

Due Friday, May 29th, before noon in the Economics Office, SS 454. The office is closed during lunch hour, noon – 1 pm.

Please remember to do a cover sheet with your name(s), the course number, number all the pages, clearly label which question and part you are answering, and staple your assignment.

1. Express each of the following sums in summation notation and then compute where possible. Let X take the values $x_1 = -2, x_2 = -1, x_3 = 0, x_4 = 1, x_5 = 2$ and Y take the values $y_1 = -1, y_2 = -0.5, y_3 = 0.5, y_4 = 1, y_5 = 1.5$.

a) $x_1 + x_2 + x_3 + x_4 + x_5 = \sum_{i=1}^5 x_i = 0$

b) $y_3 + y_4 + y_5 = \sum_{i=3}^5 y_i = 3$

c) $(x_1 - y_1) + (x_2 - y_2) + (x_3 - y_3) + (x_4 - y_4) + (x_5 - y_5) = \sum_{i=1}^5 (x_i - y_i) = \sum_{i=1}^5 x_i - \sum_{i=1}^5 y_i = 0 - 1.5 = -1.5$

d) $2(x_1 * y_2) + 2(x_2 * y_3) + 2(x_3 * y_4) = \sum_{i=1}^3 2(x_i * y_{i+1}) = 2 \sum_{i=1}^3 (x_i * y_{i+1}) = 1$

e) $(x_2/y_1) + (x_3/y_2) + (x_4/y_3) = \sum_{i=2}^4 \frac{x_i}{y_{i-1}} = 1 + 0 + 2 = 3$

f) $(x_1^2 * 2y_3) + (x_2^2 * 2y_4) + (x_3^2 * 2y_5) = \sum_{i=1}^3 (x_i^2 * 2y_{i+2}) = -2$

2. Show that $\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}$

$$\begin{aligned} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) &= \sum_{i=1}^n (x_i y_i - \bar{x} y_i - x_i \bar{y} + \bar{x} \bar{y}) \\ &= \sum_{i=1}^n x_i y_i - \sum_{i=1}^n \bar{x} y_i - \sum_{i=1}^n x_i \bar{y} + \sum_{i=1}^n \bar{x} \bar{y} \\ &= \sum_{i=1}^n x_i y_i - \bar{x} \sum_{i=1}^n y_i - \bar{y} \sum_{i=1}^n x_i + n\bar{x}\bar{y} \\ &= \sum_{i=1}^n x_i y_i - n\bar{x} \frac{\sum_{i=1}^n y_i}{n} - n\bar{y} \frac{\sum_{i=1}^n x_i}{n} + n\bar{x}\bar{y} \\ &= \sum_{i=1}^n x_i y_i - n\bar{x}\bar{y} - n\bar{y}\bar{x} + n\bar{x}\bar{y} = \sum_{i=1}^n x_i y_i - n\bar{x}\bar{y} \end{aligned}$$

3. **GRADED** An economist was trying to address the growing concern for educators about the number of students who have part-time jobs while they attend high school. To help acquire information, she recorded the average number of hours worked per week and the GPA achieved for 200 students with part time jobs. Below are four equations that were estimated by the economist with the data, where x is the average number of hours worked per week by the student and y is the GPA the student achieved that term.

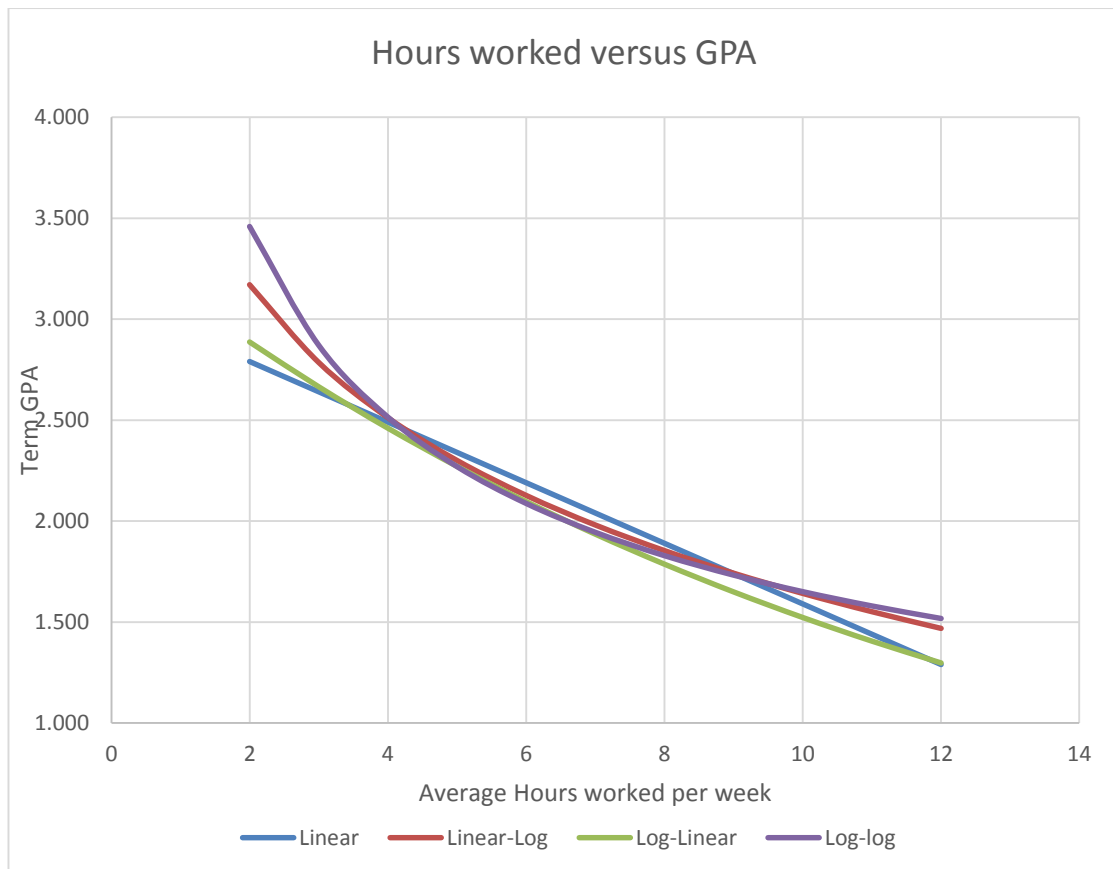
$$y = 3.09 - 0.15x$$

$$\ln(y) = 1.56 - 0.46 \ln(x)$$

$$y = 3.83 - 0.95 \ln(x)$$

$$\ln(y) = 1.22 - 0.08x$$

- a) **[8 points]** Plot each of the functions for $x = 2$ to $x = 12$ on the same graph. Comment on what you observe when you look at the functions on the graph.



All the functions are very close in the range of four to ten hours worked per week. In the case of the Log-Log function both of the extreme values, two hours worked per week and twelve hours worked per week, the GPA is higher than all the other functions, but the GPA is lower over the middle range. The lowest GPA values for the extreme values of two and ten hours per week worked are the Linear model.

- b) **[6 points – 1.5 points each equation]** Calculate the slope of each function at $x = 7$. State the interpretation of the slope.

$y = 3.09 - 0.15x$ This is a constant slope of negative 0.15. For each hour increase in the average hours worked per week the term GPA is predicted to drop by 0.15 on average.

$\ln(y) = 1.56 - 0.46 \ln(x)$ The slope is $-0.46 \cdot (y/x)$, so for $x = 7$, $-0.46 \cdot (1.944/7) = -0.1277$. A 1% increase in the average number of hours worked leads to a 0.46% decrease in the term GPA. When the average number of hours worked per week is seven hours an increase of one hour will be associated with an average decrease in GPA of 0.1277.

$y = 3.83 - 0.95 \ln(x)$ A 1% increase in the average number of hours of work per week leads to a decrease of $0.95/100 = 0.0095$ in the term GPA. The slope is $-0.95 \cdot (1/x)$, so for $x=7$, $-0.95/7 = -0.1357$. When the average number of hours worked per week is seven, an increase of one hour will be associated with an average decrease in GPA of 0.1357 or approximately 0.136.

$\ln(y) = 1.22 - 0.08x$ An increase of one hour in the average number of hours worked in a week leads to an average decrease in GPA of $0.08 \cdot 100\% = 8\%$. The slope is $-0.08 \cdot y$, so for $x = 7$, $-0.08 \cdot 1.981 = -0.1585$. When the average number of hours worked per week is seven, an increase of one hour per week is associated with an average decrease in the term GPA of 0.1585.

- c) **[6 points – 1.5 points each equation]** Calculate the elasticity of each function at $x = 7$ and give the interpretation.

$y = 3.09 - 0.15x$ The elasticity is defined as $-0.15 \cdot (x/y) = -0.15 \cdot (7/2.04) = -0.515$. Inelastic. When the average number of hours worked per week is seven, the term GPA decreases by 0.515% when the number of hours worked increases by 1%.

$\ln(y) = 1.56 - 0.46 \ln(x)$ This is a constant elasticity model, with elasticity of -0.46. A one percent increase in the average number of hours worked will on average decrease the term GPA by 0.46%. This is also inelastic.

$y = 3.83 - 0.95 \ln(x)$ The elasticity is $-0.95 \cdot (1/y)$, so for $x = 7$, $-0.95 \cdot (1/1.981) = -0.48$. This is also inelastic. When the average number of hours worked is seven, term GPA decreases by 0.48% for a 1% increase in the average number of hours worked.

$\ln(y) = 1.22 - 0.08x$ The elasticity is $-0.08 \cdot x$, so for $x = 7$, $-0.08 \cdot 7 = -0.56$. When the average number of hours worked is seven per week, term GPA will decrease by 0.56% when the average number of hours worked increases by 1%. This is inelastic as well.

4. Let X be a discrete random variable that is the value shown on a single roll of a fair die.
a) What is the expected value of X ? What is the expected value of X^2 ?

$$\begin{aligned} E(X) &= \sum_{i=1}^6 x_i f(x_i) = \left(1 \cdot \frac{1}{6}\right) + \left(2 \cdot \frac{1}{6}\right) + \left(3 \cdot \frac{1}{6}\right) + \left(4 \cdot \frac{1}{6}\right) + \left(5 \cdot \frac{1}{6}\right) + \left(6 \cdot \frac{1}{6}\right) \\ &= \frac{21}{6} = 3.5 \end{aligned}$$

The expected value of X is 3.5.

$$\begin{aligned} E(X^2) &= \sum_{i=1}^6 x_i^2 f(x_i) \\ &= \left(1^2 * \frac{1}{6}\right) + \left(2^2 * \frac{1}{6}\right) + \left(3^2 * \frac{1}{6}\right) + \left(4^2 * \frac{1}{6}\right) + \left(5^2 * \frac{1}{6}\right) + \left(6^2 * \frac{1}{6}\right) \\ &= \frac{91}{6} = 15.167 \end{aligned}$$

The expected value of X^2 is 15.167.

- b) Find the variance of X .

$$\text{var}(X) = \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2 = 15.167 - (3.5^2) = 2.9167.$$

The variance of X is 2.9167.

- c) Find the expected value and variance of W if $W = g(X) = 2X - 1$.

$$E(W) = E(2X - 1) = 2E(X) - 1 = 2 * 3.5 - 1 = 6$$

The expected value of W is 6.

$$\text{var}(W) = \text{var}(2X - 1) = 2^2 \text{var}(X) = 4 * 2.9167 = 11.6668.$$

The variance of W is 11.6668.

- d) Now, assume the die has been weighted so that $P(X=1) = 1/3$, $P(X=2) = 1/3$, and the $P(X=3) = P(X=4) = P(X=5) = P(X=6) = 1/12$. Find the answers to part a) and b) under the new assumption.

$$\begin{aligned} E(X) &= \sum_{i=1}^6 x_i f(x_i) \\ &= \left(1 * \frac{1}{3}\right) + \left(2 * \frac{1}{3}\right) + \left(3 * \frac{1}{12}\right) + \left(4 * \frac{1}{12}\right) + \left(5 * \frac{1}{12}\right) + \left(6 * \frac{1}{12}\right) \\ &= \frac{15}{6} = 2.5 \end{aligned}$$

The expected value of X is 2.5.

$$\begin{aligned} E(X^2) &= \sum_{i=1}^6 x_i^2 f(x_i) \\ &= \left(1^2 * \frac{1}{3}\right) + \left(2^2 * \frac{1}{3}\right) + \left(3^2 * \frac{1}{12}\right) + \left(4^2 * \frac{1}{12}\right) + \left(5^2 * \frac{1}{12}\right) \\ &\quad + \left(6^2 * \frac{1}{12}\right) = \frac{53}{6} = 8.833 \end{aligned}$$

The expected value of X^2 is 8.8333.

$$\text{var}(X) = \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2 = 8.833 - (2.5^2) = 2.5833.$$

The variance of X is 2.5833.

5. **GRADED** After watching a number of children playing games in a video arcade, a statistics practitioner estimated the following probability distribution of X , the number of games per visit.

x	1	2	3	4	5	6	7
f(x)	.05	.05	.15	.25	.30	.15	.05

- a) **[2 points]** What is the probability that a child will play at least four games?
 $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) = 0.25 + 0.30 + 0.15 + 0.05 = 0.75$

The probability that a child will play at least four games is 0.75

- b) **[5 points]** What is the mean and variance of the number of games played?

$$E(X) = \sum_{i=1}^7 x_i f(x_i)$$

$$= (1 * 0.05) + (2 * 0.05) + (3 * 0.15) + (4 * 0.25) + (5 * 0.3) + (6 * 0.15) + (7 * 0.05) = 0.05 + 0.1 + 0.45 + 1 + 1.5 + 0.9 + 0.35 = 4.35$$

$$E(X^2) = \sum_{i=1}^7 x_i^2 f(x_i)$$

$$= (1^2 * 0.05) + (2^2 * 0.05) + (3^2 * 0.15) + (4^2 * 0.25) + (5^2 * 0.3) + (6^2 * 0.15) + (7^2 * 0.05)$$

$$= 0.05 + 0.2 + 1.35 + 4 + 7.5 + 5.4 + 2.45 = 20.95$$

$$var(X) = \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2 = 20.95 - 4.35^2 = 2.0275 \approx 2.03$$

The mean of the number of games played is 4.35 and the variance is 2.03 games squared.

- c) **[3 points]** Suppose that each game costs \$2.25. Use the laws of expected value and variance to determine the expected value and variance of the amount of money the arcade takes in per child.

Let R be the revenue the arcade receives for each child. $R = \$2.25 * X$

$$E(R) = E(2.25X) = 2.25E(X) = 2.25 * 4.35 = \$9.79$$

$$var(R) = var(2.25X) = 2.25^2 var(X) = 5.0625 * 2.0275 = 10.26.$$

The expected value of the revenue is \$9.79 and the variance is 10.26 dollars squared.

- d) **[4 points]** Determine the probability distribution of the amount of money the arcade takes in per child.

R	2.25	4.5	6.75	9	11.25	13.5	15.75
f(R)	.05	.05	.15	.25	.30	.15	.05

- e) **[5 points]** Use the probability distribution to calculate the mean and variance of the amount of money the arcade takes in per child.

$$E(R) = \sum_{i=1}^7 r_i f(r_i)$$

$$= (2.25 * 0.05) + (4.5 * 0.05) + (6.75 * 0.15) + (9 * 0.25) + (11.25 * 0.3) + (13.5 * 0.15) + (15.75 * 0.05) = 9.7875$$

$$E(R^2) = \sum_{i=1}^7 r_i^2 f(r_i)$$

$$= (2.25^2 * 0.05) + (4.5^2 * 0.05) + (6.75^2 * 0.15) + (9^2 * 0.25) + (11.25^2 * 0.3) + (13.5^2 * 0.15) + (15.75^2 * 0.05) = 106.0594$$

$$var(R) = \sigma^2 = E(R - \mu)^2 = E(R^2) - \mu^2 = 106.06 - 9.79^2 = 10.26$$

f) **[1 point]** Are your answers in part c & part e identical? **YES.**

6. To gauge the relationship between education and unemployment an economist turned to the US Census, from which the following table was produced:

Education	Employed	Unemployed
Not a high school graduate	0.0975	0.0080
High school graduate	0.3108	0.0128
Some college, no degree	0.1785	0.0062
Associate's degree	0.0849	0.0023
Bachelor's degree	0.1959	0.0041
Advanced degree	0.0975	0.0015

a) What is the probability that a high school graduate is employed?

$$P(\text{employed} \mid \text{high school graduate}) =$$

$$\frac{P(\text{employed and high school graduate})}{P(\text{high school graduate})} = \frac{.3108}{.3108 + .0128} = \frac{.3108}{.3236} = 0.9604$$

The probability that a high school graduate is employed is 0.9604 or 96.04%.

b) Determine the probability that a randomly selected individual is unemployed.

$$P(\text{unemployed}) = .0080 + .0128 + .0062 + .0023 + .0041 + .0015 = 0.0349$$

The probability that a randomly selected individual is unemployed is 0.0349 or 3.49%.

c) Find the probability that an unemployed person possesses a Bachelor's degree.

$$P(\text{Bachelor's degree} \mid \text{unemployed}) =$$

$$\frac{P(\text{Bachelor's and unemployed})}{P(\text{unemployed})} = \frac{.0041}{.0349} = .11748 \cong 0.1175$$

The probability that an unemployed person possesses a Bachelor's degree is 0.1175 or 11.75%.

d) What is the probability that a randomly selected person did not finish high school?

$$P(\text{not a high school graduate}) = .0975 + .0080 = 0.1055$$

The probability that a randomly selected person did not finish high school is 0.1055 or 10.55%.

- e) Are employment status and education independent? Prove your answer.
If education and employment status are independent, then for all education levels and both employment statuses, the marginal probabilities multiplied together would equal the joint probabilities for every combination of education levels and employment status.
For example: $P(\text{Unemployed}) * P(\text{High school graduate}) = P(\text{Unemployed and High school graduate})$ if independent, $0.039 * 0.3236 = 0.01262 \neq 0.0128$. **So, since the test is not true, education and employment status are not independent. It takes only one example to disprove the theory.**
OR
If education and employment status are independent then for all education levels and both employment statuses, the conditional probability for an employment status or an education level must equal the marginal probability for that particular employment status or education level.
For example: $P(\text{Bachelor's degree} | \text{unemployed}) = P(\text{Bachelor's degree})$ if independent, but $0.1175 \neq 0.2000$ **So, since the test is not true, education and employment status are not independent. It takes only one example to disprove the theory.**

Data from the Office on Smoking and Health, Centers for Disease Control and Prevention, indicate that 40% of adults who did not finish high school, 34% of high school graduates, 24% of adults who completed some college (no degree), and 14% of graduates (Degree holders) smoke.

- f) Suppose that one individual is selected at random and it is discovered that the individual does not smoke. What is the probability that the individual is a graduate? (Use the probabilities from the table above to solve this question)

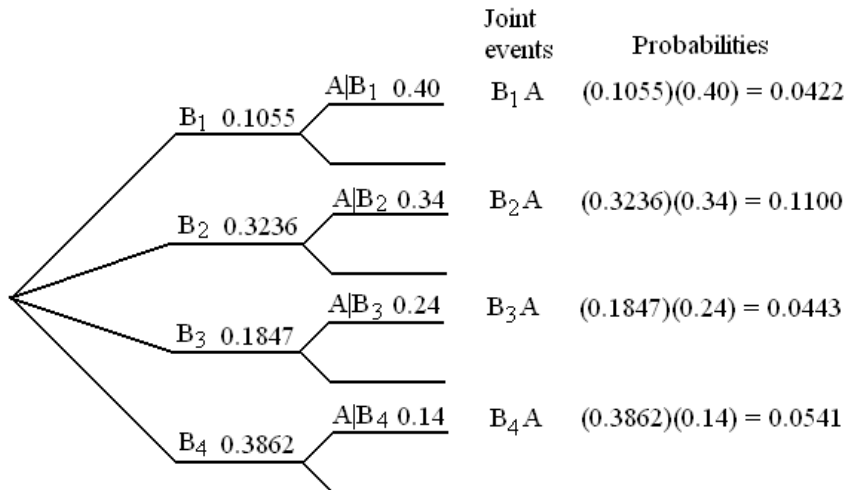
HINT: Bayes' Law Formula

$$P(A_i | B) = \frac{P(A_i)P(B | A_i)}{P(A_1)P(B | A_1) + P(A_2)P(B | A_2) + \dots + P(A_k)P(B | A_k)}$$

Define events: A = smoke, A^C = does not smoke, B_1 = did not finish high school, B_2 = high school graduate, B_3 = some college, no degree, B_4 = completed a degree

$P(A | B_1) = 0.40$ so $P(A^C | B_1) = 0.60$, $P(A | B_2) = 0.34$ so $P(A^C | B_2) = 0.66$, $P(A | B_3) = 0.24$ so $P(A^C | B_3) = 0.76$, $P(A | B_4) = 0.14$ so $P(A^C | B_4) = 0.86$.

From Exercise above: $P(B_1) = 0.1055$, $P(B_2) = 0.3236$, $P(B_3) = 0.1847$, $P(B_4) = 0.3862$



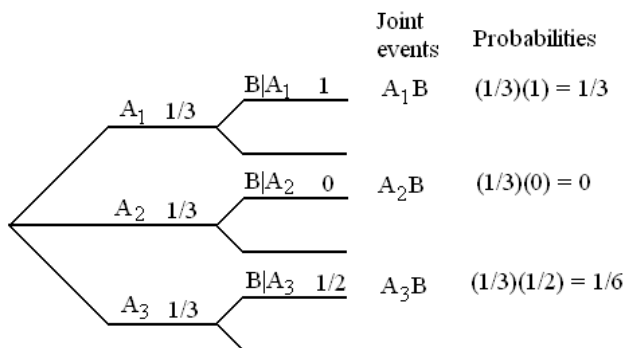
$$P(A) = .0422 + .1100 + .0443 + .0541 = 0.2506, \text{ so } P(A^C) = 1 - 0.2506 = 0.7494$$

$$P(B_4 | A^C) = (0.3862) \cdot (0.86) / 0.7494 = 0.3321 / 0.7494 = 0.4432$$

The probability that the individual is a graduate, given that the individual does not smoke, is 0.4432 or 44.32%.

7. A professor and his wife are planning to take a 2-week vacation in Hawaii, but they can't decide whether to spend 1 week on each of the islands of Maui and Oahu or 2 weeks on Maui or 2 weeks on Oahu. Placing their faith in random chance they make 3 envelopes, one envelope with a brochure for Maui and a brochure for Oahu, one envelope with 2 Maui brochures and one envelope with 2 Oahu brochures. The professor's wife selects a closed envelope randomly and the professor opens the envelope and removes one brochure to find it is for Maui. What is the probability that the other brochure in the envelope is a Maui brochure?

Define the events: A_1 = envelope containing two Maui brochures is selected, A_2 = envelope containing two Oahu brochures is selected, A_3 = envelope containing one Maui and one Oahu brochures is selected. B = a Maui brochure is removed from the selected envelope.



$$P(B) = 1/3 + 0 + 1/6 = 1/2$$

$$P(A_1 | B) = \frac{P(A_1 \text{ and } B)}{P(B)} = \frac{1/3}{1/2} = 2/3$$

The probability that the other brochure in the envelope is a Maui brochure is 2/3.

8. **GRADED** An investor holds a portfolio consisting of two stocks. He puts 33% of his money in Stock C and 67% into stock F. Stock C has an expected return of $R_C = 8\%$ and a standard deviation of $\sigma_C = 12\%$. Stock F has an expected return of $R_F = 15\%$ and a standard deviation of $\sigma_F = 22\%$. The portfolio return is $P = 0.33 R_C + 0.67 R_F$.

- a) **[5 points]** What is the expected return on the portfolio?

$$P = 0.33 R_C + 0.67 R_F$$

$$E(P) = E(0.33 R_C + 0.67 R_F) = 0.33 E(R_C) + 0.67 E(R_F) \\ = (0.33 * 8\%) + (0.67 * 15\%) = 12.69\%$$

The expected return on the portfolio is 12.69%.

- b) **[5 points]** Compute the standard deviation of the portfolio if the two stocks' returns are perfectly positively correlated.

$$var(P) = 0.33^2 var(R_C) + 0.67^2 var(R_F) + 2(0.33)(0.67)cov(R_C, R_F)$$

$$\rho = 1 = \frac{cov(R_C, R_F)}{\sigma_{R_C} * \sigma_{R_F}}, cov(R_C, R_F) = \sigma_{R_C} * \sigma_{R_F} = 12 * 22 = 264$$

$$var(P) = 0.33^2 var(R_C) + 0.67^2 var(R_F) + 2(0.33)(0.67)cov(R_C, R_F) \\ = (0.33^2 * 12^2) + (0.67^2 * 22^2) + 2(0.33)(0.67)264 = 349.69$$

$$\sigma_P = \sqrt{349.69} = 18.7$$

The standard deviation of the portfolio is 18.7%.

- c) **[5 points]** Compute the standard deviation of the portfolio if the two stocks' returns have a correlation of 0.5.

$$var(P) = var(0.33R_C) + var(0.67R_F) + 2(0.33)(0.67)cov(R_C, R_F)$$

$$\rho = 0.5 = \frac{cov(R_C, R_F)}{\sigma_{R_C} * \sigma_{R_F}}, cov(R_C, R_F) = 0.5 * \sigma_{R_C} * \sigma_{R_F} = 0.5 * 12 * 22 = 132$$

$$var(P) = 0.33^2 var(R_C) + 0.67^2 var(R_F) + 2(0.33)(0.67)cov(R_C, R_F) \\ = (0.33^2 * 12^2) + (0.67^2 * 22^2) + 2(0.33)(0.67)132 = 291.3196$$

$$\sigma_P = \sqrt{291.3196} = 17.068$$

The standard deviation of the portfolio is 17.068%.

- d) **[5 points]** Compute the standard deviation of the portfolio if the two stocks' returns are uncorrelated.

$$var(P) = var(0.33R_C) + var(0.67R_F) + 2(0.33)(0.67)cov(R_C, R_F) \\ = 0.33^2 var(R_C) + 0.67^2 var(R_F) = (0.33^2 * 12^2) + (0.67^2 * 22^2) \\ = 232.9492$$

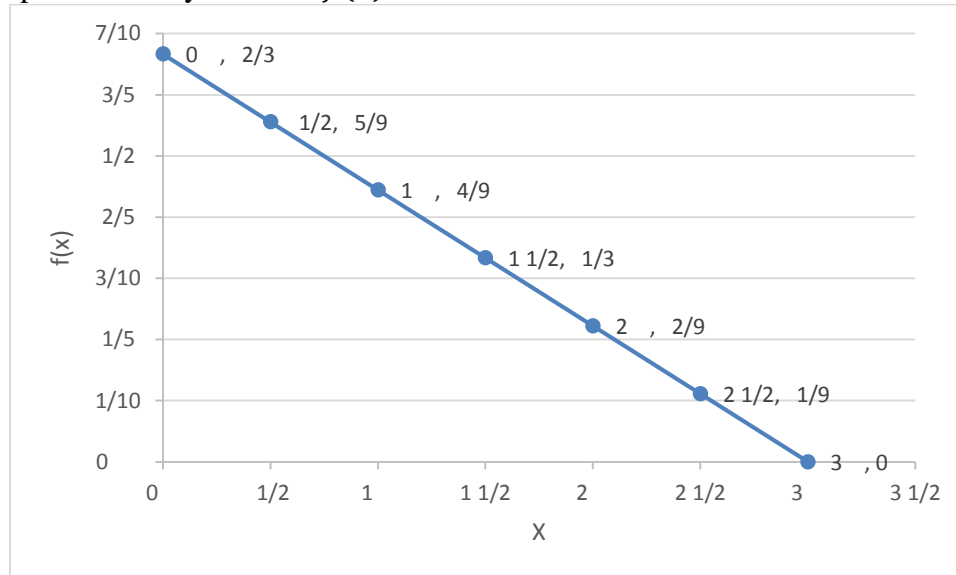
$$\sigma_P = \sqrt{232.9492} = 15.26267 \cong 15.26$$

The standard deviation of the portfolio is 15.26%.

9. Let X be a continuous random variable with probability density function given by

$$f(x) = \frac{2}{3} - \frac{2}{9}x, \quad 0 \leq x \leq 3$$

- a) Graph the density function $f(x)$.



- b) Find the total area beneath $f(x)$ for $0 \leq x \leq 3$.

$$Area = \frac{1}{2} * base * height = \frac{1}{2} * 3 * \frac{2}{3} = 1$$

The total area beneath $f(x)$ for $0 \leq x \leq 3$ is 1.

- c) What is the probability that $0 \leq x \leq 1$?

This requires that we calculate the area of the rectangle from $x = 0$ and $x = 1$ to $y = 0$ and $y = \frac{4}{9}$ and add to it the area of the triangle from $x = 0$ and $x = 1$ to $y = \frac{4}{9}$ to $y = \frac{2}{3}$.

$$area\ of\ rectangle = base * height = (1 - 0) * \left(\frac{4}{9} - 0\right) = \frac{4}{9}$$

$$area\ of\ triangle = \frac{1}{2} * base * height = \frac{1}{2} * 1 * \left(\frac{2}{3} - \left(\frac{4}{9}\right)\right) = \frac{1}{2} * 1 * \frac{2}{9} = \frac{1}{9}$$

The probability that $0 \leq x \leq 1$ is $\frac{4}{9} + \frac{1}{9} = \frac{5}{9}$ or 55.56%.

10. **GRADED** Let $Y_1, Y_2, Y_3,$ and Y_4 be independent, identically distributed random variables from a population with a mean μ and a variance σ^2 . Consider a different estimator of μ :

$$W = \frac{1}{8} Y_1 + \frac{1}{3} Y_2 + \frac{1}{6} Y_3 + \frac{3}{8} Y_4.$$

This is an example of a weighted average of the Y_i .

- a) **[6 points]** Show that W is a linear estimator.

$$W = \frac{1}{8} Y_1 + \frac{1}{3} Y_2 + \frac{1}{6} Y_3 + \frac{3}{8} Y_4.$$

$$W = \sum_{i=1}^4 a_i Y_i \text{ where } a_i \text{ are constants for } i = 1, 2, 3, 4.$$

- b) **[6 points]** Is W an unbiased estimator of μ ? Show that it is – or it isn't ($E(W) = ?$).

$$\begin{aligned} E(W) &= E\left(\frac{1}{8} Y_1 + \frac{1}{3} Y_2 + \frac{1}{6} Y_3 + \frac{3}{8} Y_4\right) = \frac{1}{8} E(Y_1) + \frac{1}{3} E(Y_2) + \frac{1}{6} E(Y_3) + \frac{3}{8} E(Y_4) \\ &= \frac{1}{8} \mu + \frac{1}{3} \mu + \frac{1}{6} \mu + \frac{3}{8} \mu = \mu \end{aligned}$$

W is an unbiased estimator of μ .

- c) **[6 points]** Find the variance of W and compare it to the variance of the sample mean \bar{Y} .

$$\begin{aligned} \text{var}(W) &= \text{var}\left(\frac{1}{8} Y_1 + \frac{1}{3} Y_2 + \frac{1}{6} Y_3 + \frac{3}{8} Y_4\right) \\ &= \frac{1^2}{8^2} \text{var}(Y_1) + \frac{1^2}{3^2} \text{var}(Y_2) + \frac{1^2}{6^2} \text{var}(Y_3) + \frac{3^2}{8^2} \text{var}(Y_4) \\ &= \frac{1}{64} \sigma^2 + \frac{1}{9} \sigma^2 + \frac{1}{36} \sigma^2 + \frac{9}{64} \sigma^2 = \frac{85}{288} \sigma^2 = 0.2951 \sigma^2 \end{aligned}$$

$$\text{var}(\bar{Y}) = \frac{\sigma^2}{N} = \frac{\sigma^2}{4} = 0.25 \sigma^2 \quad \text{var}(\bar{Y}) = 0.25 \sigma^2 < 0.2951 \sigma^2 = \text{var}(W)$$

The variance of W is larger than the variance of the sample mean \bar{Y} .

- d) **[2 points]** Is W as good an estimator as \bar{Y} ? Explain your answer.

W is not BLUE! W is not as good an estimator as \bar{Y} , as W does not have the smallest variance.