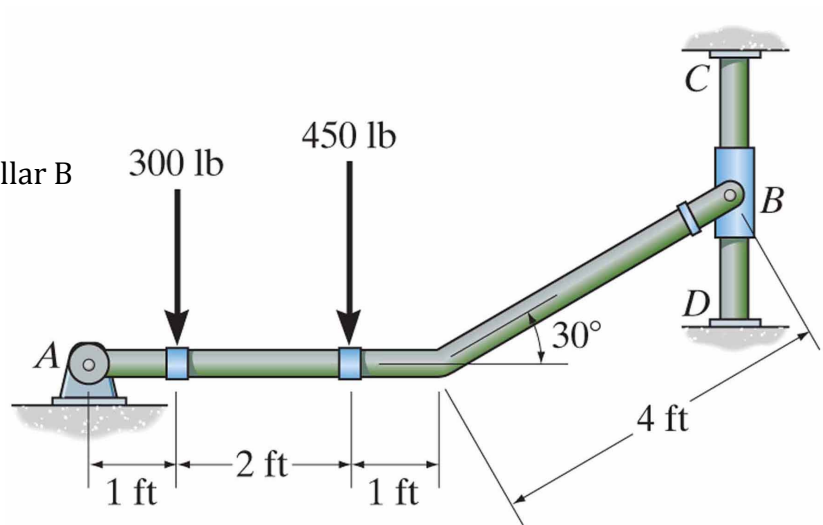


**Problem 1** (10 Marks)

Ignore the weight of the frame AB and

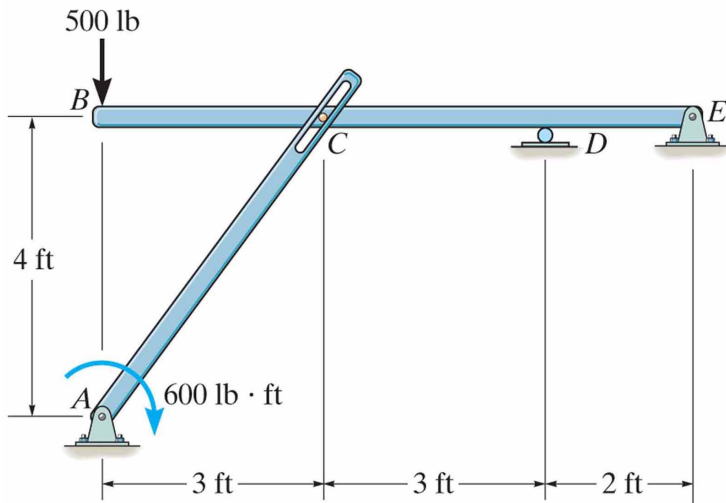
- (a) Draw its Free Body Diagram
- (b) Determine the reaction at A
- (c) Determine the reaction at the collar B



**Problem 2** (20 Marks)

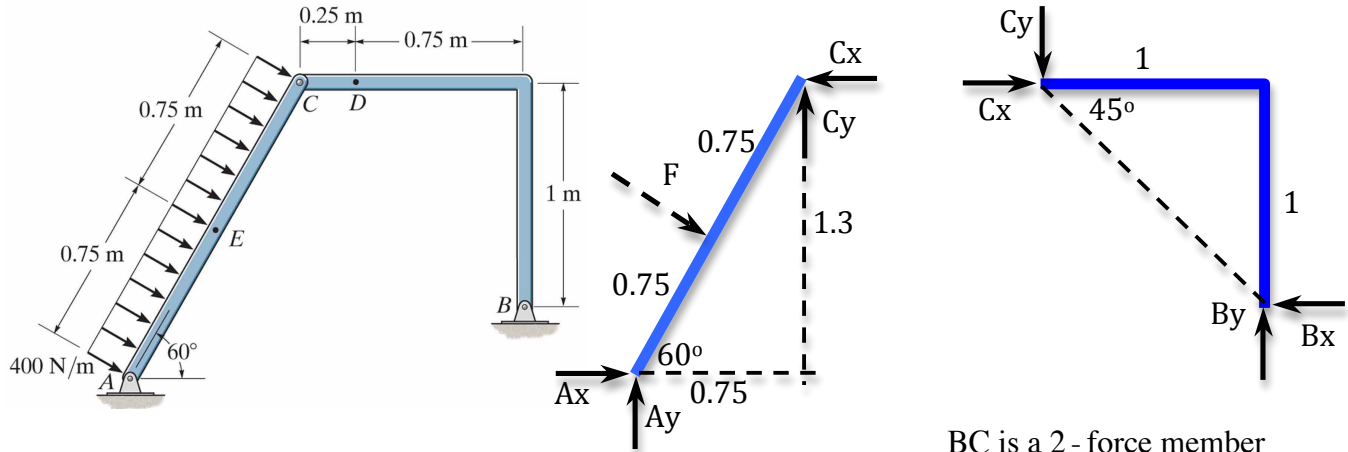
The pin at C is fixed to member BDE and passes through the smooth slot in member AC. Ignore the weight and

- (a) Draw the Free Body Diagram for AC
- (b) Draw the Free Body Diagram for ACDE
- (c) Determine the reaction at supports A, D and E



**Problem 3 (20 Marks)**

- (a) Draw the Free Body Diagram for component AC (4 marks)
- (b) Draw the Free Body Diagram for component CB (4 marks)
- (c) Determine the reactions at support A (4 marks)
- (d) Determine the reactions at support B (4 marks)
- (e) Determine the internal forces (Axial, Shear and Bending Moment) at point D. (4 marks)



$$F = 400 \text{ N/m} (0.75\text{m} + 0.75\text{m}) = 600 \text{ N}$$

$$\Sigma M_A = 0$$

$$-600(0.75) + C_x(1.3) + C_y(0.75) = 0$$

$$C_x = C_y$$

$$2.05C_x = 450$$

$$C_x = 219.5 \text{ lb} \leftarrow$$

$$C_y = C_x = 219.5 \text{ lb} \uparrow$$

$$\Sigma F_x = 0$$

$$A_x + F \sin 60 - C_x = 0$$

$$A_x = -519.6 + 219.5 = -300 \text{ lb} \leftarrow \triangleleft \text{ Ans}$$

$$\Sigma F_y = 0$$

$$A_y - F \cos 60 + C_y = 0$$

$$A_y = 300 - 219.5 = 80.5 \text{ lb} \leftarrow \triangleleft \text{ Ans}$$

$$A = \sqrt{A_x^2 + A_y^2} = 310.6 \text{ lb}$$

$$B_x = B_y = C_x = C_y$$

$$B_x = 219.5 \text{ lb} \leftarrow \triangleleft \text{ Ans}$$

$$B_y = 219.5 \text{ lb} \uparrow \triangleleft \text{ Ans}$$

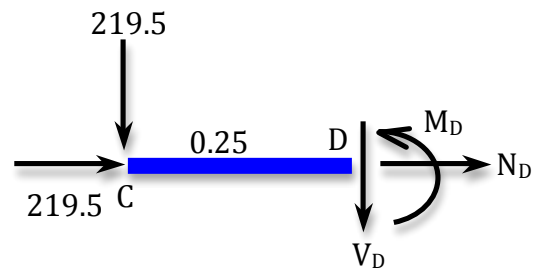
$$B = \sqrt{B_x^2 + B_y^2} = 310.4 \text{ lb}$$

BC is a 2-force member

$$C_x = C_y$$

$$B_x = B_y$$

$$C = B$$



$$\Sigma F_x = 0$$

$$N_D = -219.5 \text{ lb} \leftarrow [\text{Compression}] \triangleleft \text{ Ans}$$

$$\Sigma F_y = 0$$

$$V_D = -219.5 \text{ lb} \uparrow \downarrow \triangleleft \text{ Ans}$$

$$\Sigma M_D = 0$$

$$M_D = -219.5 (0.25) = -54.9 \text{ lb} \cdot \text{ft} \cap \triangleleft \text{ Ans}$$

**Problem 4 (10 Marks)**

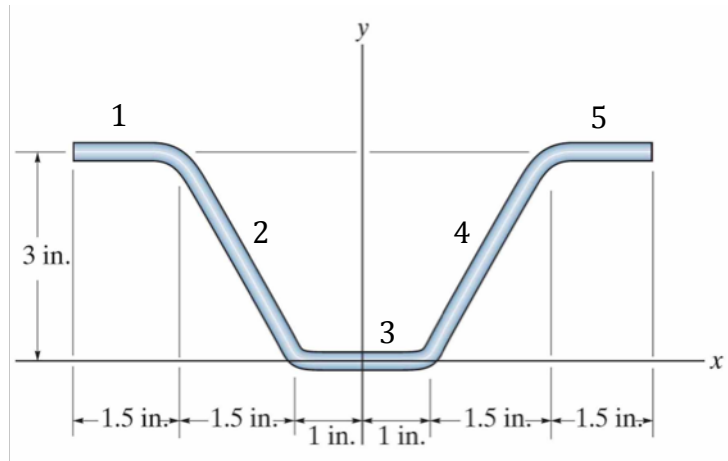
Determine the location,  $\bar{X}$  and  $\bar{Y}$ , of the center of gravity for the wire shape shown. Use the table provided to organize your calculations.

For a wire

$$\bar{X} = \frac{\sum L X_c}{\sum L} \quad \text{and}$$

$$\bar{Y} = \frac{\sum L Y_c}{\sum L}$$

where  $X_c$  and  $Y_c$  are coordinates of the centroid of each component



Note :

Components 2 and 4

$$L = \sqrt{1.5^2 + 3^2} = 3.354 \text{ in}$$

Comp	L	Xc	Yc	L Xc	L Yc
1	1.500	-3.25	3	-4.785	4.5
2	3.354	-1.75	1.5	-5.870	5.031
3	2.000	0	0	0	0
4	3.354	1.75	1.5	5.870	4.5
5	1.500	3.75	3	4.875	5.031
<b>Sum</b>	11.708			0	19.062

$$\bar{X} = \frac{\sum L X_c}{\sum L} = \frac{0}{11.708} = 0 \quad \leftarrow \text{Ans}$$

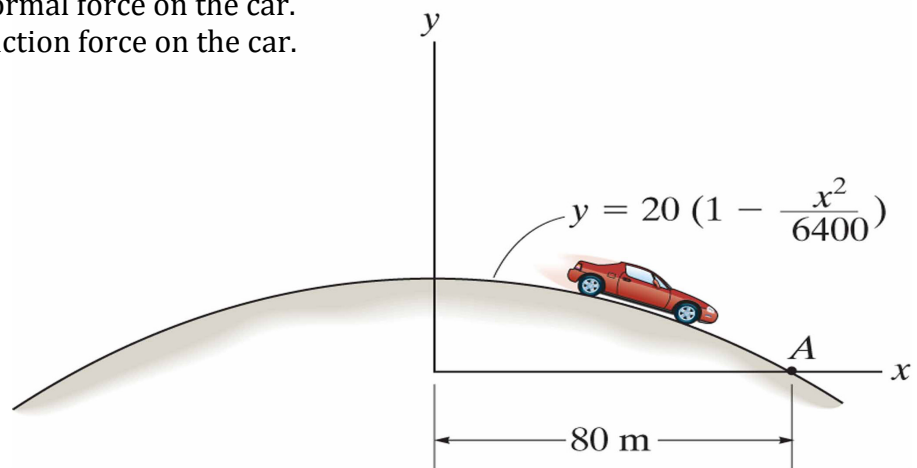
$$\bar{Y} = \frac{\sum L Y_c}{\sum L} = \frac{19.062}{11.708} = 1.628 \text{ in} \quad \leftarrow \text{Ans}$$

Marking  
 Xbar → 4 marks  
 Ybar → 6 marks

**Problem 5 (20 Marks)**

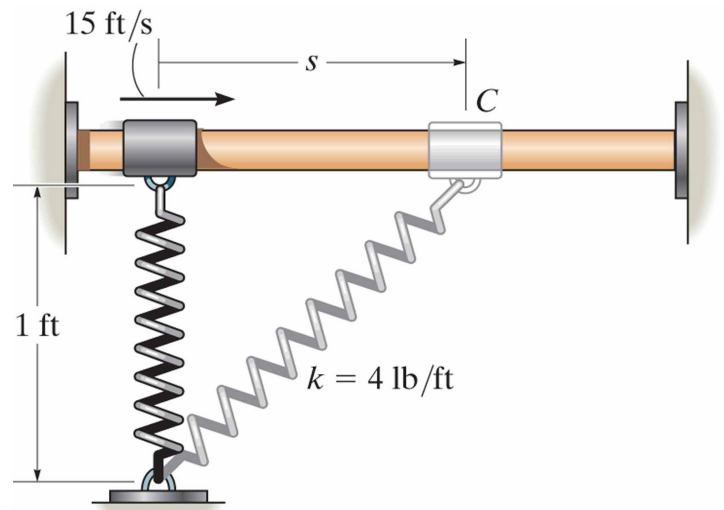
An 800 kg car travels along the vertical curve shown. At point A the car is traveling at 9 m/s and increasing its speed by  $3 \text{ m/s}^2$ .

- (a) Draw a Free Body Diagram for all forces acting on the car.
- (b) Draw a separate sketch for all velocities and accelerations acting on the car.
- (c) Determine the total normal force on the car.
- (d) Determine the total friction force on the car.



**Problem 6 (20 Marks)**

The 2-lb collar fits loosely on the smooth shaft. If the spring is unstretched when  $s = 0$  and the collar has a velocity of 15 ft/s. Use the **Principle of Work and Energy** or the **Conservation of Energy Theorem** to determine the velocity of the collar when  $s = 1$  ft



# Fundamental Equations of Dynamics

## KINEMATICS

### Particle Rectilinear Motion

Variable $a$	Constant $a = a_c$
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$
$v = \frac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$a ds = v dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

### Particle Curvilinear Motion

$x, y, z$ Coordinates	$r, \theta, z$ Coordinates
$v_x = \dot{x}$ $a_x = \ddot{x}$	$v_r = \dot{r}$ $a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$ $a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$ $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$ $a_z = \ddot{z}$	$v_z = \dot{z}$ $a_z = \ddot{z}$

### $n, t, b$ Coordinates

$v = \dot{s}$	$a_t = \dot{v} = v \frac{dv}{ds}$
	$a_n = \frac{v^2}{\rho}$ $\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{ d^2y/dx^2 }$

### Relative Motion

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

### Rigid Body Motion About a Fixed Axis

Variable $\alpha$	Constant $\alpha = \alpha_c$
$\alpha = \frac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c t^2$
$\omega d\omega = \alpha d\theta$	$\omega = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

### For Point $P$

$$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$$

### Relative General Plane Motion—Translating Axes

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$$

### Relative General Plane Motion—Trans. and Rot. Axis

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

## KINETICS

### Mass Moment of Inertia

$$I = \int r^2 dm$$

### Parallel-Axis Theorem

$$I = I_G + md^2$$

### Radius of Gyration

$$k = \sqrt{\frac{I}{m}}$$

## Equations of Motion

Particle	$\Sigma \mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G \alpha$ or $\Sigma M_P = \Sigma (\mathcal{M}_k)_P$

### Principle of Work and Energy

$$T_1 + U_{1-2} = T_2$$

### Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

### Work

#### Variable force

$$U_F = \int F \cos \theta ds$$

#### Constant force

$$U_F = (F_c \cos \theta) \Delta s$$

#### Weight

$$U_W = -W \Delta y$$

#### Spring

$$U_s = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$

#### Couple moment

$$U_M = M \Delta \theta$$

### Power and Efficiency

$$P = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v} \quad \epsilon = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_{\text{out}}}{U_{\text{in}}}$$

### Conservation of Energy Theorem

$$T_1 + V_1 = T_2 + V_2$$

### Potential Energy

$$V = V_g + V_e, \text{ where } V_g = \pm W y, V_e = +\frac{1}{2}ks^2$$

### Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$
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Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$
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### Conservation of Linear Momentum

$$\Sigma(\text{sys. } m\mathbf{v})_1 = \Sigma(\text{sys. } m\mathbf{v})_2$$

### Coefficient of Restitution

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

### Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
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Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$
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### Conservation of Angular Momentum

$$\Sigma(\text{sys. } \mathbf{H})_1 = \Sigma(\text{sys. } \mathbf{H})_2$$