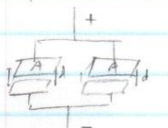
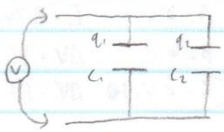
Jm Feb. 2/12

Notes for February 2nd, 2012 PHYS-1004A

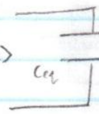
Lecture 9

Capacitors in Parallel





$\Rightarrow$



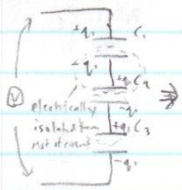
$q_1 = C_1V,\ q_2 = C_2V$

$q_1 + q_2 = C_{eq}V$

$C_1V + C_2V = C_{eq}V$

$\boxed{\therefore C_1 + C_2 = C_{eq}}$

Capacitors in Series



$\Rightarrow$

$q_1 = C_1V_1 = C_2V_2 = C_3V_3$

$V = V_1 + V_2 + V_3$

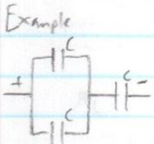
$= q_1/C_1 + q_1/C_2 + q_1/C_3 = q_1/C_{eq}$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

2 capacitors? $\boxed{C_{eq} = \dfrac{C_1C_2}{C_1 + C_2}}$

- same charge on each capacitor plate!

Example

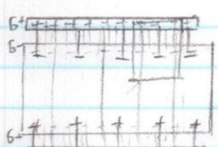


$C_{eq} =$ A. $\frac{2C}{3}$ B. C C. $\frac{3C}{2}$ D. $3C$

Parallel: $C_{eq} = C_1 + C_2 = C + C = 2C$

Series: $C_{eq} = \dfrac{2C \cdot C}{2C + C} = \dfrac{2C}{3}$

Dielectrics



$E = E_0 - E_p$

No dielectric: $E_0A = q/\varepsilon_0$

With dielectric: $EA = (q - q_p)/\varepsilon_0$

- define dielectric constant K (kappa), $E = E_0/K$
- K describes effectiveness of dielectrics (water = very high, 60 - 70)

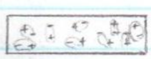
$$\frac{E_0}{K}A = \frac{(q - q_p)}{\varepsilon_0} \Rightarrow E_0A = \frac{K(q - q_p)}{\varepsilon_0} \text{ OR } \frac{q}{\varepsilon_0}$$

$\therefore K(q - q_p) = q$; $KEA = q/\varepsilon_0$ *

$E_0A = q/K\varepsilon_0$; $\boxed{q - q_p = q/K}$

Overview of Dielectrics

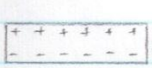
$E = 0$



$E_0 \uparrow$

$\downarrow E_p$

$E_0 \uparrow$



$\downarrow E_p$

(Theoretical Layers of Charge)

Now, Gauss' Law: $\varepsilon_0 \oint KE \cdot dA = q_{(free)}$

$* \Delta V = Ed \Rightarrow q = \left(\dfrac{K\varepsilon_0 A}{d}\right)\Delta V$ $\boxed{C_{di} = KC_0}$ dielectric; in a vacuum

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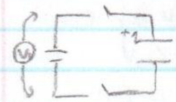
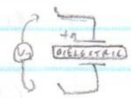
Example



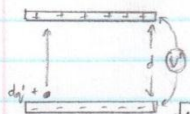


$V_2 =$ A. $> V_1$ $E = E_0/K$

B. $= V_1$ $\Delta V = \int E \cdot ds$

C. $< V_1$ $\Delta V = Ed$

Energy Stored in a Capacitor

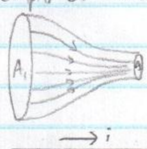


$dW' = V' dq'$ $(q = CV)$ - V': voltage increases when dq'+ reaches ⊕ plate

$dW' = \frac{q'}{C} dq'$ $(W = -U)$ $W = \int dW' = \int_{q'=0}^{q'=q_1} \frac{q'}{C} dq'$ $W = \frac{1}{2C} q^2$

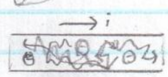
$\boxed{U = q^2/2C}$ (energy stored in capacitor)

Chapter 26 - Current and Resistance

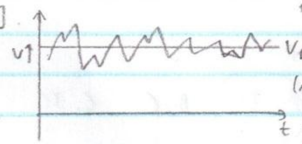


$j = i/A$ Current density = total current / cross-sectional area of conductor

$j\,[A/m^2]$, $i\,[A]$



$i = \frac{q}{\Delta t}$ $\frac{C}{s}$



$j = ne\,v_D$ (n: #/m³ carrier density, e: electric charge, v_D: drift velocity m/s)

(average velocity of charge carriers)

$i = (nALe)/\Delta t = jA$

$q = nALe$ (n = number of charge carriers/m³, A = cross-sectional area, L = length of wire)

Evan

Feb. 7/12

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Notes for February 7th, 2012

Lecture 10

Ohm's Law

$R = \frac{V}{i}$ Units: $\frac{V}{A} = \Omega$ $\rightarrow$ $R = \rho \frac{L}{A}$

$A_2 < A_1$

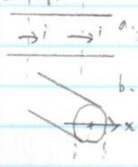
ρ = resistivity (Ω / m) $Cu = 1.69 \times 10^{-8}$, $Si = 2.5 \times 10^{3}$ (smaller = better conductor)

- n-type $Si = 8.4 \times 10^{-4}$, $SiO_2 = \sim 10^{14}$

Microscopic Ohm's Law

$\frac{V}{i} = \rho \frac{L}{A} \Rightarrow \frac{E\cancel{L}}{j\cancel{A}} = \rho \frac{\cancel{L}}{\cancel{A}} \Rightarrow \boxed{j = \frac{1}{\rho} E}$ σ = conductivity ($1/\rho$), $\therefore$ $\boxed{j = \sigma E}$

Example



a. If a section of wire is cut out and a snapshot taken while current flows, will it be charged?

A. Yes, $\ominus$ ◄ B. No C. Yes, $\oplus$

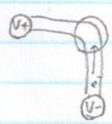
B. Same current entering/leaving wire $\therefore$ charge is neutral

b. Where does this charge reside, if it exists?

A. B. C. ◄

B. Gauss' Law only applies to electrostatics, so charge is uniform in the wire (E-fields exist in conductor)

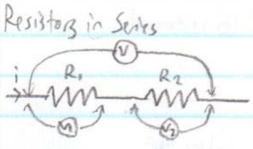
How Current Flows in Wires



What happens here?

- First electrons get stuck on outside of wire
- electric field created
- repels other electrons around the corner

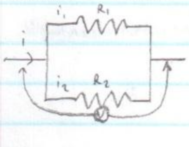
Resistors in Series



$R_{eq} = \frac{V}{i}$, $V_1 + V_2 = V$ $V = iR_1 + iR_2 = iR_{eq}$

$\therefore$ $R_{eq} = R_1 + R_2$

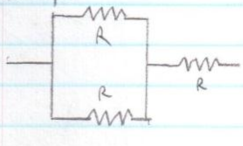
Resistors in Parallel



$V = i_1 R_1 = i_2 R_2$ $\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$ $\rightarrow$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

If only two resistors: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Example



$R_{eq} =$ A $3R$

B $\frac{3}{2}R$

C R

D $\frac{2}{3}R$ ◄

B. Parallel: $\frac{RR}{R+R} = \frac{R^2}{2R}$ OR $\frac{R}{2}$

Series: $\frac{R}{2} + R = \frac{3}{2}R$

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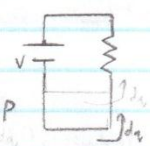
Power (Dissipation in Resistors)

- power dissipation: rate at which energy is removed from a circuit
  - power: rate when energy is supplied to a circuit

$dU = dq\,V$

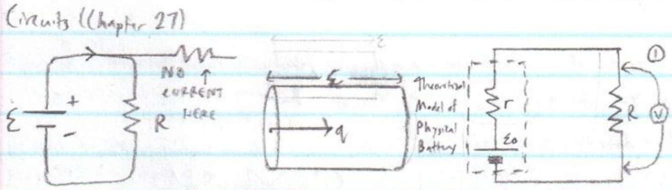
$\cancel{dU = i\,dt}$

$\frac{dU}{dt} = iV = P$



Alternative expressions: $P = i^2R = \frac{V^2}{R}$

Circuits (Chapter 27)



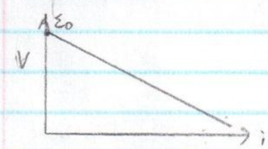
History – conventional current flows opposite electron flow
- emf = electromotive force ($\mathcal{E}$) → voltage
- but is not a force! (work required to move a charge across a sphere)

$\mathcal{E} = \frac{dW}{dq} \Rightarrow dW = \mathcal{E}\,dq = \mathcal{E}i\,dt \Rightarrow \frac{dW}{dt} = \mathcal{E}i$ = Power supplied by battery

Single-Loop Circuits

$V = iR \Rightarrow \mathcal{E}_0 - ir - iR = 0$ (see ①) (iR is measured by Ⓥ, the voltmeter)

$\boxed{V = \mathcal{E}_0 - ir}$



Example

Will the main beam give:

A. 20× more light than parking light

B. More than 20×

C. Less than 20× ◀ C. More current = less actual V.

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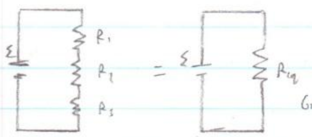
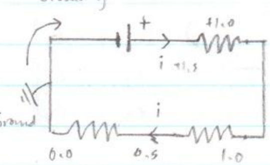
Notes for February 9th, 2012

Lecture 11

Multiple Resistors

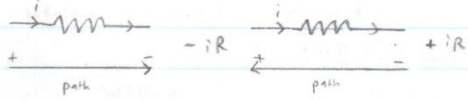
Grounding

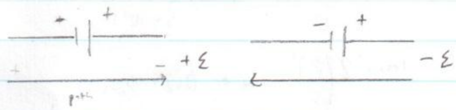




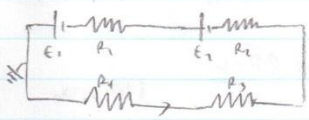
Analysis of Grounding Diagram

$+\varepsilon - iR_1 - iR_2 - iR_3 = 0$ OR $+iR_3 + iR_2 + iR_1 - \varepsilon = 0$ (P or Q)





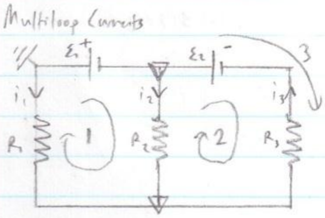
Multiple Batteries



$-\varepsilon_1 + ir_1 - \varepsilon_2 + ir_2 + iR_3 + iR_4 = 0$

$-\varepsilon_1 + (-\varepsilon_2) + i(r_1 + r_2 + R_3 + R_4) = 0$

Multiloop Circuits



Loop 1: $-\varepsilon_1 - i_2R_2 + i_1R_1 = 0$

Loop 2: $-\varepsilon_2 + i_3R_3 + i_2R_2 = 0$

Loop 3: All components included already in Loops 1 and 2

- at ∇ on diagram, sum of currents should equal zero

$\therefore i_1 + i_2 = i_3$

Solving equations: $i_1R_1 - i_2R_2 + 0 = -\varepsilon_1$ (Loop 1')

$0 + i_2R_2 + i_3R_3 = \varepsilon_2$ (Loop 2')

$i_1 + i_2 - i_3 = 0$ (i')

Elimination: (i) $i_1 + i_2 = i_3$

(i → Loop 2) $i_2R_2 + (i_1 + i_2)R_3 = \varepsilon_2$

$i_2(R_2 + R_3) - \varepsilon_2 = -i_1R_3$

$i_1 = \dfrac{1}{R_3}(\varepsilon_2 - i_2(R_2 + R_3))$ ④

④ → ① $R_1/_{R_3}(\varepsilon_2 - i_2(R_2 + R_3)) - i_2R_2 = \varepsilon_1$

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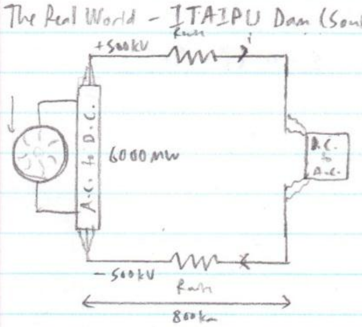
Cramer's Rule

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$\longrightarrow$ Coefficients of unknowns

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The Real World – ITAIPU Dam (South America)



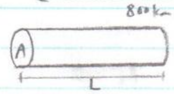
$P_{Generated} = 6000\ MW$ $\quad L = 800\ km$ $\quad V = 500kV$

$= 6 \times 10^9\ W$ $\quad = 8 \times 10^5\ m$ $\quad = 5 \times 10^5\ V$

Power loss: $2i^2 R_{wire}$

$P = iV \Rightarrow i = \dfrac{P}{V} = \text{Current} \Rightarrow \dfrac{6 \times 10^9\ W}{5 \times 10^5\ V}$

Power losses: $2\left(\dfrac{P}{V}\right)^2 R_{wire}$ or $0.25 \cdot P_{Generated}$



$R_{wire} = \rho \dfrac{L}{A}$. $\quad 2\left(\dfrac{P}{V}\right)^2 R_{wire} = 1.5 \times 10^9\ W$ $\quad$ f = fraction of power

$R_{wire} = \dfrac{1.5 \times 10^9\ (5 \times 10^5)}{2(6 \times 10^9)}$ OR $\dfrac{fV^2}{2P} = \rho \dfrac{L}{A}$ $\quad A = \dfrac{2P\rho L}{fV^2}$ $\quad \sim \$800$ million

$A = 0.698 \times 10^{-3}\ m^2 \Rightarrow A = (\pi d^2)/4 \Rightarrow d = 2.8\ cm$ Volume $(AL) \to$ Mass

44 million kilograms (4.4×10^7 kg) of copper was used for the wire! (at \$17/kg...)

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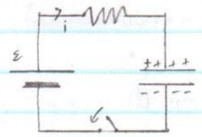
Notes for February 14^{th}, 2012

Lecture 12

RC Circuits

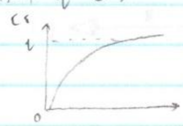


Voltage Drops

$+\varepsilon - i(t) - q(t)/C = 0$ (Note: $q = CV$)

$i(t)R + q(t)/C = \varepsilon$

$\frac{dq}{dt}R + \frac{q(t)}{C} = \varepsilon$



$t = 0$ when switch closed

(Charge increases until $V_c = \varepsilon$

- then $i = 0$

$q = C\varepsilon(1 - e^{-t/RC})$ GUESS!

$(e^x = 1 + x + x^2/2! + x^3/3! + \ldots)$

$e^0 = 1$, $e^\infty = \infty$, $1/e^\infty = 0$

$+ q(t)/C - iR = 0$

$\frac{dq}{dt} = \frac{q}{RC}$ $\left(\frac{dN}{dt} = N(t) \quad e = -t/\tau_0\right)$ $-\frac{t}{RC}$ $i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$

$q = q_0 e^{-t/RC}$

Chapter 28 – Magnetic Field

(Electric charge) → (Electric field) → (Electric charge)

$E = \frac{kq_1}{r^2}$ $F = q_2 E$ But we deal now with moving electric charges!

(Moving electric charge) → (Magnetic field B) → (Moving electric charge)

Use Ampere's Law

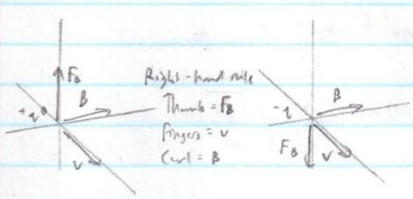
$F = qv \times B$ q = charge, v = velocity, B = magnetic field

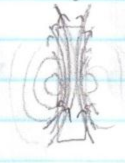
$|B| = \frac{F}{qv}$ Units: $\frac{N \cdot m}{C \cdot s}$ = Tesla

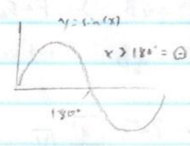
1 Tesla = very large value

1 Gauss = 1×10^{-4} T

Earth's magnetic field = 0.5 Gauss







Magnetic Field Lines → continuous

- E-field lines begin/end on electric charges, but these do not exist in magnetic field lines
  - result in continuous loops when defining field lines
- since no free charges exist here, Gauss' Law ($\oint B \cdot dA$) = 0
  - but $\oint E \cdot dA = q/\varepsilon_0$ for electric field

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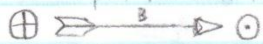
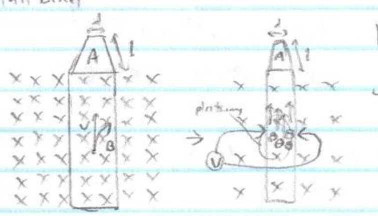
More on Magnetic Fields

- field strength is proportional to density of field lines (inversely proportional to distance from source)
- field lines point from north to south (compass tangential to field lines)



⊙ = point, ⊕ = tail! ⊙ = out of page, ⊕ = into page

Hall Effect



Voltage $V = Ed$ (Note: E = Volts/meter)

$j = ne v_D$ v_D = drift velocity

n = charge-carrying density, e = electric charge

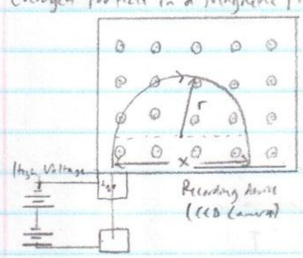
$v_D = \frac{j}{ne} \rightarrow \frac{i}{Ane}$

Force due to magnetic field

$F_B = e v_D B \Rightarrow e\left(\frac{i}{Ane}\right)B$ $F_E = eE = e\frac{V}{d}$ Set $F_B = F_E$

$\cancel{e}\frac{V}{d} = \cancel{e}\left(\frac{i}{Ane}\right)B \Rightarrow \boxed{n = \frac{Bi}{V(A/d)e} \text{ or } \frac{Bi}{Vle}}$

Charged Particle in a Magnetic Field



How does this spectrometer work?

$\Delta K + \Delta U = 0$

$\frac{mv^2}{2} - qV = 0$

$\underset{\text{velocity}}{v} = \sqrt{\frac{2q\overset{\text{voltage}}{V}}{m}}$ This describes velocity.

To describe the circle, there must be centripetal force: $F = \frac{mv^2}{r}$ (from Mechanics)

$F_B = qvB$ Set $F = F_B$

$\frac{mv}{r} = qB$ Insert velocity expression... $x = 2r = 2\frac{m}{qB}\sqrt{\frac{2qV}{m}}$

$x = 2\sqrt{2}\sqrt{\frac{q}{m}}\sqrt{V}$

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Notes for February 16th, 2012

Lecture 13

Chapter 28 - Continued

see charge-to-mass ratios on WebCT; check links

Example

A uniform magnetic field is directed out of the page, and a charged particle is travelling on the plane of the page as shown.

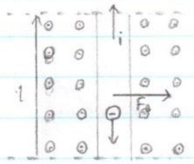
a. Is the charge positive or negative?

b. Is the particle slowing down, speeding up or maintaining a constant velocity?

A. Apply right-hand rule; centripetal force reveals charge to be POSITIVE

B. Particle is SLOWING DOWN (centripetal force equation: mv/r, as r decreases, less speed is needed)

Magnetic Force on Current-Carrying Wire



$F = q\vec{v} \times \vec{B}$ $\quad i = q/t$ [Coulombs/second]

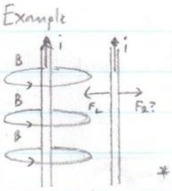
$v_D \Rightarrow j = nev_D \quad t = l/v_D$

$i = \dfrac{q\,v_D}{l} \Rightarrow \boxed{q = \dfrac{li}{v_D}}$

- substitute q-equation into F-equation: $F = \dfrac{li}{\cancel{v_D}} \cancel{v_D} \times B$ $\quad \boxed{\vec{F} = i\vec{l} \times \vec{B}}$

$F = il \times B$ (magnitude)

Example

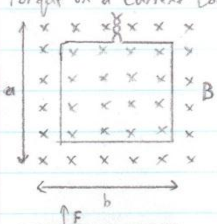


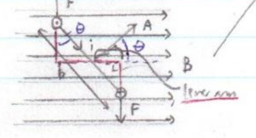
Does the right-hand wire experience a force to the left or the right?

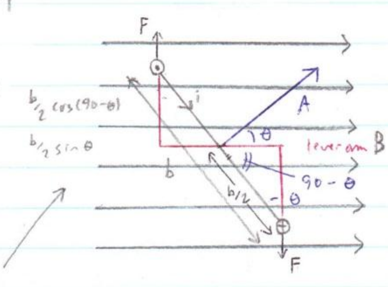
- using the right-hand rule, the wire experiences a force TO THE LEFT

* seems counterintuitive in electrostatics, but is explained by magnetism

Torque on a Current Loop







$\tau = r \frac{b}{2} \sin\theta$ ~~or~~ F_B

$= \frac{b}{2} \sin\theta\, i a B$

$\tau = 2(\frac{b}{2}) \sin\theta\, i a B$

$\tau = iab\sin\theta$

$\tau = iAB\sin\theta$

$\tau = i\vec{A} \times \vec{B}$

For n coils: $\tau = niA \times B$

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Notes for February 16th, 2012

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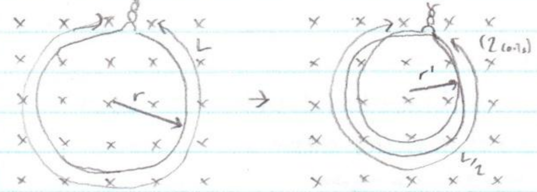
Torque on a Current Loop (continued)

Example (Problem 28-56)



Show max. torque equals:

$\tau = \frac{1}{4\pi} L^2 i B$

$A = \pi r^2$

$L = 2\pi r \Rightarrow r = L/2\pi$

$$A = \pi\left(\frac{L}{2\pi}\right)^2 = \frac{L^2}{4\pi} \qquad \tau = i\,\frac{L^2}{4\pi}\,B$$

For smaller dual-loop: $A' = \pi r'^2$ $\quad r' = \frac{L}{2}\frac{1}{2\pi}$ $\quad \frac{L}{2} = 2\pi r'$

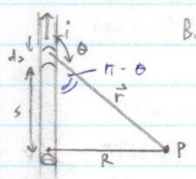
$$A' = \pi\left(\frac{L}{2}\,\frac{1}{2\pi}\right)^2 \Rightarrow A' = \frac{L}{16\pi} \quad \text{and} \quad \tau = i\,\frac{L^2}{8\pi}\,B$$

(Moving charges) → (Magnetic field) → (Moving charges)

$F = iL \times B$
$= qv \times B$

- Biot-Savart Law describes field generated by moving charge
- Ampere's Law describes field generated by current

Biot-Savart Law



Biot-Savart: $dB = \left(\frac{\mu_0}{2\pi}\right)\frac{i\,ds\,\sin\theta}{r^2} \Rightarrow B = \int_0^\infty dB = \int_0^\infty \frac{i\,ds\,\sin\theta}{r^2}\left(\frac{\mu_0}{2\pi}\right)$

$r = (R^2 + s^2)^{1/2}$, $\sin\theta = \sin(\pi - \theta)$ * Recall sine graph

$= R/r \Rightarrow R/(\sqrt{R^2+s^2})$

$$= \frac{\mu_0 i}{2\pi}\int \frac{R}{(R^2+s^2)^{3/2}}\,ds$$

*Standard Integral: $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{(x^2+a^2)^{1/2}}$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{1}{R}\frac{s}{\sqrt{s^2+R^2}}\right]_0^\infty \Rightarrow \boxed{B = \frac{\mu_0 i}{2\pi R}}$$

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Notes for February 28th, 2012

Lecture 14

Previous class - correction: charged particle in a magnetic field = $2r = \frac{2\sqrt{2}}{B} \sqrt{\frac{q}{m}} \sqrt{V}$

magnetic field $\Rightarrow B$; charge : mass ratio ; Accelerating voltage V

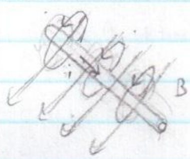
Torque on a loop of wire: $\tau = n\, i\, A \times B$ . . . Magnetic dipole moment: $n\, i\, A = \mu$

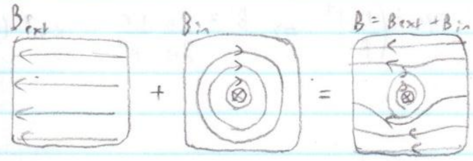
Number of turns, current, area × magnetic field

Biot - Savart Law: $d\vec{B} = \left(\frac{\mu_0}{4\pi}\right) \frac{dq\,\vec{v} \times \hat{r}}{r^2}$ $\qquad dE = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{dq\,\hat{r}}{r^2}$ $\qquad (dq\vec{v} = i\,d\vec{s})$

Extrinsic and Intrinsic Magnetic Fields

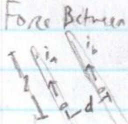
Force on a current-carrying wire: $F = i\,l\,B$





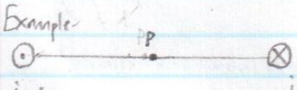
However, we only use B_{ext}, therefore $F = i\,l\,B_{ext}$.

Force Between Two Parallel Conductors



$B_a = \left(\frac{\mu_0}{2\pi}\right)\frac{i_a}{d}$ $\qquad F_b = i_b\, l\, B_a \Rightarrow F_b = \left(\frac{\mu_0}{2\pi}\right)\frac{i_a i_b}{d}\, l$

Example

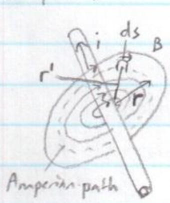


Given $|i_{out}| = |i_{in}|$ and P is halfway between them, what is the direction of the magnetic field at P?

A. Upward B. Right C. Downward D. Left E. Zero (no direction)

* use right-hand rule on both wires to find solution; both wires exert upward force at P

Ampere's Law



$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,ds \times \hat{r}}{r^2}$ Biot-Savart Law

Ampere's Law, current-carrying wire:

$\uparrow B = \frac{\mu_0}{2\pi}\frac{i}{R}$ $\qquad \oint \vec{B}\cdot d\vec{s} \Rightarrow B\oint d\vec{s}$ (B is a constant)

But integration is cumbersome, so:

$B = \frac{\mu_0}{2\pi}\frac{i}{R} \Rightarrow B' 2\pi r' \Rightarrow \frac{\mu_0}{\cancel{2\pi}}\frac{i}{\cancel{r'}}\cancel{2\pi r'} = \mu_0 i$

$\therefore \boxed{\oint \vec{B}\cdot d\vec{s} = \mu_0 i \text{ (Ampere's Law)}}$

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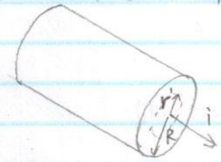
Notes for February 28^{th}, 2012

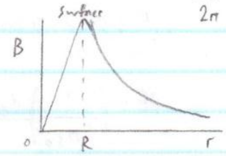
(continued from previous page)

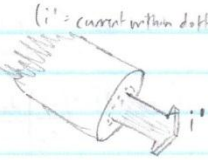
Application of Ampere's Law

Field inside a current-carrying wire : Outside wire : $B = \frac{\mu_0}{2\pi}\frac{i}{R}$ $\oint \vec{B}\cdot d\vec{s} = \mu_0 i'$

(i' = current within dotted path)





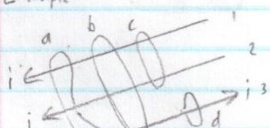


Current inside the loop : $jA' = j\pi r'^2$ (* Recall : $j = \frac{i}{\pi R^2}$

$i' = \frac{i}{\pi R^2}\pi r'^2 \Rightarrow i\left(\frac{r'}{R}\right)^2$ and then $\oint B \cdot ds = \mu_0 i (r'/R)^2$

$\Rightarrow B' 2\pi r' = \mu_0 i \left(\frac{r'}{R}\right)^2 \Rightarrow B' = \frac{\mu_0}{2\pi}\frac{i r'}{R^2} = \propto r'$ (proportional to r')

Example



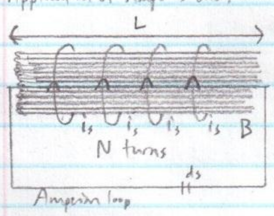
4 Amperian loops are wrapped around 3 wires carrying the same value for current, i. Which loop has the largest value for $\oint B \cdot ds$?

(a encloses wires 1 and 3; b encloses all; c encloses 1 and 2; d encloses 3)

A. a B. b C. c D. d E. All have the same value

* use Ampere's Law (B = net current through loop); look for maximum net current possible with loops ($\mu_0 i$)

Application of Ampere's Law, Continued



Solenoid assumptions : magnetic field is constant within solenoid

- magnetic field outside solenoid is zero

$\oint B \cdot ds = \mu_0 i$ i = total current $\Rightarrow \oint B \cdot ds = \mu_0 N i_s$

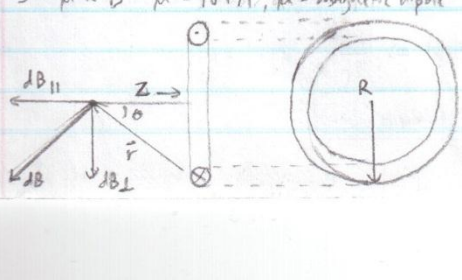
$BL = \mu_0 N i_s \Rightarrow B = \mu_0 (N/L)\, i_s$ (N/L) = turns ratio

$B = \mu_0 n i_s$ n = turns ratio N/L, n = turns/m

Inside a solenoid

Current Loop as a Dipole

$\tau = \mu \times B$ $\mu = NiA$, μ = magnetic dipole



(current loop : $(B(z) = B_{\parallel})$:

Magnetic field on axis B : $B_{\parallel} = \frac{\mu_0 i R^2}{2(R^2+z^2)^{3/2}}$

If $z \gg R$: $B_{\parallel} = \frac{\mu_0 i R^2}{2z^3} \sim \frac{1}{z^3}$

(continued on next sheet)

Notes for February 28^{th}, 2012

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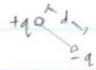
(continued from previous sheet)

Current Loop as a Dipole

$$\Rightarrow B_{\parallel} = \frac{\mu_0 i \pi R^2}{2\pi z^3} \Rightarrow \left(\frac{\mu_0}{2\pi}\right)\frac{\mu}{z^3} \quad (\mu = iA) \Rightarrow \boxed{E_{dp} = \left(\frac{1}{2\pi\varepsilon_0}\right)\frac{p}{z^3}} \leftarrow qd$$

Dipole field



We can therefore associate current loops with magnetic dipoles.

Evan

Notes for January 5^{th}, 2012 PHYS-1004A

Introduction

- final exam: 20 multiple-choice, 3/5 long-answer (tentative)
- tests: 3 questions (1-2 shorter types, 1 longer)
- purchase lab manual and four beige booklets (title: Carleton University Lab Report)
  - lab manual: SC 118, booklets: University Bookstore (sounds expensive!)
- check 'toolboxes' in WebCT (review concepts relevant to course)

Review - Vectors and Integration (Math Toolbox)

- vectors defined by magnitude and direction
- unit vector: vector with magnitude 1
  - component vectors are unit vectors, i.e.: $\hat{i}, \hat{j}, \hat{k}$ OR i, j, k
  - i.e.: $\vec{A} = 6i + 3j$, magnitude of $\vec{A}$ or $|\vec{A}| = \sqrt{(6)^2 + (3)^2}$
  - angle of $\vec{A}$ (consider x-axis as axis of origin): $\theta = \tan^{-1} 3/6$ (y/x)

Adding and Subtracting Vectors

$\vec{A} = 6i + 3j$, $\vec{B} = 2i + 2j$

$\vec{A} + \vec{B} = 8i + 5j$, $\vec{A} - \vec{B} = 4i + j$ (magnitudes and angles can now be measured as above)

Multiplying Vectors

$\vec{A} = 6i + 3j$, $\vec{B} = i$

- dot product $\vec{A} \cdot \vec{B}$: $\vec{A} \cdot i = (6i + 3j) \cdot i$ (recall: $i \cdot i = 1$, $i \cdot j = i \cdot k = 0$)

  $\vec{A} \cdot i = 6(1) + 3(0) = 6$ (used to find x- and/or y- components of vectors)

- cross/vector product: area of parallelogram created by two vectors

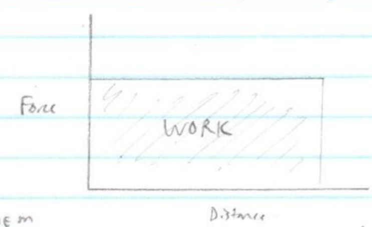
$|\vec{A} \times \vec{B}| = ab\sin\theta$ (conceptually, this turns the parallelogram into a rectangle)

- direction: perpendicular to $\vec{A}$ and $\vec{B}$ $\quad$ $i \times j = +k$ $\quad$ $i \times i = 0$

  $j \times i = -k$ $\quad$ $j \times j = 0$

$\vec{A} = 6i + 3j$, $\vec{B} = 2i + 2j$

$$\vec{A} \times \vec{B} = (6i + 3j) \times (2i + 2j)$$
$$= 12(0) + 6(-k) + 12(k) + 6(0)$$
$$= -6k + 12k = 6k$$



Application in Physics

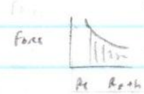
Work = Force × Distance (if force is constant)

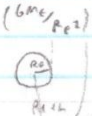
- if force changes (i.e.: gravity): $F(r) = G\frac{M_E m}{r^2}$

$$W = \sum_{i=1}^{N} F_i \Delta r = \int_{r=R_E}^{r=R_E+h} F(r)\,dr$$
$$= \int \frac{GM_E m}{r^2}\,dr$$
$$= GM_E m \int dr/r^2 \qquad \int dr/r^2 = -1/r$$
$$\int_{R_E}^{R_E+h} dr/r^2 = \frac{h}{R_E(R_E+h)} GM_E m$$

If h is small: $GM_E m (h/R_E^2)$

$= m(GM_E/R_E^2)h$ $\quad$ "g" = 9.81 m/s²





Jan

Complete questions before exam

Jan. 10/12

To bring to labs:
- Physics binder
- Physics textbook (7.6cm binder)
- Lab manual

|cross product| = $ab\sin\theta$

|dot product| = $ab\cos\theta$

Notes for January 10th, 2012 PHYS-1004A

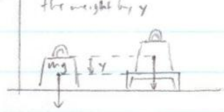
Lecture 2 - The Physics Toolbox

Work and Energy

- investigate relationship between potential energy and electrical potential energy (U and V)
  - set equal by writing $U = qV$ (q = amount of charge; V is per unit charge)
- Work-Energy Theorem: $\Delta U + \Delta K$ (kinetic energy) $= 0$, or $\Delta U = -\Delta K$

"System" is the weight

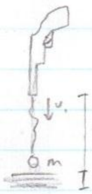
External work done to lift the weight by y



Example (see left)

$W = F \cdot d = -mgy$

- angle = 180°; $\cos 180° = -1$
- fortunately, $\Delta U = -W$ $= +mgy$
- kinetic energy is zero because object begins and ends stationary



$h_2 = 21$ cm

$h = 18$ m

$m = 5.2g = 5.2 \times 10^{-3}$ kg

18 m $v_1 = 14$ m/s

21cm $v_2 = 0$

$\Delta E = \Delta U + \Delta K$ losing potential energy and kinetic energy (hence ⊖)

$= -(mgh_1 + mgh_2) + (\frac{1}{2}mv_1^2 + 0)$

$= -0.0052(9.81)(18 + 0.21) - (0.0052)(14^2)(\frac{1}{2})$

$= -1.44$ J

∴ the energy change is 1.44 J lost.

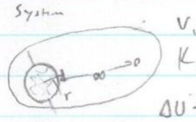
Newton's Gravitation Law

$F_g = \frac{GM_Em}{r^2}$ $\quad \Delta U = -W = -\int_{R_E}^{R_E+h} \vec{F}_g(r)\,\vec{dr}$

$\Delta U = \int_{R_E}^{R_E+h} \vec{F}_g(r)\,dr \rightarrow \Delta U = GM_Em \int \frac{dr}{r^2} \rightarrow \Delta U = GM_Em - \left[\frac{1}{r}\right]_{R_E}^{R_E+h}$

$\Delta U = GM_Em\left[\frac{1}{R_E} - \frac{1}{R_E+h}\right] \rightarrow GM_Em\left[\frac{(R_E+h) - R_E}{R_E(R_E+h)}\right] \rightarrow GM_Em\left[\frac{h}{R_E(R_E-h)}\right]$

System



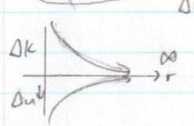
$v_1 = 0$

$K_1 = 0$

$\Delta U = -G\left[\frac{M_Em}{r^1}\right]_\infty^r \rightarrow -G\frac{M_Em}{r}$

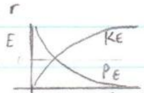
(loss of energy)

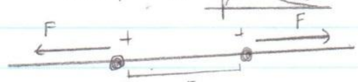
$\Delta U + \Delta K = 0$



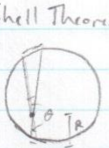
Compare with:

Corresponds to:





Shell Theorem



- radius of 'scratchlight' from object within shell proportional to distance from shell ($r = R$)

$F_g = G\frac{M_s m_{object}}{R^2} \rightarrow G\frac{M(r^3)m_{object}}{R^2}$

Area ~ r^2, $F_g = 1/R^2$ - review Shell Theorem in textbook

Check WISE for results sheet in 'Laboratory'

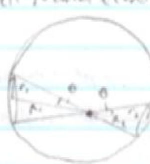
$U = -\frac{GMm}{r}$ $F = \frac{GMm}{r^2}$

Notes for January 12th, 2012

PHYS-1004A

Lecture 3

Shell Theorem (continued)



Volume density: ρ kg/m³ Surface density: σ kg/m²

Line density: λ kg/m (use surface density - lowercase sigma)

Area = πr_1^2, Mass = $\sigma \pi r_1^2$ note that $r_1 = \tan\theta R_1$

$\therefore$ Mass = $\sigma \pi R_1^2 \tan\theta$

$F_2 = \frac{G\sigma\pi \cancel{R_2^2} \tan^2\theta m}{\cancel{R_2^2}}$

$F_1 = \frac{G\sigma\pi \cancel{R_1^2} \tan^2\theta m}{\cancel{R_1^2}}$ $F_1 = G\sigma\pi \tan^2\theta m$

$F_2 = G\sigma\pi\tan^2\theta m$. $F_1 = F_2 \therefore$ net force = zero!

- this is a Faraday cage, used to protect electrical equipment

Coulomb's Law and Electric Charges

Example

1a. If we want to charge a 5kg block of iron to 0.1C, what fraction of its electrons need to be removed?

a. $n_e = \frac{0.1}{1.6\times10^{-19}}$ $n_e = 6.25\times10^{17}$ b. How many electrons are in the block? (mass number 56, atomic number 26)

b. $^{56}_{26}Fe$: 26 protons, 30 neutrons Molar mass $\doteq$ 56 g, 6.02×10^{23} atoms in 56 g

$n = \frac{m}{M}A_0 \rightarrow \frac{5\text{ kg}}{0.056\text{ kg}}(6.02\times10^{23}) \rightarrow 1.39\times10^{27}$ $\frac{6\times10^{17}}{1.39\times10^{27}} \doteq 4.3\times10^{-10}$ ← number of electrons in 0.1C

Which of these spheres has the most positive charge?

What if these same spheres were made of insulating material instead of conductor material? None!

- the presence of the positively-charged rod may polarize them, but no net charge is produced

Coulomb's Law

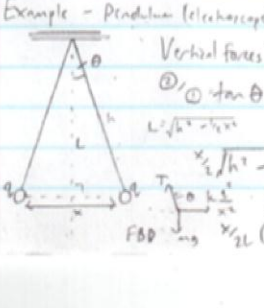
$F = k\frac{q_1 q_2}{r^2}$ k = Coulomb's Constant (9.0×10^9) q_1, q_2 = charges 1 and 2

r = distance between charges multiply by $\hat{r}$ to make it a vector force

Scenarios

$q_1 \oplus \xrightarrow{r} \oplus q_2$ $\xrightarrow{\hat{r}}$ for q_2 $\xleftarrow{\hat{r}}$ for q_1

Example - Pendulum (electroscope) formula



Vertical forces: $T\cos\theta = mg$ ① Horizontal forces: $kq^2/x^2 = T\sin\theta$ ②.

②/① $\tan\theta = \frac{kq^2}{x^2 mg}$ $\rightarrow \frac{x}{2L} = \frac{kq^2}{x^2 mg}$

$L = \sqrt{h^2 - \frac{1}{4}x^2}$ $x^3 = \frac{2kq^2L}{mg}$

$\frac{x}{2}\sqrt{h^2 - (\frac{x}{2})^2} = \tan\theta$ $x = \sqrt[3]{\frac{2kq^2L}{mg}}$ ↗ $x = \sqrt[3]{\frac{2kq^2L}{mg}}$

FBD $\frac{x}{2L}\left(1 - \frac{x^2}{8L^2}\right)^{-1/2} = \tan\theta \rightarrow \tan\theta = \frac{x}{2L}$ (if θ is small) (continued on next page)

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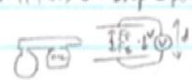
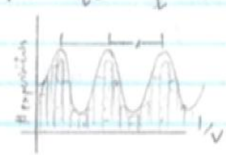
Quantization of Charge

Millikan's Oil Drop Experiment



- d = oil drop charged by friction, entered capacitor plates after being sprayed by atomizer
- F_E = electrostatic force, F_G = force of gravity, V = velocity, d = distance of plates

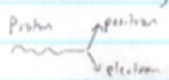
$F_E \propto qE = q\,V/d \quad F_G = mg = q\,V/d \quad V = mgd/q, \; q = mgd/V$



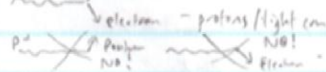
Histogram: number of times when a certain voltage was observed in experiments

- results: elementary charge = 1.59×10^{-19} C
- today, we know it as 1.60×10^{-19} C

Conservation of Charge



- no single charge cannot be created, instead making opposite charges in pairs
- protons/light can be produced by combining electrons and positrons



- to create a positive charge, one must also create a negative charge

Sam Jan. 17/12

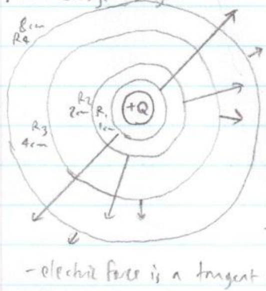
Notes for January 17th, 2012 PHYS-1004A

Lecture 4

Electric Field

Equation Definition: $\vec{E} = \frac{\vec{F}}{q}$ Force / Unit charge   Using Coulomb's Law: $\vec{E} = \frac{\vec{F}}{q} = k\frac{q_2}{r^2}\hat{r}$

Point Charge - Diagram



R_1: 1   This diagram will not be very useful when considering all of the forces around the charge. We therefore use electric field lines.

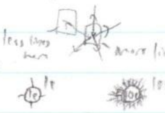
R_2: 1/4

R_3: 1/16

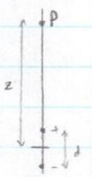
R_4: 1/64   Electric Field Lines

$1/r^2$ - begin on charge, end at negative charge or infinity

- lines never cross or intersect each other in diagrams
- electric force is a tangent to an electric field line
- magnitude of electric force is proportional to the density of the field lines
- number of field lines in vicinity of electric charge depends on the charge



Dipole Field Around an Axis

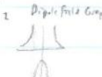


$F_p = \frac{kQq^{\oplus}}{(z-d/2)^2} - \frac{kQq^{\ominus}}{(z+d/2)^2}$ $E = \frac{F_p}{Q}$ Force at point P / Test charge $= kq\left(\frac{1}{(z-d/2)^2} - \frac{1}{(z+d/2)^2}\right)$

$E = \frac{kq}{z^2}\left(\frac{1}{(1-\frac{d}{2z})^2} \ominus \frac{1}{(1+\frac{d}{2z})^2}\right)$ Given: $(1-\frac{d}{2z})^{-2} \approx 1 - \frac{2d}{2z} + (\frac{d}{2z})^2$

$\ominus\ 1 + \frac{2d}{2z} + (\frac{d}{2z})^2$

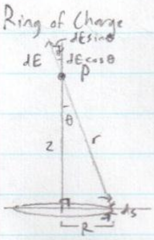
Dipole Field Graph



$\boxed{E = \frac{2kqd}{z^3}}$ $qd = p$ (i.e. $2kp$) The electric field due to a dipole

- formula works at large distances
- for shorter distances, a new equation is required

Ring of Charge



Charge on ring: λ = Coulombs/m $\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+R^2}}$

Charge in ds: $\lambda\, ds$

Test charge: +Q

$F = k\frac{Q\lambda ds}{r^2}$ $E = \frac{F}{Q} = k\frac{\lambda ds}{(z^2+R^2)}$ $E\cos\theta = k\frac{\lambda ds}{(z^2+R^2)}\frac{z}{\sqrt{z^2+R^2}}$

total charge on ring

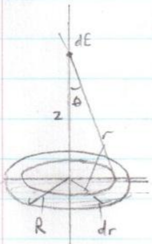
$E\cos\theta = \frac{kz\lambda ds}{(z^2+R^2)^{3/2}}$ $E_{total} = \frac{kz\lambda}{(z^2+R^2)^{3/2}}\oint_{\text{around the ring}} \underbrace{ds}_{2\pi R} \rightarrow \boxed{\frac{kz\lambda 2\pi R}{(z^2+R^2)^{3/2}} = E_p}$ or $\frac{kzq}{(z^2+R^2)^{3/2}} = E_z$

Jan 19/12

Notes for January 19th, 2012 PHYS - 1004A

Lecture 5

Disc of Charge



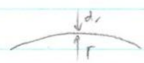
R is now the fixed outer radius of the disc

Ring

λ C/m

$q = \lambda ds$

$Q = \int dq = \lambda 2\pi r$



σ C/m²

$q = \sigma 2\pi r dr$

$Q = \int dq = 2\pi \int r dr$

(re-label $R \to r$)

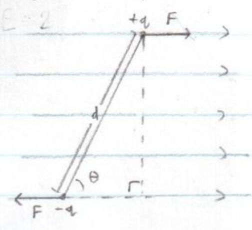
derivative of denominator?

$$dE = \int_{r=0}^{r=R} \frac{k\sigma 2\pi r dr z}{(z^2+R^2)^{3/2}} \Rightarrow k\sigma\pi z \left[\frac{1}{-\frac{1}{2}}(z^2+r^2)^{-\frac{1}{2}}\right]_0^R \quad \boxed{E = 2\pi k\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)}$$

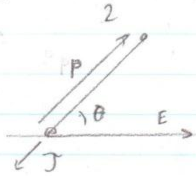
* in text, Coulomb's constant replaced by permittivity of free space $\left(\frac{1}{4\pi\varepsilon_0}\right) \to \left(\frac{\sigma}{2\varepsilon_0}\right)$

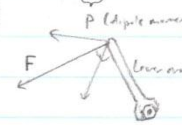
If $z \ll R$, $\sqrt{z^2+R^2} = z/\sqrt{R^2} \approx 0$, and $E = 2\pi k\sigma$ or $\sigma/2\varepsilon_0$

Torque on a Dipole in Electric Field



$$\underset{(\text{tau})}{\tau} = Eq\frac{d\sin\theta}{2} + Eq\frac{d\sin\theta}{2} = E\underbrace{qd}_{p\ (\text{dipole moment})}\sin\theta \quad \vec{\tau} = \vec{p}\times\vec{E}$$

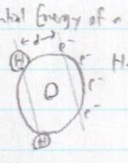


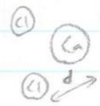


When $\theta = 0°$, there is no torque

- a dipole is parallel to the electric field

Potential Energy of a Dipole





$p = qd$ = dipole moment

- as the dipole rotates, it stores potential energy
  - when field is shut off, dipole can revert to original position

$$U = -W = -\int_{90°}^{0°} \tau\, d\theta \to -\int_{90°}^{0°} pE\sin\theta\, d\theta \Rightarrow +pE[\cos\theta]_{90}^{0}, \boxed{U = pE}$$

$$U = pE\cos\theta \Rightarrow U = \vec{p}\cdot\vec{E} \text{ and } \tau = \vec{p}\times\vec{E}$$

Signs in Potential Energy

1. $U = -W$
2. $\int pE\sin\theta\, d\theta$ → $-\cos\theta$
3. $[\cos\theta]_{90}^{0}$
4. Convention for torques: ↻ = ⊖ ↺ = ⊕

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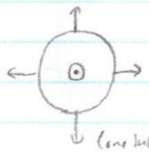
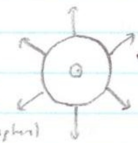
Gauss' Law (Chapter 23)

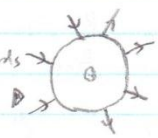




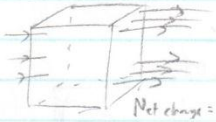
Gaussian surface (sphere)

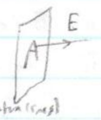
6 lines of E - field outwards

3 lines in, 6 lines out



To 'see' field lines:





A = area

E = electric field

AE = flux ; charge inside volume

Calculate density ; extra lines (AE)

* no electric field exists outside capacitors

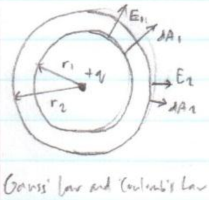
Notes for January 24th, 2012 PHYS-1004A

Lecture 6

Gauss' Law (Continued)

Net flux $= E \times A$ (E is constant, A = area)

- if E is not constant use $\oint E\,dA \rightarrow q_{enclosed}/\varepsilon_0$



$\oint E\,dA \rightarrow E_1 4\pi r_1^2 = \dfrac{q_{enclosed}}{\varepsilon_0}$

$E_1 = \dfrac{1}{4\pi r_1^2}\dfrac{q}{\varepsilon_0}$ - consistent with Coulomb's Law

$E_1 = \dfrac{q}{4\pi\varepsilon_0 r_1^2}$ $E_2 = \dfrac{q}{4\pi\varepsilon_0 r_2^2}$

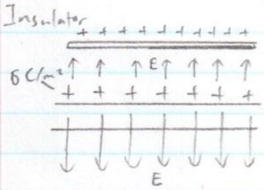
Gauss' Law and Coulomb's Law

If a cube replaces a sphere surrounding a charge of $+q$, under what conditions will net flux be the same?

- the flux is the same under all conditions, because flux is the product of surface area and charge
  - flux = electric field emanating from charge itself (same regardless of Gaussian shape)

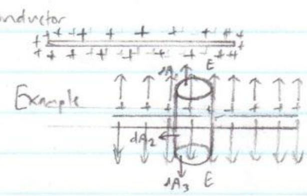
Gauss' Law and Planar Geometry

Insulator



Conductor

Example



Example Calculations

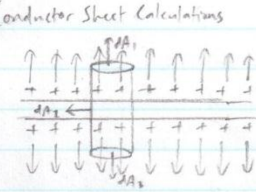
$\oint E \cdot dA = \int E\,dA_1 + \cancel{\int E\,dA_2} + \int E\,dA_3$ (no electric field acting in direction of dA_2)

$= E\pi r^2 + E\pi r^2$ net flux $= 2E\pi r^2$

Charge $= \sigma A \rightarrow \sigma\pi r^2$ Set flux = charge/ε_0

$2E\cancel{\pi r^2} = \dfrac{\sigma}{\varepsilon_0}\cancel{\pi r^2} \longrightarrow E = \dfrac{\sigma}{2\varepsilon_0}$ (charge on an insulating sheet; close to surface)

Conductor Sheet Calculations



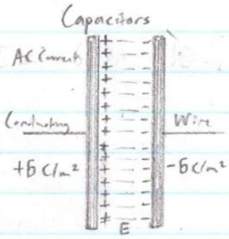
$\oint E \cdot dA = \int E\,dA_1 + \cancel{\int E\,dA_2} + \int E\,dA_3$

net flux $= 2E\pi r^2$

Charge: $2\sigma A \rightarrow 2\sigma\pi r^2$

$E = \sigma/\varepsilon_0$ (twice that of an insulator)

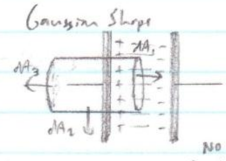
Capacitors



$E\pi r^2 = \dfrac{\sigma\pi r^2}{\varepsilon_0}$

$\boxed{E = \dfrac{\sigma}{\varepsilon_0}}$ (Charge on a capacitor plate)

Gaussian Shape



NO FIELD

$\oint E\,dA = \int E\,dA_1 + \cancel{\int E\,dA_2} + \cancel{\int E\,dA_3}$

$= E\pi r^2$

Evan

Jan. 26/12

Flux: $\frac{N \cdot m^2}{C}$

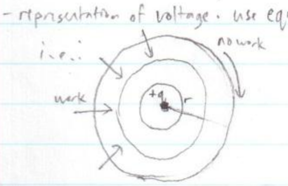
Notes for January 26th, 2012

PHYS-1004A

Lecture 7

Electric Potential

- scalar quantity (compare to vector nature of electric field); easier to sum components' contributions
  - Potential energy $(U) = qV$ (V = electrical potential per unit charge; $\Delta U = -W = U_f - U_i$
  - Similarly, $\Delta V = V_f - V_i = \Delta U/q = -W/q$
- representation of voltage: use equipotentials
  - i.e.:

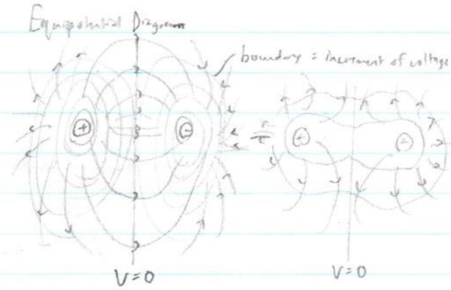


- red lines = increasing potential; blue = decreasing
- associated with positive and negative charges, respectively
- traversing lines represents the same amount of voltage changed

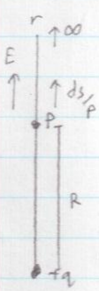
Example

* Drop a device * 20 min.

Equipotential Diagram



Point Charge



$E \cdot d_s = |E||d_s|\cos\theta$ ($0° = 1$)

$V_f - V_i = -\int_R^{\infty} E\,dr$ (in direction)

$= -kq \int_p^{\infty} dr/r^2$

$0 - V_p = -kq\left[-1/r\right]_R^{\infty}$

$-V_p = -kq/r$

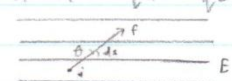
$V_p = kq/r$ (potential close to a point charge)

$V_f - V_i = -\int_i^f E \cdot ds$

Set zero for potential → at ∞

Calculating E from V

Work done $= -q\,dV = qE \cdot ds$

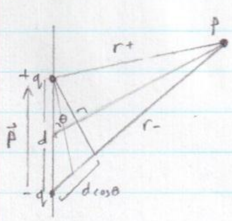


$-\cancel{q}\,dV = \cancel{q}E\,ds\cos\theta$

$\frac{dV}{ds} = -E\cos\theta$ ← component of E along ds

$\frac{dV}{dx} = -E_x$, $\frac{dV}{dy} = -E_y$, $\frac{dV}{dz} = -E_z$

Dipole



$V_{dp} = \frac{kq}{r_+} - \frac{kq}{r_-}$

$V_{dp} = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$

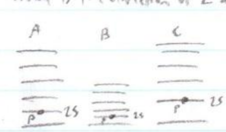
$= kq\left(\frac{r_+ - r_-}{r_+ r_-}\right)$

$= \frac{kqd\cos\theta}{r^2}$ (distance to centre of dp)

$V_{dp} = \frac{kp\cos\theta}{r^2}$

Which exhibits the greatest electric field? B

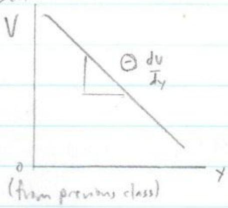
What is the direction of E at P? Down



Jan. 31/12

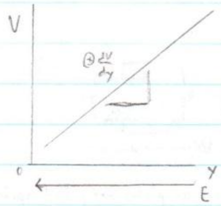
Notes for January 31st, 2012 PHYS-1004A

Lecture 8



(from previous class)

$$E_y = -\frac{dV}{dy} = -\left(-\frac{dV}{dy}\right) = \oplus \text{ (up)}$$

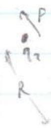


$$E_y = -\frac{dV}{dy}$$

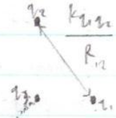
(in example, field is always up!)

Electric Potential Energy

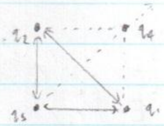
No potential energy



$$U = q_2 V_p = \frac{k q_1 q_2}{R}$$



$$\frac{k q_1 q_2}{R_{12}} + \frac{k q_1 q_3}{R_{13}} + \frac{k q_2 q_3}{R_{23}}$$



$$\frac{k q_1 q_2}{R_{12}} + \frac{k q_1 q_3}{R_{13}} + \frac{k q_2 q_3}{R_{23}} + \frac{k q_1 q_4}{R_{14}} + \frac{k q_2 q_4}{R_{24}} + \frac{k q_3 q_4}{R_{34}}$$ Energy of a system of charges

* Exponents: $\frac{10^{-13}}{10^{-19} \times 10^{9}} \rightarrow 10^{-13+19-9} = 10^{-3}$

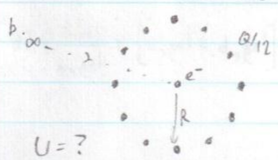
$0.012 \times 10^{-3} \rightarrow 1.2 \times 10^{-5}$

Example

a. $W = 2.16 \times 10^{-13}\ J$, $R = 8\ cm$

∞ - - - - - - - - - - e^- ← R → $-Q$ $Q = ?$

b.



a. $W = -\Delta U = -eV = -e\frac{kQ}{R}$ $Q = \frac{WR}{ek}$

$$Q = \frac{(2.16 \times 10^{-13}\ J)(0.08\ m)}{(1.6 \times 10^{-19}\ C)(9.0 \times 10^{9}\ \frac{Nm^2}{C^2})}^{*} \rightarrow Q = -12\ \mu C$$

b. $\Delta U = \sum_{i=1}^{12} \frac{k e\, Q/12}{R} \overset{\text{(simplified)}}{\Longrightarrow} \Delta U = \frac{k e\, Q/12}{R} = -2.16 \times 10^{-13}\ J$ ($\because \Delta U = -W$)

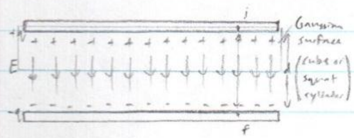
Capacitors (Chapter 25)

- useful circuit elements (tuning receivers and transmitters, store digital information, data acquisition...)
  - also good for signal conditioning and storing energy

Definition: $q_{capacitor} = CV$ C = capacitance (Farads)

- very large unit; use mF, μF, pF ($pF = 10^{-12}$, $nF = 10^{-9}$, $\mu F = 10^{-6}$, $mF = 10^{-3}$)

Parallel-Plate Capacitor



Gaussian Volume: $\oint E \cdot dA = q/\varepsilon_0$ $EA = q/\varepsilon_0$

$$V_f - V_i = -\int_i^f E \cdot ds = -Ed = \Delta V$$

$$\Delta V = -\left(\frac{q}{A\varepsilon_0}\right) d \quad q = \left(\frac{\varepsilon_0 A}{d}\right)\Delta V$$

capacitance

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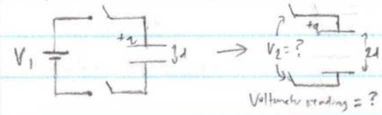
Example



$V_2 =$ A. $0.5 V_1$

B. V_1

C. $2V_1$ ✓ ◀ SELECTION (see equations on previous sheet)

- more energy is put into system as plates are pulled apart

$$q_1 = C_1 V_1 \Rightarrow C_2 = \frac{\varepsilon_0 A}{2d} = \frac{C_1}{2} \Rightarrow q_1 = \frac{C_1}{2} V_2 \Rightarrow V_1 = \left(\frac{q_1}{C_1}\right) \Rightarrow V_2 = 2V_1$$

Derivation of Capacitance

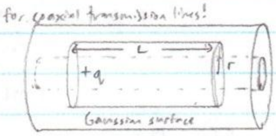
1. Use Gauss' Law to relate E to unit charge
2. Calculate work done per unit charge by calculating $-\int_i^f E \cdot ds = \Delta V$
3. Eliminate E between the two expressions

Capacitors in Series: $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

Parallel: $C_{eq}^{-1} = C_1^{-1} + C_2^{-1}$

Coaxial Cylinders

- useful for coaxial transmission lines!



$\oint E \cdot dA = q/\varepsilon_0 \quad E \sim 1/r$

If r = constant, E = constant

$E_r 2\pi r L$ (no endcaps; E only radiates)

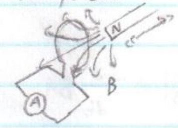
$E_r = q/(2\pi r L \varepsilon_0)$

$$V_f - V_i = -\int_{r=A}^{r=B} E_r\,dr = -\int \frac{q}{2\pi r L \varepsilon_0}\,dr \rightarrow \frac{q}{2\pi r L \varepsilon_0}\left[\log_e b - \log_e a\right] \text{ OR } \frac{\log_e b}{\log_e a} = \Delta V$$

Notes for March 1st, 2012 PHYS-1004A

Lecture 15

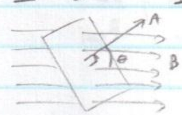
Faraday's Law

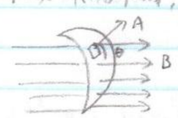


1. Current flows in the circuit when the magnetic field changes
2. Size of current increased as change became more rapid
3. Direction of current depends on polarity of magnet

Magnetic Flux

$\Phi = \vec{B} \cdot \vec{A}$ Φ = flux (letter Phi), $\vec{B}$ = magnetic field, $\vec{A}$ = area





$= \int \vec{B} \cdot d\vec{A}$ (for a curved surface)

Through a closed surface: $\oint \vec{B} \cdot d\vec{A} = 0$

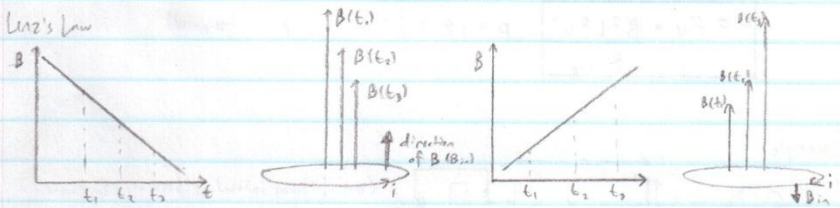
This is Gauss' Law for Magnetism.

So Faraday's Law: $\varepsilon = -\dfrac{d}{dt}(\Phi)$ negative sign = Lenz's Law (direction of current)

ε = emf

Lenz's Law is a conservation law.

Lenz's Law



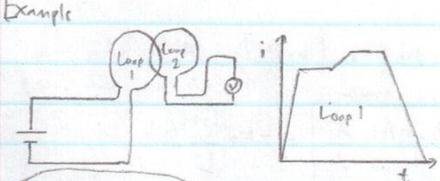
Faraday's Law, continued: $\varepsilon = -\dfrac{d}{dt}(\vec{B} \cdot \vec{A}) = -\dfrac{d}{dt}(BA\cos\theta)$ $\dfrac{dB}{dt}A\cos\theta$ $\dfrac{dA}{dt}B\cos\theta$ ω = omega, $\theta = \omega t$

The equation can be restated as: $\varepsilon = BA\,{}^{d}/_{dt}\cos(\omega t)$.

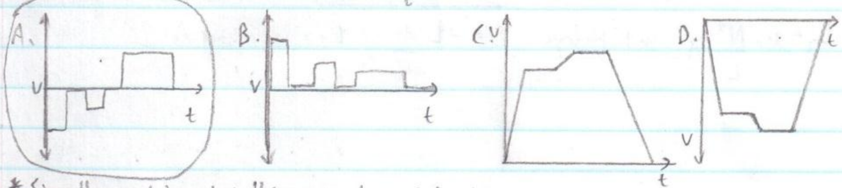
Electrostatics: $V = \int \vec{E} \cdot d\vec{s}$ $E_x = -{}^{dV}/_{dx}$ $E = [\frac{V}{m}]m = V$

Therefore, Faraday's Law can be stated as: $\oint \vec{E} \cdot d\vec{s} = {}^{d\Phi_B}/_{dt}$

Example



Which of the below graphs is correct for voltage in Loop 2 given the information to the left?



* Since the current is constant, there are no changes induced

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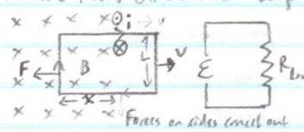
Force and Power Generated in a Loop



$F = iLB$

Hur, Faraday's Law: $\mathcal{E} = -\frac{d}{dt}(\Phi_B) \Rightarrow -\frac{d}{dt}BA$

$(\cos\theta,\ \theta = 0,\ \cos 0 = 1)$

(current direction determined by Lenz's Law) $= -B\frac{dA}{dt} \Rightarrow -B\frac{d}{dt}(xL) = -BL\frac{dx}{dt}$

(right-hand rule determines force's direction)

$\mathcal{E} = -BLv$ (v = velocity as in diagram, not voltage!) $i = \frac{\mathcal{E}}{R_{Loop}} = \frac{BLv}{R_{Loop}}$

$F = iLB \Rightarrow \frac{BLv}{R_{Loop}}LB \Rightarrow \frac{B^2L^2v}{R_{Loop}}$

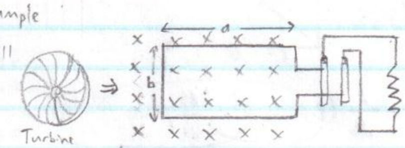
Power: $\boxed{P = Fv}$ F = force, v = velocity $P = mav = m\,{}^{dv}/_{dt}\,v$

$P = \frac{d}{dt}\left(\frac{mv^2}{2}\right) \Rightarrow \frac{dv}{dt}\frac{d}{dv}\left(\frac{mv^2}{2}\right)$ Integrate with respect to velocity instead of time and we get: $P = \frac{dv}{dt}mv$

$\boxed{P = F\cdot v = \frac{B^2L^2v^2}{R_{Loop}}}$ $p = i\mathcal{E} = i^2R$ (p = power dissipated)

Example

30-11

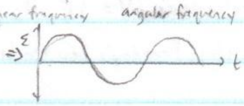


Coil: N turns rotated at frequency f, connected by slip rings to external circuit. Show that $\mathcal{E} = 2\pi f N ab B \sin(2\pi f t)$.

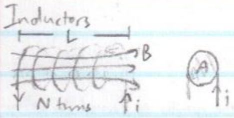
$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(BA\cos\theta)$ $\mathcal{E} = -NBA\frac{d}{dt}(\cos\theta) \Rightarrow \theta = (2\pi f)t = \omega t$

linear frequency, angular frequency

$\mathcal{E} = -NBA\,2\pi f \sin(2\pi f t) \Rightarrow \mathcal{E} = NBab\,2\pi f \sin(2\pi f t) \Rightarrow$



Inductors



$B = \mu_0 \frac{N}{L} i_s = \mu_0 n i_s$ (turns/m) Flux $= \Phi_B = BA = \mu_0 n A i_s$ (constant - depends on geometry)

$\mathcal{E} = -\frac{Nd\Phi_B}{dt} = \frac{Nd}{dt}(\mu_0 n A i_s)$ $\boxed{\mathcal{E} = -N\mu_0 n A i_s \frac{di}{dt}} = -\mu_0 \frac{N^2}{L} A \frac{di}{dt}$

$L_I = \mu_0 \frac{N^2}{L} A$ unit: Henrys $\mathcal{E} = -L_I \frac{di}{dt}$ L_I = inductance (or L)

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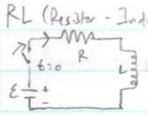
Lecture 16

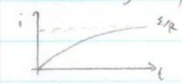
RL (Resistor - Inductor) Circuits



$+\varepsilon - iR - L\frac{di}{dt} = 0 \Rightarrow L\frac{di}{dt} + iR = \varepsilon$ (Guess and check: $i = \varepsilon/R\,(1 - e^{-(R/L)t})$)

At $t = 0$, $i = 0$ At $t = \infty$, $i = \varepsilon/R$ (voltage of resistance)

- current is given by voltage of resistor (inductor = short circuit for DC current; infinite resistance to AC)



- compare with RC circuit $\varepsilon = L\frac{di}{dt} + iR$

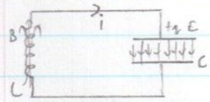
Multiply by i. $\varepsilon i = Li\frac{di}{dt} + i^2R$

εi = power dissipated by battery, i^2R = power dissipated by resistor, $Li\frac{di}{dt}$ = power dissipated by inductor

We obtain: $\frac{dU_B}{dt} = Li\frac{di}{dt} \Rightarrow U_B = \int Li\frac{di}{dt} = \frac{1}{2}Li^2 \Rightarrow U_B = \frac{1}{2}Li^2$
energy in inductor

Chapter 30 - Induction and Inductance

LC Oscillations



$\frac{dU}{dt} = 0$ (constant) $\Rightarrow \frac{dU_B}{dt} + \frac{dU_E}{dt} \Rightarrow \frac{d}{dt}(\frac{1}{2}Li^2) + \frac{d}{dt}(\frac{1}{2}q^2/C) = 0$

$U_B = \frac{1}{2}Li^2$ (L)

$\Rightarrow \frac{di}{dt}\frac{d}{di}(\frac{1}{2}Li^2) + \frac{dq}{dt}\frac{d}{dq}(\frac{1}{2}q^2/C) = 0$

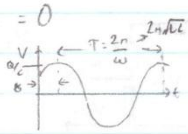
$U_E = \frac{1}{2}q^2/C$ (C)

$\Rightarrow \frac{di}{dt}Li + \frac{dq}{dt}q/C = 0 \Rightarrow L\frac{di}{dt} + \frac{q}{C} = 0$

$U_B + U_E = U$ = constant

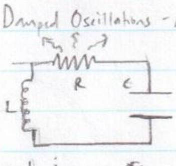
dq/dt $\therefore di/dt = -\frac{1}{LC}q$

$\Rightarrow \boxed{\frac{d^2q}{dt^2} = -\frac{1}{LC}q}$ (simple harmonic motion!) $q = Q\cos(\omega t + \emptyset)$



Q = initial charge on capacitor, $\emptyset$ = phase constant, ω = angular frequency ($1/\sqrt{LC}$)

Damped Oscillations - Adding a Resistor

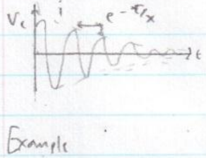


$L\boxed{i}\frac{di}{dt} + \frac{q}{C}\boxed{\frac{dq}{dt}} = -i\boxed{i}R \Rightarrow L\frac{di}{dt} + \frac{q}{C} = -i^2R$

$\Rightarrow L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$ Solve for q: $q = Q \cdot e^{-t/(2L/R)}\cos(\omega' t + \emptyset)$

$\omega' = \sqrt{(1/LC)^2 - (R/2L)^2}$

removes energy from circuit



Example

In the RLC circuit with damped oscillation as shown has a maximum voltage across the capacitor, what is the current through the resistor? A. I_R = maximum B. $I_R = 0$ C. I_R = minimum

* Current = time-rate change of charge (dq/dt); differentiating cos = -sin, but $\omega t + \emptyset = 0$ and $\sin 0 = 0$, meaning $I_R = 0$.

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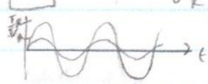
AC Power AC

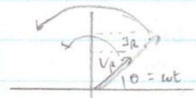
- driving currents with AC; most common form of power (can step up or step down using transformer)

Resistive Load

$V(t) = V_0 \sin(\omega t)$ - apply Ohm's Law for the current

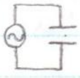
$V_R = I_R R, \; I_R = V_R/R \cdot \sin(\omega t)$

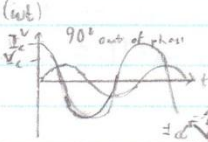




Capacitive Load

$V = V_0 \sin(\omega t)$



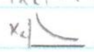


$q = CV_0 \sin \omega t$

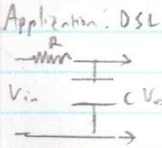
$i = dq/dt = \omega C V_0 \cos \omega t$

current <u>leads</u> voltage (90°)

$X_C = \frac{V}{i} = \frac{V_0 \sin \omega t}{\omega C V_0 \cos \omega t}$ Reactance $X_C = \left(\frac{1}{\omega C}\right) \frac{\sin \omega t}{\sin(\omega t + 90°)}$ $|X_C| = |1/\omega C| = |1/2\pi f C|$



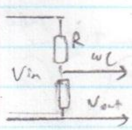
Application: DSL Filter Line



- high frequencies shorted out by capacitor; low frequencies only pass through

speech $f \lesssim 20$ kHz

1/2 power point $\Rightarrow V_{out}/V_{in} = 1/\sqrt{2}$ i.e. $100\,\Omega$, $0.16\,\mu F$



$$\frac{V_{out}}{V_{in}} = \frac{X_C}{\sqrt{X_C^2 + R^2}} \quad (\because \text{not in phase}) \Rightarrow \frac{X_C}{\sqrt{1 + (R/X_C)^2}} \Rightarrow \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Ivan Mar. 8/12

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Lecture 17

Examples

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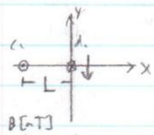
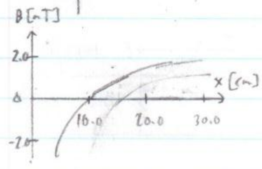


a. $i_1/i_2 = 4.0$ $\quad B_1 = \frac{\mu_0}{2\pi}\frac{i_1}{L+x}$ $\quad B_2 = -\frac{\mu_0}{2\pi}\frac{i_2}{x}$



$$\frac{\mu_0}{2\pi}\left(\frac{i_1}{L+x} - \frac{i_2}{x}\right) = 0 \Rightarrow \frac{i_1}{L+x} = \frac{i_2}{x}$$

$$\frac{i_1}{i_2} = 4 = \frac{L+x}{x} \Rightarrow 4x = L+x \Rightarrow L = 3x$$

$$L = 3x = 3 \cdot 0.1\,m = 0.3\,m$$

b. $\frac{\mu_0}{2\pi}\left(\frac{i_1}{L+x} - \frac{i_2}{x}\right)$ = total field $\Rightarrow \frac{dB}{dx} = \frac{d}{dx}\left[\frac{\mu_0}{2\pi}\left(i_1(L+x)^{-1} - i_2(x)^{-1}\right)\right]$

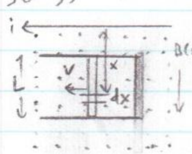
$$= \frac{\mu_0}{2\pi}\left(-i_1(L+x)^{-2} + i_2(x)^{-2}\right) = 0 \Rightarrow \frac{-i_1}{(L+x)^2} + \frac{i_2}{x^2} = 0 \Rightarrow \frac{i_2}{x^2} = \frac{i_1}{(L+x)^2}$$

$$\Rightarrow (L+x)^2 = \frac{i_1}{i_2} = 4x^2 \Rightarrow x^2 + 2Lx + L^2 = 4x^2 \text{ or } 3x^2 - 2Lx - L^2 = 0$$

Use quadratic equation ..., $x = 0.3\,m$ or $30\,cm$

To find if this answer is the maximum or minimum, use the second-differential test and the sign

30-33



$\mathcal{E} = BLV$ (but only if the field is constant, which it is not)

$$d\mathcal{E} = BV\,dx \Rightarrow \int d\mathcal{E} = \int_{x=a}^{x=a+L} Bv\,dx$$

$$= \frac{\mu_0 i v}{2\pi}\int_{x=a}^{x=a+L} dx/x \Rightarrow \frac{\mu_0 i v}{2\pi}\left[\log_e(a+L) - \log_e(a)\right]$$

$L = 10\,cm = 0.1\,m$

$v = 5\,m/s \quad i = 100\,A$

$R_{loop} = 0.4\,\Omega$

$a = 10\,mm = 0.01\,m$

$$\mathcal{E} = \frac{\mu_0 i v}{2\pi}\log_e\left(\frac{a+L}{a}\right)$$

$$= \frac{(4\pi \times 10^{-7})(100)(5)}{2\pi}\log_e\left(\frac{0.01+0.1}{0.01}\right) \Rightarrow \mathcal{E} = 2.4 \times 10^{-4}\,V \text{ or } 240\,\mu V$$

Reactance

Recalling the previous class, the instantaneous reactance is not very useful, we are more interested in the average. But the average of $\sin x$ and $\cos x$ is zero, resulting in an indeterminate figure. So we take the average of the square, which is the midway point between the amplitudes.

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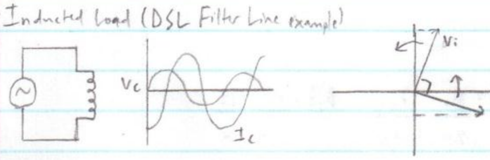
Inducted Load (DSL Filter Line example)

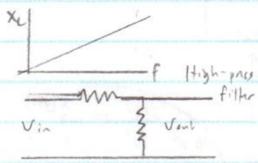


Resistance: $\frac{V}{i} = \frac{V_m \sin\omega t}{-\frac{V_m}{\omega L}\cos\omega t}$

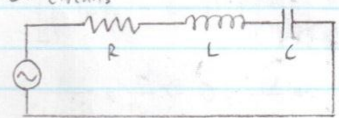
$\langle X_L \rangle = \omega L$

$V = V_m \sin\omega t$, $V_L = L\frac{di}{dt}$, $\therefore \frac{di}{dt} = \frac{V_L}{L} = \frac{V_m \sin\omega t}{L}$

$i = -\frac{V}{\omega L}\cos(\omega t)$ Mnemonic (?): ELI the ICE man (Inductive load / Capacitive load; $X_C = 1/\omega C$, $X_L = \omega L$)
(to remember if current is behind voltage)

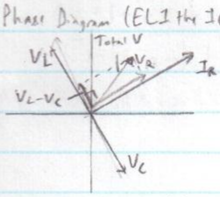


RLC Circuits



(Common variable is current (same at all points here!)
Voltage may change, but current is the same

Phase Diagram (ELI the ICE man!)



$$\mathcal{E} = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$
$$= \sqrt{(iR)^2 + i^2(X_L - X_C)^2}$$
$$= i\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \; *$$

* If $\omega L = \frac{1}{\omega C}$, then $\mathcal{E} = iR$

$\omega^2 = \frac{1}{LC}$ $\omega = \frac{1}{\sqrt{LC}}$

Resonance ($\omega = \frac{1}{\sqrt{LC}}$)

Example

21P (C-31)

$R = 5\Omega$, $C = 20\mu F$, $L = 1.0H$ $\mathcal{E}_m = 30V$

What is the current of resonance (I_m) and the frequency when $I = I_m/2$?

$I_m = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow \frac{30V}{5\Omega}$ $I_m = 6A$ $\frac{I_m}{2} = 3A$ $f = ?$

Resonance: $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow 224$ rad/s (radians per second) (angular frequency) $f = \frac{\omega_0}{2\pi}$ or $\frac{224}{2\pi}$ (linear)

$\frac{I_m}{2} = \frac{\mathcal{E}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$ $\omega = 219$ rad/s Compare with 224 rad/s = narrow peak

Q = quality factor ($Q = \frac{f}{\Delta f}$)

$Q = \frac{224}{2 \cdot 5} \Rightarrow \frac{224}{10}$ $Q = 22.4$

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Notes for March 13th, 2012

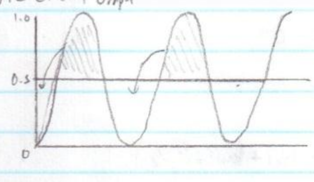
Lecture 18

Power

From DC circuits: $P = i^2 R$ In AC circuits: $P = [I_{max} \sin(\omega t - \phi)]^2 R$

These give power at time t (instantaneous)

AC Circuit Graph



$\langle P \rangle = \frac{I_m^2}{2} R$ ← uniform amplitude of current

AC Motor $\longrightarrow I_{rms} = \frac{I_m}{\sqrt{2}}$

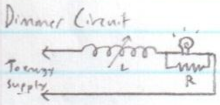
$\langle P \rangle = I_{rms}^2 R$

$I_{rms} = \frac{\mathcal{E}_{rms}}{\sqrt{R_2 + (\omega L - 1/\omega C)^2}}$ $V_{rms} = \frac{V_{max}}{\sqrt{2}}$ $\Rightarrow \langle P \rangle = I_{rms}\mathcal{E}_{rms} \frac{R}{\sqrt{R_2 + (\omega L - 1/\omega C)^2}}$

$\Rightarrow \langle P \rangle = I_{rms}\mathcal{E}_{rms}\cos\phi$ power factor

Example

Dimmer Circuit



$V_{rms} = 120\,V$ 60Hz Lightbulb: 120 V, 1000 W

Calculate L_{max} so that energy dissipation can be varied by a factor of 5

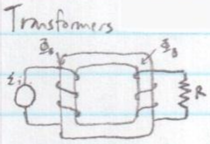
$P_{max} = 1000\,W$ $P_{min} = 200\,W$ $L_{max} = ?$ $L_{min} = 0$ R represents resistance of lightbulb

$R = \frac{V^2}{P} = \frac{(120\,V)^2}{1000\,W} = 14.4\,\Omega$ $L_{max} = \frac{V_{rms}}{R} = \frac{120}{14.4} = 8.33\,A$

$L_{min} = \sqrt{\frac{P}{R}} = \sqrt{\frac{200\,V}{14.4\,\Omega}} = 3.73\,A$ $V_{rms} = \sqrt{V_L^2 + V_R^2} = i\sqrt{(\omega L)^2 + R^2}$

$L = 0.074\,H$ $(\omega L = 2\pi/60)$

Transformers



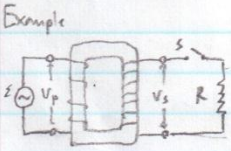
$\mathcal{E}_p = N_p \frac{d\Phi_B}{dt}\Big|_p = V_p$ $\mathcal{E}_s = N_s \frac{d\Phi_B}{dt}\Big|_s = V_s$ p = primary, s = secondary $\frac{V_p}{V_s} = \frac{N_p}{N_s}$

$V_s = V_p (N_s/N_p)$

For an ideal transformer: $P_p = P_s \Rightarrow i_p V_p = i_s V_s$ $\Rightarrow \frac{i_p}{i_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow i_s = \left(\frac{N_p}{N_s}\right) i_p$

Example

Step up current = step down voltage, vice-versa



a. $V_s = V_p (N_s/N_p)$
$= 120\,V_{rms} (10/500)$
$= 2.4\,V_{rms}$

b. $i_s = (N_p/N_s)\,i_p$ $i = V/R$

$(V_p i_p) = V_s i_s)$ $= 2.4/15$

$i_p = (V_s/V_p)\,i_s$ N_s/N_p $= 0.16$

$i_s = (N_s/N_p)^2 V_p = 0.16\,A$

$n_p = 500$ $n_s = 10$ $V_p = 120\,V_{rms}$ $V_s = ?$

a. V_s with switch open (no current) b. i_s, switch closed

TO BE CONTINUED (edits pending)

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Chapter 32 - Maxwell's Equations

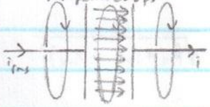
- summarize all characteristics of electrical and magnetic phenomena
- form the laws of predicting existence of E.M. waves → light

Gauss' Law

- electrostatics: $\oint E \cdot dA = q_{enc}/\varepsilon_0$ - magnetism: $\oint B \cdot dA = 0$

Faraday's Law: $\mathcal{E} = \dfrac{-d\Phi_B}{dt} \Rightarrow \oint E \cdot ds = \dfrac{d}{dt}\left[\int B \cdot dA\right]$

Amper's Law: $\oint B \cdot ds = \mu_0 i$ - Maxwell's addition: $\oint B \cdot ds = \mu_0 i + \mu_0 \boxed{\varepsilon_0 \, d\Phi_E/dt}$



- "displacement current": $i_d = \varepsilon_0 \, d\Phi_E/dt$ "displacement current"
- "displacement current" is actually a rate of change, but has similar units

In a vacuum: $\oint E \cdot dA = 0$, $\oint B \cdot dA = 0$, $\oint E \cdot ds = -d\Phi_B/dt$, $\oint B \cdot ds = \mu_0 \varepsilon_0 \, d\Phi_E/dt$

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Notes for March 15th, 2012

Lecture 19

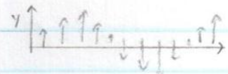
Waves

- can be longitudinal, transverse ; travelling, standing
  - standing : peak of the wave remains in same position

	Transverse	Longitudinal
Travelling	E.M. waves	Sound waves
Standing	Stringed instrument	*Instrument bodies/pipes
	Laser cavity	

- note that water and p-waves are both transverse, longitudinal
- discuss E.M waves today

Travelling Transverse Wave

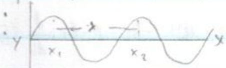


$y(x,t) = y_m \sin(kx - \omega t)$

- due to Fourier's Theorem, trig. ratio selection is important

y_m = amplitude, wave number = $2\pi/\lambda = k$ ω = angular frequency = $2\pi f$

At $t = t_1$:



The Argument of sine

At maximum when $(\ ) = \pi/2$

$\sin(kx_1 - \omega t_1) = \sin(\pi/2) = 1$

$\sin(kx_2 - \omega t_1) = \sin(\pi/2) = 1$

$kx_1 - \omega t_1 = \pi/2$

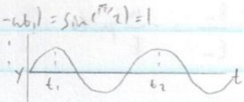
$kx_2 - \omega t_1 = 2\pi + \pi/2$

$k(x_1 - x_2) = 2\pi$

Connection between k and λ

$$\boxed{(x_1 - x_2) = \frac{2\pi}{k} = \lambda}$$

At $x = x_1$:

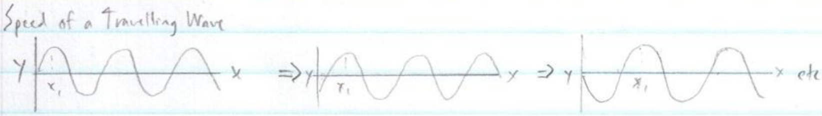


$\sin(kx_1 - \omega t_1) = \sin \pi/2$

$\sin(kx_2 - \omega t_1) = \sin(2\pi + \pi/2)$

$\omega(t_2 - t_1) = 2\pi$, $\boxed{(t_2 - t_1) = \frac{2\pi}{\omega} = T}$

Speed of a Travelling Wave



At $t = 0$ At $t = \pi/2$ At $t = \pi$

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T} = V \Rightarrow V = \frac{\lambda}{T} = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} \Rightarrow \boxed{V = \frac{\omega}{k}} \quad \sin(\omega t \overset{*}{-} kx)$$

* - $\ominus$ = velocity in $+x$ direction; $\oplus$ = velocity in $-x$ direction $(kx - \omega t)$ = ∿ $(\omega t - kx)$ = ∿

More importantly: $T = \frac{2\pi}{\omega}$, $\lambda = \frac{2\pi}{k}$, $V = \frac{\omega}{k}$

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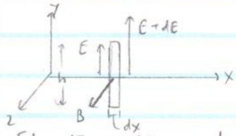
Relation Between Time Change of B and E (Faraday)



$$\oint E \cdot ds = -\frac{d\Phi_B}{dt}$$ (electric field only dependent on x, not y)

∃ magnetic field varies with time

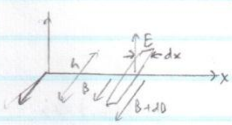
$$Eh - (E + dE)h = -\frac{d}{dt} Bh\,dx \Rightarrow -h\,dx \frac{dB}{dt} = -dE\,h \Rightarrow \underbrace{\frac{dE}{dx}}_{\text{spatial derivative}} = \underbrace{\frac{dB}{dt}}_{\text{time derivative}}$$

Application to Existing Equations: $E_y = E_m \sin(\overset{\text{direction of travelling wave}}{k}x - \omega t)$, $\overset{\perp \text{ to } E_y \text{ and } x}{B_z} = B_m \sin(kx - \omega t)$ ° m subscript = max.

$$\frac{dE}{dx} \Rightarrow kE_m \cos(kx - \omega k) \overset{\perp \text{ to location of motion}}{=} -\omega B_m \cos(kx - \omega t) \Rightarrow \boxed{\frac{E_m}{B_m} = -\frac{\omega}{k} = -c}$$

Ratio of electric field : magnetic field = c ($c \neq$ capacitance!)

Relation Between Time Rate of Change of E and B (Faraday's Law) (Maxwell)



$$\oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \Rightarrow (B + dB)h - Bh = \mu_0 \varepsilon_0 \frac{d}{dt}(Eh\,dx)$$

$$h\,dB = \mu_0 \varepsilon_0 h\,dx \frac{dE}{dt} \Rightarrow \underbrace{\frac{dB}{dx}}_{\text{spatial derivative}} = \underbrace{\frac{dE}{dt}}_{\text{time derivative}}$$

Application: $kB_m \cos(kx - \omega t) = -\overset{\mu_0 \varepsilon_0}{\omega} E_m \cos(kx - \omega t) \Rightarrow \frac{B_m}{E_m} = -\frac{\omega}{k} \mu_0 \varepsilon_0 = \frac{k}{\omega}$

$$\Rightarrow \left(\frac{\omega^2}{k^2}\right) = \frac{1}{\mu_0 \varepsilon_0} = \underbrace{c^2}_{\text{speed of light}} \qquad \boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

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Notes for March 20th, 2012

Lecture 20

Energy Transfer

Rate of energy transfer per unit area: Poynting vector $\vec{S} = (1/\mu_0)\,\vec{E} \times \vec{B}$

- this value is instantaneous; units: $\frac{Power}{Area} = \frac{W}{m^2}$ $|S| = \frac{1}{\mu_0} EB$ $B = \frac{E}{c} \Rightarrow S = \frac{1}{c\mu_0} E^2$

- time-average: $\langle |S| \rangle = \frac{1}{c\mu_0}\left[E_m \sin(kx - \omega t)\right]^2 \Rightarrow \frac{E_m^2}{c\mu_0 2} \Rightarrow \frac{E_{rms}}{c\mu_0}$ $\left(E_{rms} = \frac{E_m}{\sqrt{2}}\right)$

- intensity I: $(E^2_{rms})/(c\mu_0)$

Example 33-87

$P = 180\ kW$

$A = 0.22\ m^2$

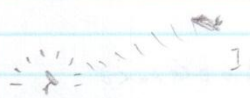
$d = 90\ km$

Intensity = Power/Area

$= P/2\pi r^2$ (hemisphere)

$= \frac{180 \times 10^3\ W}{2\pi(90 \times 10^3\ m)^2}$

$I = 3 \times 10^{-6}\ W/m^2$



$P = IA$

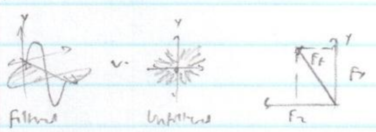
$= (3 \times 10^{-6})(0.22)$

$= 0.66\ \mu W$

$I = \frac{(0.66 \times 10^{-6})W}{2\pi(90 \times 10^3\ m)^2}$

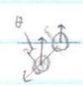
$I = 1.5 \times 10^{-17}\ W/m^2$

Polarization



Filtering light: $I = 0.5\, I_0$

Polarizing light:



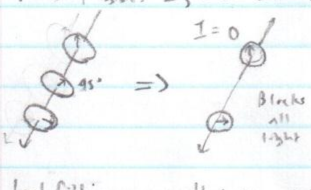
Polarized light through filter: $I = I_0 \cos^2\theta$

- θ = angle between incoming light and filter

Examples

$I_1 = 0.5\, I_0$, $I_2 = I_1 \cos^2\theta \Rightarrow I_2 = I_1 \cos 100°$, $I_3 = I_2 \cos^2(100°)$ [$\theta_1 - (-\theta_2)$]

Full expression: $I_3 = 0.5\, I_0 \cos^2(100°)\cos^2(100°)$



The left-hand 3-filter system yields an intensity of:

A. $I = 0$ B. $I \neq 0$

* as light passes through 45° filter, component of vertically-oscillating light is reoriented and its component goes through the last filter, meaning that some intensity is measured in the left-hand system! (use $\cos^2\theta$...)

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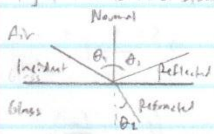
Chapter 34 - Geometrical Optics

- light travels in a straight line macroscopically



- Fermat's Last Theorem proves that incident angle = reflected angle
- refractive index = c/v (light in vacuum/in medium)
- by this logic, refractive index $n > 1$

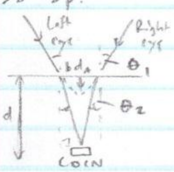
$c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$, $v = \dfrac{1}{\sqrt{\varepsilon\varepsilon_0 \mu\mu_0}}$

- Fermat's Last Theorem minimizes ~~distance~~ light travels

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Snell's Law)

Example

38 - 8p.



Apparent depth of coin = d_A?

$\sin\theta_2 = \dfrac{b}{\sqrt{b^2 + d^2}}$ $\sin\theta_1 = \dfrac{b}{\sqrt{b^2 + d_A^2}}$

$\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{\sqrt{b^2 + d^2}}{\sqrt{b^2 + d_A^2}} = \dfrac{n_1}{n_2}$ As $\theta \to 0^\circ$, $b \to 0 \ll d_A < d$

$\dfrac{d}{d_a} = \dfrac{n_1}{n_2}$ $d_A = \dfrac{n_2}{n_1} d$

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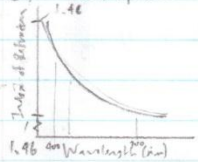
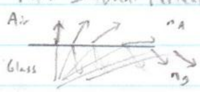
Lecture 21

Chromatic Dispersion



- refractive index = function of wavelength

Total Internal Reflection

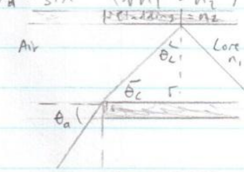


- $\theta_{critical} = \sin^{-1}(n_a/n_1)$
- recall critical angle: $n_g \sin\theta_c = 1$

$\theta_c = \sin^{-1}(1/n_g)$

Fibre-Optic Cables

$\theta_a = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$ - uses critical angles to transport light/frequency



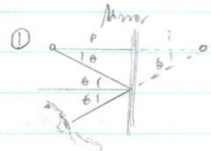
$\bar{\theta}_c = 90° - \theta_c$

$n_1 \sin\theta_c = n_2 \Rightarrow \sin\theta_c = \boxed{n_2/n_1 = \cos\bar{\theta}_c}$

$\cos(90 - \bar{\theta}_c) = \sin\theta_c$ (properties of sines and cosines)

$1.0 \sin\theta_a = n_1 \sin\bar{\theta}_c$ ($\sin^2\theta + \cos^2\theta = 1$, $\sin\theta = \sqrt{1-\cos^2\theta}$)

$\sin\theta_a = n_1\sqrt{1-\cos^2\bar{\theta}_c} \Rightarrow n_1\sqrt{1-\left(\frac{n_2}{n_1}\right)^2} \Rightarrow \sqrt{n_1^2 - n_2^2}$

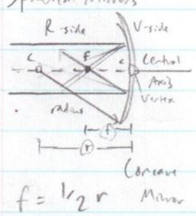


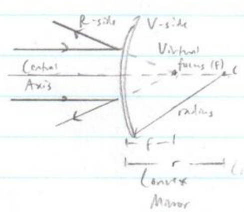
Chapter 34 - Images

- brain conceives images; makes straight-line extrapolation to location of object
- virtual image: no light rays at the image, as in a mirror
- real image: light rays do reach the object (all the way to the image, can be formed on solid objects)
- right image: right way up; inverted image: inverted (images)

① $p = i$ (p = object distance, i = image distance)

Spherical Mirrors

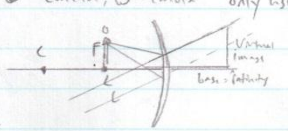


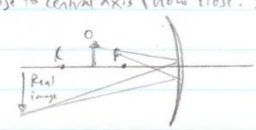


- we make a few assumptions, one of which is:
  - all rays pass through one point with curved mirrors (there are different positions where light can intersect)
- ideal shape for lens is parabolic, not spherical
- see automotive headlight lenses/buckets

$f = \frac{1}{2}r$

⊕ = concave, ⊖ = convex • only use rays very close to central axis (how close? $\sin\theta \doteq \tan\theta$, $< 10°$)





When the object is at the focal point, virtual image is an infinite distance away (virtual, right)
- size of image is very large (∞)

When the object is between the focal point and F, the real image is inverted and magnified (beyond C, object not magnified)
magnification: $1/p$ ($1/p + 1/i = 1/f$) (continued on next page)

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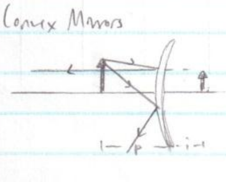
Example (p, f given)

1.

p	f	r	i	m	R/V	I/NI	Side
18cm	12cm	24cm	36cm	(inverted) −2	Real	Inverted	Object

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$ (finding i)

Convex Mirrors



p	f	r	i	m	R/V	I/NI	Side
8cm	−10cm	−20cm	$-40/9$ (4.44)		Virtual	Right	Virtual

Case	Image Type	Orientation	Magnification
$p < f$	Virtual	Non-inverted	Enlarged
$p = f$	Real	Non-inverted	∞
$f < p < r$	Real	Inverted	Enlarged
$p > r$	Real	Inverted	Reduced

Quantity		When positive	When negative	(Mirrors and Lenses)
Object distance	p	Usually	Almost never	
Image	i	Real image	Virtual image	
Focal length	f	Converging	Diverging	
Magnification	m	Non-inverted	Inverted image	

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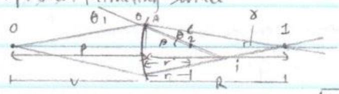
Lecture 22

Spherical Refracting Surface



$n_1 \sin\theta_1 = n_2 \sin\theta_2$ ($\theta_1, \theta_2 < 10°$: $n_1\theta_1 \approx n_2\theta_2$)

$\theta_1 = \alpha + \beta$ $\quad \beta = \theta_2 + \gamma$

$n_1(\alpha + \beta) = n_2(\beta + \gamma) \Rightarrow \boxed{n_1/p + n_2/i = (n_2 - n_1)/r}$ (expressing α, β, γ in tan-ratios)

Example

$n_1 = 1.0$ Glass lens

$n_2 = 1.5$ $\quad i = ?$

$p = 10$ cm

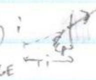
$r = +30$ [Convex]

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{(n_2 - n_1)}{r} \Rightarrow \frac{1}{10} + \frac{1.5}{i} = \frac{0.5}{30} \Rightarrow \frac{1-6}{60}$$

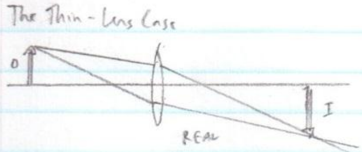
$$\Rightarrow \frac{1.5}{i} = -\frac{5}{60} \Rightarrow i = -12(1.5)$$

$i = -18$ cm

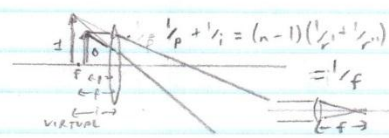
VIRTUAL IMAGE



The Thin-Lens Case



OR



$1/p + 1/i = (n-1)(1/r' + 1/r'')$

$= 1/f$

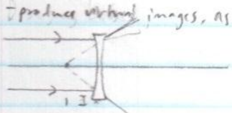
Image Characteristics of a Converging Lens (Same as Concave Mirrors!)

When $p < f$: Virtual, non-inverted, magnified $\quad f < p < c$: Real, inverted, magnified

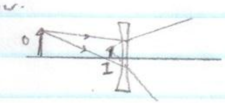
$p = f$: Real, image at infinity $\quad p > 2f$: Real, inverted, reduced

Diverging Lenses

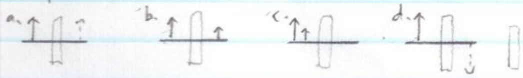
- produce virtual images, as seen below:



OR



Example



= mirror: plane, concave (converging), convex (diverging)
lens: converging, diverging

a. Plane mirror, b. Convex mirror c. Diverging lens d. Converging lens

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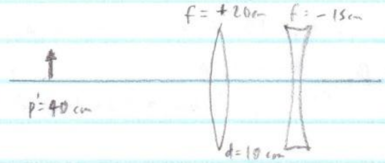
Dual-Lens Case (Example 66 p)



Variables relating to convex lens = ', concave = "

1. $\frac{1}{p'} + \frac{1}{i'} = \frac{1}{f'}$

   $\frac{1}{40} + \frac{1}{i'} = \frac{1}{20}$

   $\frac{1}{i'} = \frac{(2-1)}{40} = \frac{1}{40}$ $\quad i' = +40\text{ cm}$

2. $\frac{1}{p''} + \frac{1}{i''} = \frac{1}{f''}$ (note that $p'' = i' - d$ or $40 - 10 = 30$)

   $\frac{1}{-30} + \frac{1}{i''} = \frac{1}{-15} \Rightarrow \frac{1}{i''} = \frac{(-2+1)}{30} \Rightarrow i'' = -30\text{ cm}$

Result

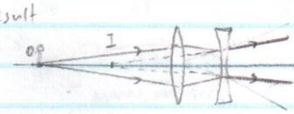


Image is located 20 cm in front of lens

This configuration used for variable-focus instruments (zoom lens)

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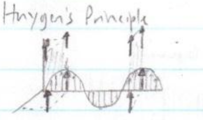
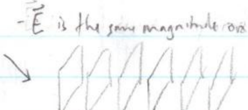
Lecture 23

Huygen's Principle



locus of points with equal phase

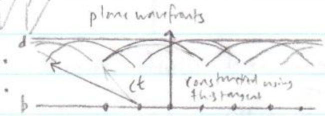
- $\vec{E}$ is the same magnitude on all points on the plane (same phase)



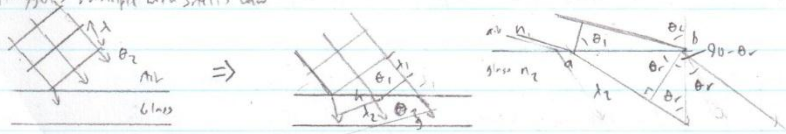
$E_y = E_m \sin(kx - \omega t)$

Only dependent on x

Huygen's construction of other wavefronts



Huygen's Principle with Snell's Law



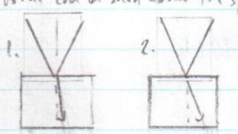
$\sin\theta_1 = \frac{\lambda_1}{ab}$ $\quad \sin\theta_r = \frac{\lambda_2}{ab}$ $\quad ab = \frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_r}$ $\quad \lambda = \lambda_1 n_1 = \lambda_2 n_2$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\Rightarrow \frac{\lambda}{n_1 \sin\theta_1} = \frac{\lambda}{n_2 \sin\theta_2} \Longrightarrow$ Snell's Law $(n_1 \sin\theta_1 = n_2 \sin\theta_2)$

Example

What can be said about the speed of light in each block?



a. The speed of light is the same

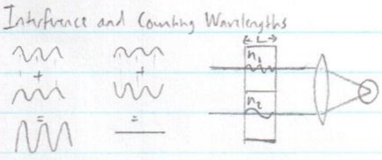
b. The speed is greater in block 1

c. The speed is greater in block 2

d. Not enough information is given to answer

* larger bending angle = slower speed of light in block; more of this in 1

Interference and Counting Wavelengths



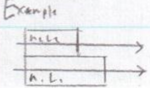
In a vacuum: $f = \frac{c}{\lambda}$ In a medium: $f = \frac{v}{\lambda_1}$ $\quad \lambda_1 < \lambda$ $\quad v < c$

$\therefore \frac{c}{\lambda} = \frac{v}{\lambda_1}, \quad \lambda_1 = \frac{v}{c}\lambda = \frac{\lambda}{n_1}$

In medium 1: $N_1 = \frac{L}{\lambda_1} = \frac{n_1 L}{\lambda}$ $\qquad$ In medium 2: $N_2 = \frac{L}{\lambda_2} = \frac{n_2 L}{\lambda}$ $\qquad (N_2 - N_1) = \frac{L}{\lambda}(n_2 - n_1)$

If $N_2 - N_1$ = whole number, $\Rightarrow$ in phase (bright spot) If $N_2 - N_1 = x.5$, $\Rightarrow$ out of phase (dark spot)

Example



$N_1 = \frac{L_1 n_1}{\lambda}$ $\qquad (N_2 - N_1) = \frac{1}{\lambda}(L_2 n_2 - L_1 n_1)$

$N_2 = \frac{L_2 n_2}{\lambda}$ $\qquad = \frac{1}{0.6\,\mu m}[(3.5 \times 1.6) - (4.0 \times 1.4)] = 0$

No difference in wavelength; in phase, bright spot

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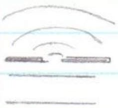
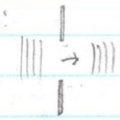
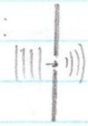
Diffraction



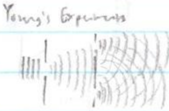
Large aperture:



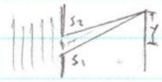
Small aperture:



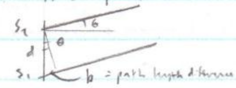
Young's Experiment



In Young's experiments, waves overlapping = points of constructive interference



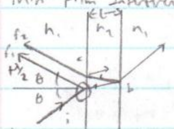
$\Rightarrow$



$$\sin\theta = \frac{S_1 b}{d} \qquad \frac{S_1 b}{\lambda} = \frac{d\sin\theta}{\lambda} = N$$

$d\sin\theta = N\lambda$ If N = whole number $\Rightarrow$ bright spot N = anything else $\Rightarrow$ dark spot

Thin-Film Interference



Path difference $a \to b \to c$

For thin-film:

Suppose $\theta \to 0°$; then $a \to b \to c = \frac{2L}{\lambda_2} = N = \frac{2Ln_2}{\lambda}$ $\quad \frac{2Ln_2}{\lambda} = N + \frac{1}{2}$ (constructive)

But the formula is ineffective for thin films!

- $\lambda/2$ wavelength shift occurs on reflection in a medium of low refractive index when reflected off a material of high refractive index (only occurs at front face) * see fixed-end string wave demonstration *

Example

A soap film of $n = 1.35$ reflects mostly red light (682 nm). What is the film's minimum thickness?

$$N + \frac{1}{2} = \frac{2Ln_2}{\lambda} = \frac{2L(1.35)}{0.682\,\mu m} \Rightarrow \frac{1}{2} = \frac{2L\,1.35}{0.682} \qquad L \doteq 126\text{ nm}$$