UNIVERSITY OF VICTORIA
MIDTERM EXAMINATIONS Spring 2015
MATHEMATICS 102 A01-A03
Calculus for Students in the Social and Biological Sciences
Version A

Name: SOLUTION Student No.:______________

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TO BE ANSWERED ON THE PAPER and the BUBBLE SHEET Duration: 2 hours

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS QUESTION PAPER HAS 19 QUESTIONS ON 10 PAGES, PLUS THIS COVER PAGE.

Problem	Marks	Score
Multiple Choice	30	
#16	4	
#17	5	
#18	6	
#19	5	
TOTAL	50	

Multiple Choice

Choose the answer that is closest to yours (you may or may not find the exact answer). If your **unrounded** answer is exactly in the middle of two choices, round up.

[2] 1. Given the function $f(x) = x^5 + 2x^2 - 12x + 1$, compute $f'(1)$.

(A) −9; (B) −8; (C) −3; (D) −2; (E) 0; (F) 1; (G) 2; (H) 3; (I) 4; (J) Does not exist

$f'(x) = 5x^4 + 4x - 12$

$f'(1) = 5 + 4 - 12$

$= -3$

$\boxed{C}$

[2] 2. Find the derivative of $g(x) = \sqrt{x}\left(x - \frac{1}{x}\right)$, when $x = 1$.

(A) −2; (B) −1.5; (C) −1; (D) −0.5; (E) 0; (F) 0.5; (G) 1; (H) 1.5; (I) 2; (J) Does not exist.

$g(x) = x^{1/2}(x - x^{-1})$

$g'(x) = \frac{1}{2}x^{-1/2}(x - x^{-1}) + x^{1/2}(1 - (-x^{-2}))$

$= \frac{1}{2\sqrt{x}}\left(x - \frac{1}{x}\right) + \sqrt{x}\left(1 + \frac{1}{x^2}\right)$

$g'(1) = \frac{1}{2}(1 - 1) + 1(1 + 1)$

$= 0 + 2 = 2.$

$\boxed{I}$

[2] 3. Find the derivative of $h(x) = \dfrac{x^2 + 1}{x - 2}$, when $x = 1$.

(A) −4; (B) −3; (C) −2; (D) −1; (E) 0; (F) 1; (G) 2; (H) 3; (I) 4; (J) Does not exist

$h'(x) = \dfrac{(2x)(x-2) - (x^2+1)(1)}{(x-2)^2}$

$h'(1) = \dfrac{(2)(1-2) - (1^2+1)(1)}{(1-2)^2}$

$= \dfrac{2(-1) - (2)(1)}{(-1)^2} = \dfrac{-4}{1} = -4.$

$\boxed{A}$

Questions 4, 5 and 6 refer to the following setup:

A company finds that the cost (in dollars) of producing x items is given by the function

$$C(x) = \frac{x^2}{3x+5}.$$

[2] 4. Find the the marginal cost when 10 units have been produced.

(A) −2.9; (B) −0.7; (C) −0.3; $\boxed{\text{(D) 0.3;}}$ (E) 0.7; (F) 2.9;
(G) 11.4; (H) 25.7; (I) 28.6; (J) Does not exist

$$C'(x) = \frac{2x(3x+5) - x^2(3)}{(3x+5)^2}$$

$$= \frac{6x^2+10x-3x^2}{(3x+5)^2}$$

$$= \frac{3x^2+10x}{(3x+5)^2}$$

$$C'(10) = \frac{3(10)^2+10(10)}{(3(10)+5)^2}$$

$$= \frac{300+100}{(35)^2}$$

$$\approx 0.326531$$

$\boxed{D}$

[2] 5. Find the average cost when 10 units have been produced.

(A) −2.9; (B) −0.33; (C) −0.28; $\boxed{\text{(D) 0.28;}}$ (E) 0.33; (F) 2.9;
(G) 11; (H) 26; (I) 29; (J) Does not exist

$$\bar{C}(x) = \frac{x^2}{3x+5}\left(\frac{1}{x}\right) = \frac{x}{3x+5}$$

$\boxed{D}$

$$\bar{C}(10) = \frac{10}{3(10)+5} = \frac{10}{35} \approx 0.285714.$$

[2] 6. Find the marginal average cost when 10 units have been produced.

(A) −2.9; (B) −0.33; (C) −0.28; (D) −0.14; $\boxed{\text{(E) 0;}}$ (F) 0.14;
(G) 0.28; (H) 0.33; (I) 2.9; (J) Does not exist

$$\bar{C}'(x) = \frac{(1)(3x+5) - x(3)}{(3x+5)^2}$$

$$= \frac{3x+5-3x}{(3x+5)^2} = \frac{5}{(3x+5)^2}$$

$\boxed{E}$

$$\bar{C}'(10) = \frac{5}{(35)^2} \approx 0.00408$$

[2] 7. Find the derivative of $f(x) = (x^2+1)^4$, when $x = 1$.

(A) 0; (B) 0.5; (C) 1; (D) 2; (E) 4; (F) 8; (G) 16; (H) 32; (I) 64; (J) Does not exist.

$f'(x) = 4(x^2+1)^3 \cdot (2x)$

$f'(1) = 4(1+1)^3 \cdot 2(1)$

$= 8(2)^3$

$= 64.$

$\boxed{I}$

[2] 8. Find the derivative of $g(x) = \sqrt{\frac{x}{x+1}}$, when $x = 1$.

(A) -0.71; (B) -0.35; (C) -0.18; (D) -0.09; (E) 0; (F) 0.09;
(G) 0.18; (H) 0.35; (I) 0.71; (J) Does not exist

$g(x) = \left(\frac{x}{x+1}\right)^{1/2}$

$g'(x) = \frac{1}{2}\left(\frac{x}{x+1}\right)^{-1/2}\left[\frac{1(\cancel{x}+1) - \cancel{x}(1)}{(x+1)^2}\right]$

$= \frac{1}{2}\sqrt{\frac{x+1}{x}}\left[\frac{1}{(x+1)^2}\right]$

$g'(1) = \frac{1}{2}\sqrt{\frac{2}{1}}\left(\frac{1}{(2)^2}\right)$

≈ 0.17678

$\boxed{G}$

[2] 9. Find the derivative of $h(x) = \frac{1}{1+e^{-x}}$, when $x = 1$.

(A) -1; (B) -0.73; (C) -0.27; (D) -0.19; (E) 0; (F) 0.19;
(G) 0.27; (H) 0.73; (I) 1; (J) Does not exist

$h(x) = (1+e^{-x})^{-1}$

$h'(x) = (-1)(1+e^{-x})^{-2}(-e^{-x})$

$= \frac{e^{-x}}{(1+e^{-x})^2}.$

$h'(1) = \frac{1/e}{(1+1/e)^2} \approx 0.196619$

$\boxed{F}$

[2] 10. Find the derivative of $f(x) = e^{x+2}e^{x-1}$, when $x = 1$.

(A) -40; (B) -20; (C) -3; (D) -1; (E) 0; (F) 1;
(G) 3; (H) 20; (I) 40; (J) Does not exist

$f(x) = e^{(x+2)+(x-1)} = e^{2x+1}$

$f'(x) = (e^{2x+1})(2)$

$f'(1) = e^3(2)$

≈ 40.17

$\boxed{I}$

[2] 11. Find the derivative of $g(x) = \ln(2x)$, when $x = 1$.

(A) -2; (B) -1; (C) -0.7; (D) -0.5; (E) 0; (F) 0.5; (G) 0.7; (H) 1; (I) 2; (J) Does not exist.

$g'(x) = \left(\frac{1}{2x}\right)(2)$

$g'(1) = \left(\frac{1}{2}\right)2$

$= 1$

$\boxed{H}$

[2] 12. Find the derivative of $h(x) = \log_2 5$, when $x = 1$.

$\boxed{\text{(A) 0;}}$ (B) 0.1; (C) 0.2; (D) 0.3; (E) 0.5; (F) 1; (G) 2; (H) 5; (I) 10; (J) Does not exist.

$h(x) = \log_2 5$ is a constant.

$h'(x) = 0$.

$h'(1) = 0$.

$\boxed{A}$

[2] 13. Find the derivative of $p(x) = \ln(x^2+1)^2$, when $x = 1$.

(A) 0; (B) 0.25; (C) 0.5; (D) 0.7; (E) 1; (F) 1.4; $\boxed{\text{(G) 2;}}$ (H) 4; (I) 8; (J) Does not exist.

$p(x) = 2\ln(x^2+1)$.

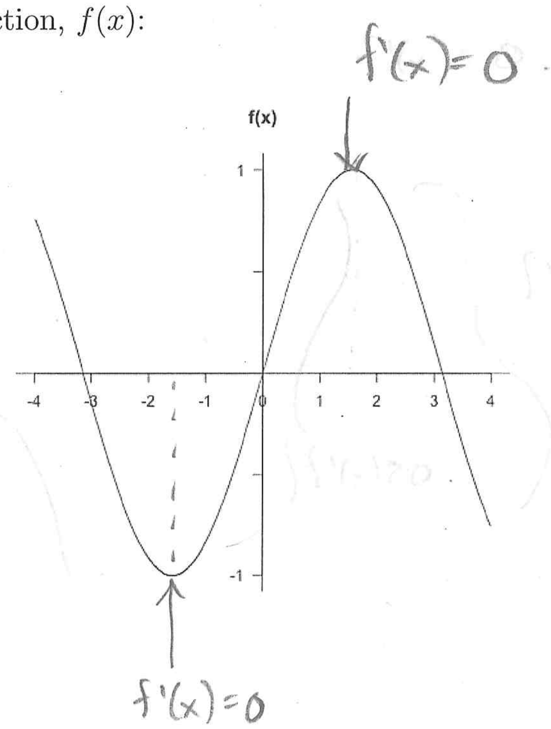
$p'(x) = 2\left(\frac{1}{x^2+1}\right)(2x)$

$= \frac{4x}{x^2+1}$

$p'(1) = \frac{4}{2} = 2$.

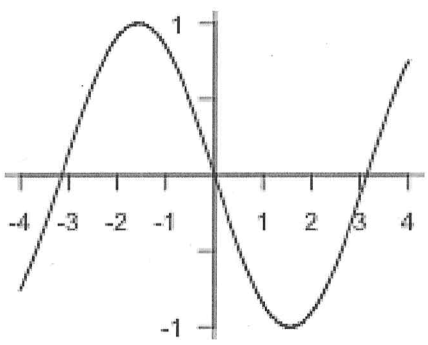
$\boxed{G}$

[2] 14. Consider the following function, $f(x)$:

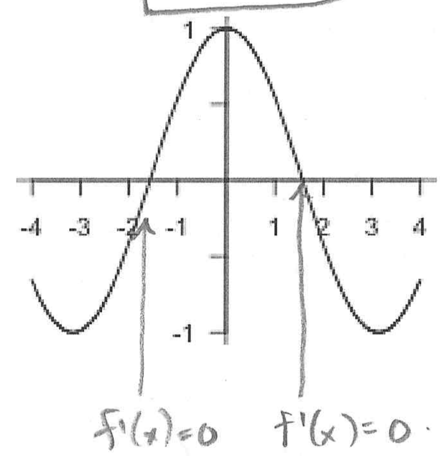


Which of the following is the graph of the *derivative* of $f(x)$?

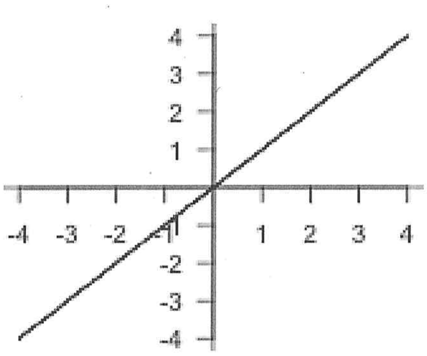
(A)



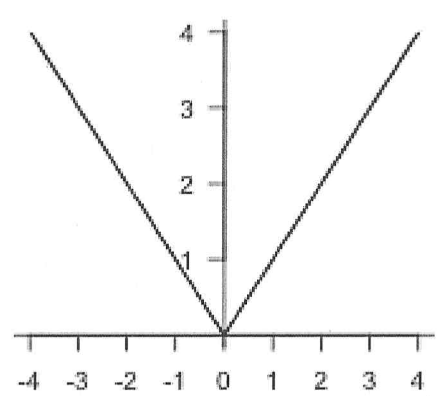
(B)



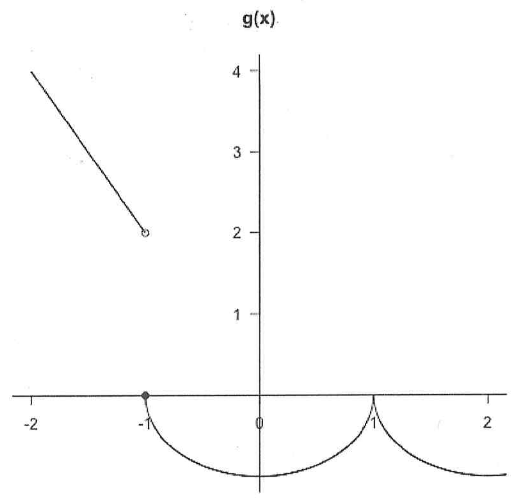
(C)



(D)



[2] 15. Consider the following function, $g(x)$:



$f'(x)$ DNE at $x = -1$ & $x = 1$

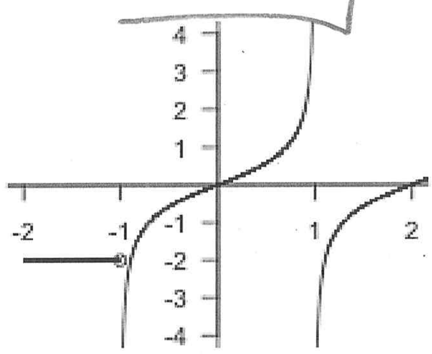
$f'(x) < 0$ for $x < -1$

$f'(x) = 0$ at $x = 0, x = 2$.

$f'(x) \to -\infty$ as $x \to -1^-$

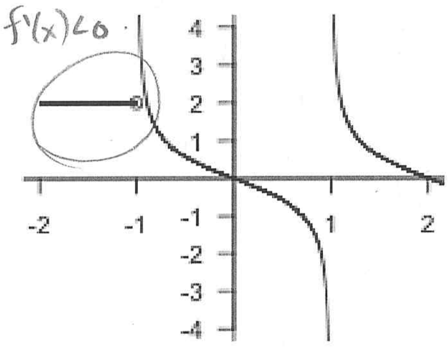
Which of the following is the graph of the *derivative* of $g(x)$?

(A)

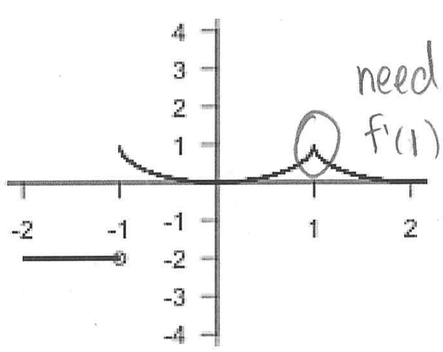


need $f'(x) < 0$

(B)

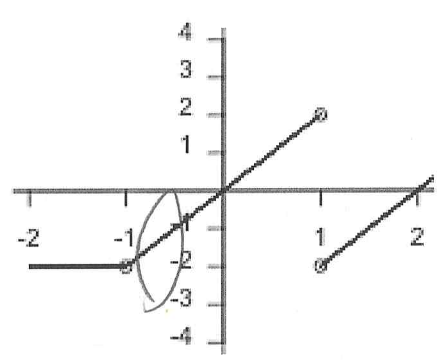


(C)



need $f'(1)$ DNE

(D)

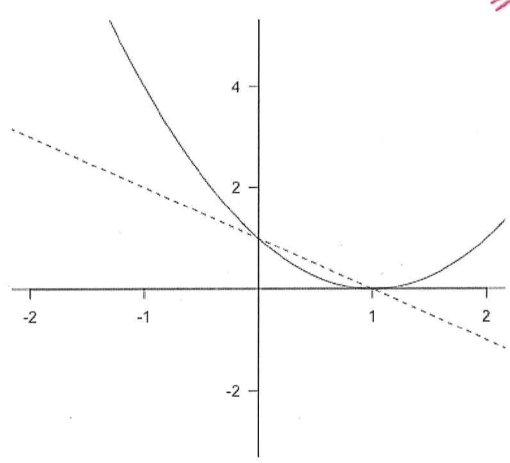


need $\lim_{x \to 1^-} f'(x) = -\infty$.

Long Answer Problems

You must clearly justify your answers. Correct answers without supporting work will be given a mark of zero.

[4] 16. Is the dashed (broken) line the graph of the derivative of the solid line? Justify your answer; simple answers of "Yes" or "No" will receive zero marks.



No. 2 The parabola (solid line) slopes down for $x<1$, so the derivative should be negative for $x<1$. 1 However, the dashed line has positive y-values for $x<1$, so it cannot be the derivative.

[5] 17. The number of bacteria, $G(t)$, in a population is given by the exponential growth function

$$G(t) = G_0 e^{rt},$$

where t is measured in days.

[2] (a) If the initial population size $G_0 = 100$, and $G(2) = 500$, what is the value of r?

$G(2) = 500 \Rightarrow 500 = 100e^{r(2)}$ 1

$\Rightarrow 5 = e^{r2}$.

$\Rightarrow \ln(5) = r2$. 1

$\Rightarrow r = \frac{\ln(5)}{2} \approx 0.8047$.

[3] (b) If the initial population size $G_0 = 100$, and $r = 4$, what is the *instantaneous rate of change* of the population after 2 days (i.e., when $t = 2$)?

$G(t) = 100e^{4t}$ 1 for knowing to compute $G'(t)$

$G'(t) = 100e^{4t}(4)$ 1

$= 400e^{4t}$

$G'(2) = 400e^{(8)}$ 1

≈ 1192383.195

[6] 18. Consider the following function,

$$f(x) = -\sqrt{2 - x^2}.$$

[2] (a) Find the derivative, $f'(x)$. You do not need to simplify your answer.

$f(x) = -(2-x^2)^{1/2}$.

$f'(x) = -\frac{1}{2}(2-x^2)^{-1/2}(2x)$ 2

$= -\dfrac{x}{\sqrt{2-x^2}}$

(0 for partially correct answers)

[2] (b) Find the equation of the line tangent to $f(x)$ at the point $(1, -1)$. Write your answer in slope-intercept form ($y = mx + b$).

$f'(1) = -\dfrac{1}{\sqrt{2-1^2}} = -1$ (so $m=-1$) 1

$y-(-1) = -1(x-1)$

$y+1 = -x+1$ 2

$y = -x$

[2] (c) Find the equation of the line tangent to $f(x)$ at the point $(\sqrt{2}, 0)$.

$f'(\sqrt{2}) = -\dfrac{1}{\sqrt{2-(\sqrt{2})^2}}$ DNE. 1

The derivative does not exist at $x=\sqrt{2}$, So we cannot write the equation of the line tangent to $f(x)$ at $(\sqrt{2}, 0)$. 1

There is a vertical tangent line at $x=\sqrt{2}$.

[5] 19. Consider the function

$$y = e^{(x^2+1)^2}.$$

[2] (a) Identify the *three* functions, $f(x), g(x)$, and $h(x)$ such that $y = f(g(h(x)))$; *i.e.* write y as a composition of three functions.

$f(x) = e^x$

$g(x) = x^2$.

$h(x) = x^2 + 1$.

1 for ~~partially~~ One or two correct answers

2 for all three to be correct

[3] (b) Use the chain rule to find $\frac{dy}{dx}$.

$y = f(g(h(x)))$

$\frac{dy}{dx} = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$. 1

$= e^{(x^2+1)^2} \cdot 2(x^2+1) \cdot 2x$ 1

$= 4x(x^2+1)e^{(x^2+1)^2}$. 1

or, chain rule for $e^{\boxed{(x^2+1)^2}}$ 1

chain rule for $(x^2+1)^2$ 1

final answer 1