

MAT 2377, Probability and statistics for engineers

Assignment 5 - solutions

1. We compute the sample mean $\bar{x} = \sum_{i=1}^n x_i/n = 0.39125$ and the sample standard deviation

$$s = \sqrt{\frac{(\sum_{i=1}^n x_i^2) - (\sum_{i=1}^n x_i)^2/n}{n-1}} = 0.048576.$$

A 95% confidence interval for the mean corrosion loss is

$$\bar{x} \pm t_{0.025;7} \frac{s}{\sqrt{n}} = [0.355, 0.428],$$

where $t_{0.025;7} = 2.365$.

2. a) A 90% confidence interval for the mean porosity of specimens of this type of concrete is

$$\bar{x} \pm z_{0.05} \frac{\sigma}{\sqrt{n}} = [20.88, 22.32],$$

where $z_{0.05} = 1.645$, $\bar{x} = 21.6$, $\sigma = 3.2$ and $n = 53$.

- (b) A 95% confidence interval for the mean porosity of specimens of this type of concrete is

$$\bar{x} \pm z_{0.025} \frac{\sigma}{\sqrt{n}} = [20.74, 22.46],$$

where $z_{0.025} = 1.96$

- (c) The length of the interval is $22.2 - 21 = 1.2$, which is

$$(\bar{x} + z_{0.025} \frac{\sigma}{\sqrt{n}}) - (\bar{x} - z_{0.025} \frac{\sigma}{\sqrt{n}}) = 2 z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = (0.87911) z_{\alpha/2}.$$

Thus, $z_{\alpha/2} = 1.2/0.87911 \approx 1.37$. Which means that $\alpha/2 = 1 - 0.9162$. Thus, $1 - \alpha = 0.8324$. Therefore the confidence level is about 83%.

- (d) Solving

$$n \geq \left(\frac{z_{0.025} \sigma}{E} \right)^2 = \left(\frac{1.96 (3.2)}{0.3} \right)^2 = 437.0887.$$

We require 438 observations.

3. (a) A 95% upper confidence bound for the mean weight loss to answer the question is

$$\bar{x} + t_{0.05;24} \frac{s}{\sqrt{n}} = 3.27 + 1.711 \left(\frac{0.75}{\sqrt{25}} \right) = 3.527.$$

So we are 95% confident that $\mu < 3.527$. At a level of significance of $\alpha = 5\%$, we cannot conclude that $\mu < 3.5$.

(b) The observed value of the t -test statistic is

$$t_0 = \frac{\bar{x} - 3.5}{s/\sqrt{n}} = \frac{3.27 - 3.5}{0.75/\sqrt{25}} = -1.533.$$

Since it is a left-sided alternative, then the p -value is

$$p = P(T < -1.533), \quad \text{where } T \sim t(24).$$

By the symmetry of the t distribution, we can reexpress the p -value as follows. $p = P(T > 1.533)$. Since $t_{0.10;24} = 1.318$ and $t_{0.10;24} = 1.711$, then

$$0.05 < p\text{-value} < 0.10.$$

4. (a) Let μ be the mean wall thickness (in mils). We want to test

$$H_0 : \mu = 4 \quad \text{against} \quad H_1 : \mu \neq 4.$$

(b) We conclude that the bottles do not meet specifications, but in reality the specifications are met.

(c) We conclude that the bottles meet the specifications, but in reality the specifications are not met.

(d) The observed value of the t -test statistic is

$$t_0 = \frac{\bar{x} - 4}{s/\sqrt{n}} = 1.3732,$$

where $\bar{x} = \sum_{i=1}^n x_i/n = 4.03414$ and

$$s = \sqrt{\frac{(\sum_{i=1}^n x_i^2) - (\sum_{i=1}^n x_i)^2/n}{n-1}} = 0.06578.$$

We will reject H_0 only if $|t_0| > t_{0.025;6} = 2.447$.

Since $|t_0| = 1.3732$ did not fall in the critical region, then the evidence that the wall thickness does not meet the specifications is **not** significant.

5. (a) Let μ be mean octane rating (in %). We want to test

$$H_0 : \mu = 90 \quad \text{against} \quad H_1 : \mu > 90.$$

(b) The observed value of the test statistic is

$$t_0 = \frac{\bar{x} - 90}{s/\sqrt{n}} = 0.5199,$$

where $\bar{x} = \sum_{i=1}^n x_i/n = 90.3$ and

$$s = \sqrt{\frac{(\sum_{i=1}^n x_i^2) - (\sum_{i=1}^n x_i)^2/n}{n-1}} = 1.2903.$$

We will reject H_0 , only if $t_0 > t_{0.05;4} = 2.132$. Since $t_0 \leq 2.132$, then we cannot conclude that the mean octane rating is greater than 90.

(c) It is a right-sided alternative, the the p -value is

$$p = P(T > 0.5199), \quad \text{where } T \sim T(4).$$

Since $t_{0.4;4} = 0.271$ and $t_{0.25;4} = 0.741$, then

$$0.25 < p\text{-value} < 0.4.$$

6. (a) If $\mu = 11.5$, then

$$Z_0 = \frac{\bar{X} - 12}{\sigma/\sqrt{n}} \sim N(\phi, 1),$$

where

$$\phi = \sqrt{n} \frac{(11.5 - 12)}{\sigma} = \sqrt{10} \frac{(11.5 - 12)}{2} = -0.7906.$$

Not rejecting H_0 , means that $z_0 \geq -z_{0.05}$, where $-z_{0.05} = -1.645$. We want

$$\begin{aligned} \beta(11.5) &= P(\text{not rejecting } H_0 | \mu = 11.5) \\ &= P(Z_0 \geq -1.645 | \mu = 11.5) \\ &= 1 - \Phi\left(\frac{-1.645 - (-0.7906)}{1}\right) \\ &= 1 - \Phi(-0.85) = 1 - 0.1977 = 0.8023 \end{aligned}$$

(b) If $\mu = 10$, then

$$Z_0 = \frac{\bar{X} - 12}{\sigma/\sqrt{n}} \sim N(\phi, 1),$$

where

$$\phi = \sqrt{n} \frac{(10 - 12)}{\sigma} = \sqrt{10} \frac{(10 - 12)}{2} = -3.1622.$$

Not rejecting H_0 , means that $z_0 \geq -z_{0.05}$, where $-z_{0.05} = -1.645$. We want

$$\begin{aligned}\beta(10) &= P(\text{not rejecting } H_0 | \mu = 10) \\ &= P(Z_0 \geq -1.645 | \mu = 10) \\ &= 1 - \Phi\left(\frac{-1.645 - (-3.1622)}{1}\right) \\ &= 1 - \Phi(1.52) = 1 - 0.9357 = 0.0643.\end{aligned}$$

(c) Find the required number n of painted specimens so that our risk of not rejecting the null hypothesis is 1%, if the mean drying is $\mu = 10$. It is a one sided alternative, so

$$n = \frac{(z_\beta + z_\alpha)^2}{(\mu_1 - \mu_0)^2} \sigma^2 = \frac{(z_{0.01} + z_{0.05})^2}{(10 - 12)^2} 2^2 = 15.77,$$

where $z_{0.01} = 2.326$ and $z_{0.05} = 1.645$. So we require $n = 16$ observations.