Math 102 Spring 2015 (A01-A03) Review

Note: this review sheet is intended to *support* your preparations for the final exam, not replace them! You must do practice problems from the textbook/MML, review midterms and class notes, in addition to ensuring your understanding of each topic on this paper.

1. **Limits**
   - understand the concept of a limit - this will help you apply the formulas and interpret questions
   - compute $\lim\limits_{x \to a}$ of sums, differences, powers, multiples, and compounded functions
   - compute $\lim_{x\to\infty}$ of functions (including rational functions)
   - know and use the **limit definition of the derivative**

     $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

     ** Note: you may have seen slightly different, but equivalent, formulas for this in your class
   - understand the concept of continuity and how this relates to limits

2. **Derivatives**
   - understand that the derivative is also interpreted as a rate of change, or as the slope of the tangent line
   - understand how the graph of a function is related to the graph of the *derivative* of that function
   - know and use the limit definition of the derivative (above)
   - know and use the rules for finding the limit of sums and differences of functions, and constant multiples of functions
   - know and use the power rule, product rule, and quotient rule for derivative
   - know and use the rules for finding derivatives of exponential and logarithmic functions

- be able to notice the differences between similar looking functions such as the following, and apply the derivative rules appropriately

$$f_1(x) = e^x; \quad f_2(x) = a^x \quad f_3(x) = x^a$$

- know and be able to use the chain rule to compute the derivative of compound functions. These include (but are not limited to) expression such as

$$e^{g(x)}; \quad [f(x)]^{10}; \quad \ln[h(x)]$$

- know and use the correct procedures for sketching the curve of a function
  - identify domain, $x-$ and $y-$intercepts
  - identify increasing/decreasing intervals, and critical points using the first derivative
    * the *first derivative test* will tell you where the relative minima/maxima are
  - identify concave up/down intervals, and inflection points using the second derivative
    * the *second derivative test* will also tell you where the relative minima/maxima are

3. **Anti-derivatives and Integrals**

- and *anti-derivative* of a function is another function whose derivative is your original function.
  - e.g. x^3 is an anti-derivative of $2x^2$ because the derivative of x^3 is $2x^2$
  - notice that $x^3 + 4$ and $x^3 + 12/75$ are also anti-derivatives of $2x^2$
  - the family of antiderivatives of $2x^2$ is $x^3 + c$, where c is a constant (c may be 0)
- the *indefinite integral* is another way of referring to the family of antiderivatives of a function:

$$\int f(x)dx = F(x) + c; \quad \text{where f is the derivative of F}$$

- the *definite integral* has numbers (limits) at the top and bottom of the integral sign:

$$\int_a^b f(x)dx$$

  - we use the Fundamental Theorem of Calculus to compute (get a value for) the indefinite integral:

$$\int_a^b f(x)dx = F(a) - F(b); \quad \text{F is anti-derivative of f}$$

- the *area under a graph* and the *area between two curves* are both applications of the definite integral
  - for basic shapes, we may determine the are under a curve by counting the squares in graph paper, or using formulas from geometry
  - if f is a positive function, the area between f and the $x-$axis, on the interval $a \leq x \leq b$ is

$$\int_a^b f(x)dx$$

  - if f and g are functions, and $f \geq g$, the area *between* f and g on the interval $a \leq x \leq b$ is

$$\int_a^b f(x)-g(x)dx; \text{ this is also equal to } \int_a^b f(x)dx - \int_a^b g(x)dx$$

4. **Averages and Similarly worded calculations**

- read questions carefully - be sure you understand what is being asked before you apply a formula!
- instantaneous rate of change of $f(x)$

$$f'(x)$$

- average rate of change of $f(x)$

$$\frac{f(b) - f(a)}{b - a}$$

- average of a function $f(x)$

$$\frac{F(b) - F(a)}{b - a}$$

where $F(x)$ is an antiderivative of the function

- total change in a function $f(x)$ is the integral of the derivative of that function

$$\int_a^b f'(x)dx$$

5. **Word Problems**

- read questions carefully - be sure you understand what is being asked before you apply a method or formula!
- Maximize/minimize Problems - there will be multiple variables. You need to identify one equation for the quantity to be maximized/minimized (e.g. cost, area, volume) and a second equation that will relate two of the variables with a number (e.g. volume = 1000).
- Related Rates Problems - there will be multiple variables and rates of change. You will need to identify one equation relating the variables, and then use *implicit differentiation* (with respect to time) to write another equation that contains your unknown rate.
- Other word problems containing derivatives and anti-derivatives: always keep in mind the definitions and formulas we have covered in this class when you are trying to figure out which to apply. For example, if we have a cost function $C(x)$ we can compute this such as the average rate of change in cost, the instantaneous rate of change in cost, the marginal cost, the total change in cost over an interval, the average cost over an interval, and the cost of making a particular number of items. Think of these not as a huge list of calculations you need to memorize for each business, biology, and physics application you saw this term, but as a much smaller list of straightforward calculations using derivatives and anti-derivatives.
- Read. Questions. Carefully.