

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

| Course | Number | Section(s) |
|--|-----------------|---------------|
| Mathematics | 209 | All except EC |
| Examination | Date | Pages |
| Final | April 2010 | 3 |
| Instructors | Course Examiner | |
| M. Amir, L. Dube, E. Duma, H. Greenspan, B. Rhodes, J. Ruddy, C. Santana, U. Tiwari | R. Raphael | |

Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Only approved calculators are allowed.

MARKS

[9] 1. (i) Find $\lim_{x \rightarrow -\infty} \frac{-5x^7 + 3x^2 + 2}{4 - x^2}$

(ii) Given $\lim_{x \rightarrow 3} f(x) = -5$ and $\lim_{x \rightarrow 3} g(x) = 4$, find

(a) $\lim_{x \rightarrow 3} [-3g(x)]$ (b) $\lim_{x \rightarrow 3} \sqrt{g(x)}$ (c) $\lim_{x \rightarrow 3} [g(x)/2f(x)]$

(iii) Find the value of each of the following:

(a) $\lim_{x \rightarrow -3} \frac{x^2 - 3x + 2}{(x - 1)}$ (b) $\lim_{x \rightarrow 5} \frac{x^2 - 16}{(x - 5)}$

[18] 2. (a) If $f(x) = 4 - 6x^{10} - 4x^3$, find $f'(x)$.

(b) If $f(x) = \frac{x^2 - 3x + 4}{x^2 + 1}$ find $f'(x)$.

(c) If $f(x) = \frac{x^2 + 5x}{e^x - 7}$ find $f'(x)$.

(d) If $y = 2e^{(x^2 - 5)}$, then $y' = ?$

(e) If $y = \ln[(x^2 + 3x)^2]$, then $\frac{dy}{dx} = ?$

(f) If $y = \sqrt[5]{x + 5}$, then $\frac{dy}{dx} = ?$

(g) Find y' given $xy = e^y - 2$.

- [9] 3. A manufacturer currently sells sunglasses for \$4 a pair. The price p and the demand x for these glasses are related by

$$x = f(p) = 7,000 - 500p$$

If the current price is increased, will revenue increase or decrease? Explain why.

- [5] 4. A sphere with a radius of 5 centimeters is coated with ice 0.1 centimeters thick. Use differentials to estimate the volume (V) of the ice. [Recall that $V = \frac{4}{3}\pi r^3$.]

- [5] 5. Give a function $f : R \rightarrow R$ that is continuous at 0 but not differentiable at 0. Explain.

- [9] 6. The price p (in dollars) and the demand x for a particular steam iron are related by the equation

$$x = 1,000 - 20p$$

- (A) Express the price p in terms of the demand x , and find the domain of this function.
- (B) Find the revenue $R(x)$ from the sale of x clock radios. What is the domain of R ?
- (C) Find the marginal revenue at a production level of 400 steam irons, and interpret the results.
- (D) Find the marginal revenue at a production level of 650 steam irons, and interpret the results.

- [9] 7. Boyle's law for enclosed gases states that if the volume is kept constant, the pressure P and the temperature T are related by the equation $P/T = k$ where k is a constant. If the temperature is increasing at 3 kelvins per hour, what is the rate of change of pressure when the temperature is 250 kelvins and the pressure is 500 pounds per square inch?

[12] 8. Compute the following:

(a) $\int (x - 4)^{-3} dx$

(b) $\int e^{5x} dx$

(c) $\int \frac{x}{\sqrt{x-7}} dx$

(d) $\int (3x^2 + 5x) dx$

(e) $\int \frac{x^2}{4 + x^3} dx$

(f) $\int (x^2 + 1)^{12} x dx$

[6] 9. Evaluate the following integrals [accurate to 2 decimals].

(a) $\int_0^5 (t^2 - 4) dt$

(b) $\int_2^3 e^{h^2} h dh$

[9] 10. Find the area bounded by $f(x) = x^2 - x$ and $g(x) = 2x$ for $-2 \leq x \leq 3$.

[9] 11. Consider the function $f(x) = x^4 - 2x^3$. Show its intercepts, where it is increasing, decreasing, concave up and concave down. Sketch its graph.