Problems for Chapter 5

Problem 1

The production function of widgets can be expressed as: $Q = 50K^{1.25}L^{0.5}$, where Q = units of widgets per month, K = capital input (units of machine hour), and L = labour input (units of worker hour). The company can rent its equipment and hire workers at competitive rates. Equipment needed for this operation can be rented at $100 per hour, and labour can be hired at $25 per worker hour. The

a) Find the marginal rate of technical substitution.
b) Determine the optimal input mix to produce an output of 20,000 units. Compute the cost of production.
c) Suppose the rate of worker hour increases to $40. Determine the optimal input mix to produce 20,000 units. Calculate the cost of production.
d) Suppose the cost of production remains constant as in part (b) and the worker hour rate is $40. Determine the greatest output that the firm can produce.
e) Calculate the substitution effect and the scale effect in the employment of labour to the increase in the worker hour rate.
f) Draw the isoquants and isoexpenditure lines in parts (b) to (d), and illustrate the substitution and scale effect.

Answer:

a) $MP_L = 50*0.5*K^{1.25}L^{-0.5}$

$MP_K = 50*1.25*K^{0.25}L^{0.5}$

MRTS = $\frac{MP_L}{MP_K} = \frac{50*0.5*K^{1.25}L^{-0.5}}{50*1.25*K^{0.25}L^{0.5}} = \frac{2K}{5L}$

b) Optimal K/L ratio: $\frac{MP_L}{MP_K} = \frac{w}{c} = \frac{25}{100} \rightarrow \frac{2K}{5L} = \frac{1}{4} \rightarrow \frac{K}{L} = \frac{5}{8} \rightarrow K = \frac{5L}{8} = 0.625L$

$Q = 20000 = 50*(0.625L)^{1.25}*L^{0.5} = 50*0.625^{1.25}*L^{1.75}$

$L = [20000/(50*0.625^{1.25})]^{1/1.75}$ = 42.9231 units

K = 0.625*42.9231 = 26.8270 units

Cost of production = 25L + 100K = 25*42.9231 + 100*26.8270 = $3,755.7775

c) The worker hour rate = $40

$$\frac{MP_L}{MP_K} = \frac{w}{c} = \frac{40}{100} \rightarrow \frac{2K}{5L} = \frac{2}{5} \rightarrow K = L$$

Q = 20000 = $50K^{1.25}L^{0.5}$ = $50L^{1.25}L^{0.5}$ = $50L^{1.75}$

L = 20000/$50L^{1.75}$ = 30.6825 units = K

New cost of production = 40*30.6825 + 100*30.6825 = $4,295.5567

d) Production cost = $3,755.7775 and w = $40

3755.7775 = 40L + 100K = 40L + 100L = 140L

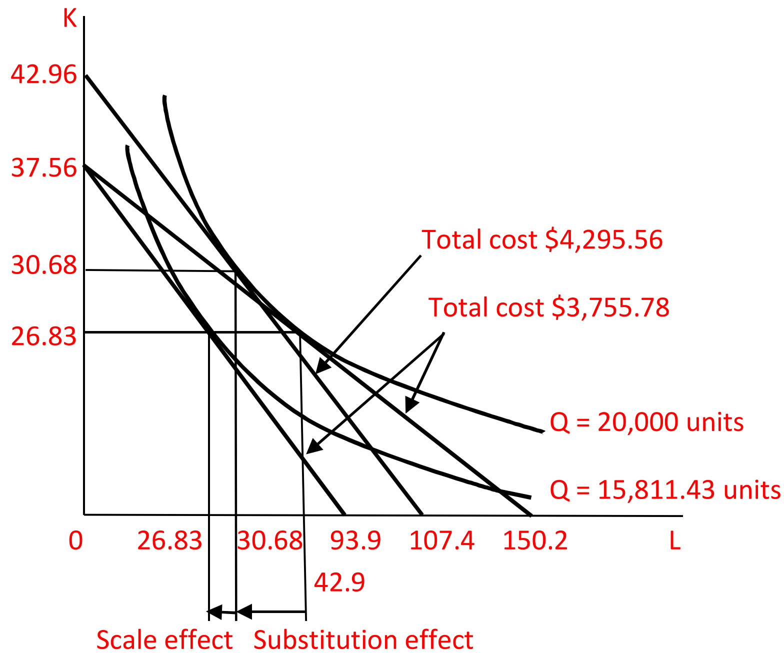
L = 3755.7775/140 = 26.8270 units = K

The greatest output Q = 50*$26.8270^{1.25}$*$26.8270^{0.5}$ = 15,811.4317 units

e) The substitution effect in the employment of labour = 42.9231 – 30.6825 = 12.2406 units

The scale effect in the employment of labour = 30.6825 – 26.8270 = 3.8555 units

f)



Problem 2

The JMSB Press produces notepads in its local shop. The company can rent its equipment and hire workers at competitive rates. Equipment needed for this operation can be rented at $24 per hour, and labour can be hired at $12 per worker hour. The company has allocated $100,000 for the initial run of notepads. The production function using available technology can be expressed as: $Q = 0.25K^{0.2}L^{0.8}$, where Q represents notepads (boxes per hour), K denotes capital input (units per hour), and L denotes labour input (units of worker time per hour).

a) Find the optimal input ratio in the long term.
b) Determine the appropriate input mix to get the greatest output for a cost of $100,000 for a production run of notepads. Compute the level of output.
c) Determine the labour demand if production were increased by 200 boxes per hour in the short term. Calculate the new cost of production.
d) Suppose the increase in the production became permanent (i.e., in the long term). Determine the optimal input mix. Calculate the cost of production and the scale effect in the employment of labour.
e) Draw the isoquants and isoexpenditure lines in parts (b) and (d), and illustrate the scale effect.

Answer:

a) $MP_L = 0.25*0.8*K^{0.2}L^{-0.2}$

   $MP_K = 0.25*0.2*K^{-0.8}L^{0.8}$

   MRTS = $\frac{MP_L}{MP_K} = \frac{0.25*0.8*K^{0.2}L^{-0.2}}{0.25*0.2*K^{-0.8}L^{0.8}} = \frac{4K}{L}$

   $$\frac{MP_L}{MP_K} = \frac{w}{c} = \frac{12}{24} \rightarrow \frac{4K}{L} = \frac{1}{2} \rightarrow \frac{K}{L} = \frac{1}{8} \rightarrow L = 8K$$

b) 100,000 = 12L + 24K = 12(8K) + 24K = 120K

   K = 100000/120 = 833.3333 units

   L = 8*833.3333 = 6666.6667 units

   $Q = 0.25*833.3333^{0.2}*6666.6667^{0.8}$ = 1,099.59 boxes

c) New Q = 1099.59 + 200 = 1299.59 boxes

   In the short term, K = 833.3333 units and $L = [1299.59/(0.25*833.3333^{0.2})]^{1/0.8}$ = 8,215.3913 units

   The new cost of production = 12*8215.3913 + 24*833.3333 = $118,584.6948

d) In the long term, $Q = 1299.59 = 0.25K^{0.2}L^{0.8} = 0.25K^{0.2}(8K)^{0.8} = 0.25*8^{0.8}*K$

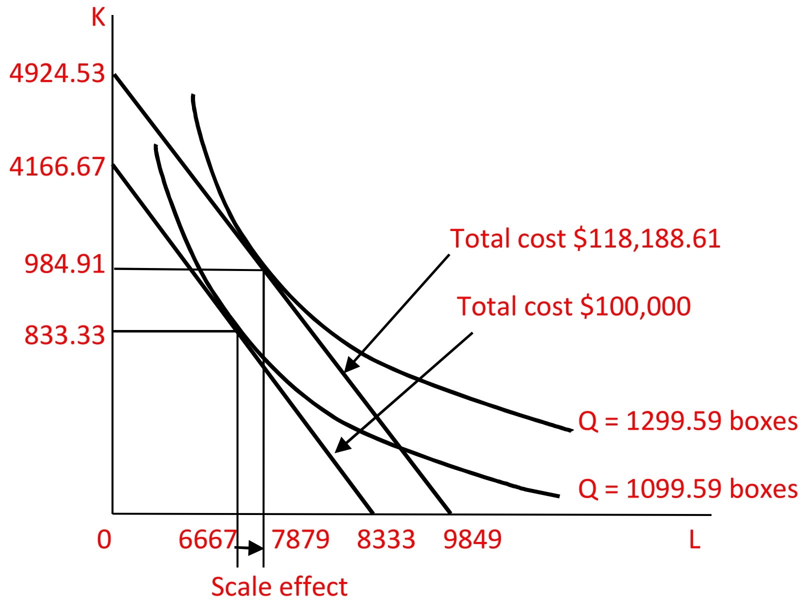
$K = 1299.59/(0.25*8^{0.8}) = 984.9050$ units

L = 8*984.9050 = 7,879.2404 units

Cost of production = 12*7879.2404 + 24*984.9050 = $118,188.61

Scale effect in the employment of labour = 7879.2404 – 6666.6667 = 1,212.5737 units

e)



Problem 3

Consider the following production function of DVDs: $Q = K^{0.5}L^{0.5}$, where Q represents DVDs (boxes per hour), K denotes capital input (units per hour), and L denotes labour input (units of worker time per hour). The unit cost of capital and labour are $40 and $20, respectively.

a) What is the optimal input ratio of labour and capital for the production?
b) Determine the appropriate input mix to produce 800 boxes of DVDs. Compute the cost of production.
c) Suppose the government decided to offer a subsidy that would make the cost of labour $15. What is the optimal input mix to produce the same level of output, and the new cost of production? Also, compute the substitution effect in the employment of labour.
d) Draw the isoquant and isoexpenditure lines in parts (b) and (c), and illustrate the substitution effect.

Answer:

a) $MP_L = 0.5*K^{0.5}L^{-0.5}$

$MP_K = 0.5*K^{-0.5}L^{0.5}$

MRTS = $\frac{MP_L}{MP_K} = \frac{0.5*K^{0.5}L^{-0.5}}{0.5*K^{-0.5}L^{0.5}} = \frac{K}{L}$

$$\frac{MP_L}{MP_K} = \frac{w}{c} = \frac{20}{40} \rightarrow \frac{K}{L} = \frac{1}{2} \rightarrow K = 0.5L$$

b) $Q = 800 = K^{0.5}L^{0.5} = (0.5L)^{0.5}L^{0.5} = 0.5^{0.5}L$

$L = 800/0.5^{0.5}$ = 1,131.3709 units

K = 0.5*1131.3709 = 565.6854 units

Cost of production = 20L + 40K = 20*1131.3709 + 40*565.6854 = $45,254.835

c) The cost of labour = $15

$$\frac{MP_L}{MP_K} = \frac{w}{c} = \frac{15}{40} \rightarrow \frac{K}{L} = \frac{3}{8} \rightarrow K = 0.375L$$

$Q = 800 = K^{0.5}L^{0.5} = (0.375L)^{0.5}L^{0.5} = 0.375^{0.5}L$

$L = 800/0.375^{0.5}$ = 1,306.3945 units

K = 0.375*1306.3945 = 489.8979 units

New cost of production = 15*1306.3945 + 40*489.8979 = $39,191.8335

Substitution effect in the employment of labour = 1131.3709 – 1306.3945 =175.0236 units

d)