\infty} \frac{2x^2 - 3x - 1}{x^2 - 4} = \lim_{x\to-\infty} \frac{2x^2 - 3x - 1}{x^2 - 4} = 2$, the graph of this function has a horizontal asymptote $y = 2$.

4. (2 marks) Match the following graphs with functions (a) $y = 2^x$, (b) $y = \left(\frac{1}{2}\right)^x$, (c) $y = \log_2 x$,

(d) $y = \log_{(1/2)} x$:



Answer. (I) -> (c), (II) -> (a), (III) -> (d), (IV) -> (b).

5. (3 marks) Suppose an amount \$10000 is deposit to an account with interest compounded continuously. After 2 years, the balance becomes §11000. What is the annual interest rate?

Solution. $A(t) = A(0)\,e^{rt}$. $e^{2r} = 1.1$. $2r = \ln 1.1$, $r = \ln 1.1 / 2$.

6. (6 marks) Find derivatives of the following functions:

(a) $y = 2x^4 - 2\sqrt{x} + 3$. (b) $y = \dfrac{\ln x}{x}$. (c) $y = \sqrt{e^{2x}+1}$.

Solution. (a) $y' = 8x^3 - \dfrac{1}{\sqrt{x}}$.

(b) $y' = \dfrac{\frac{1}{x}x - \ln x}{x^2} = \dfrac{1-\ln x}{x^2}$.

(c) $y' = \dfrac{1}{2}(e^{2x}+1)^{-1/2}(e^{2x})' = \dfrac{e^{2x}}{\sqrt{e^{2x}+1}}$.