MAT 2377, Probability and statistics for engineers

Assignment 4

Deadline : Before 3 pm on Friday, March 20

Submit the assignment in the drop box at 585 King Edward

UNless otherwise stated, solve the following exercises with a TI-30, TI-34, Casio FX-260 or Casio FX-300 calculator.

1. (based on 4.12) Compute the following, given the following sample data:

$$13 \quad 49 \quad 36 \quad 3 \quad 6 \quad 38 \quad 38 \quad 30 \quad 8 \quad 40 \quad 31 \quad 5 \quad 36:$$

   (a) the sample mean
   (b) The median
   (c) The first and third quartiles (i.e. 25th and 75 percentile)

2. (based on 4.16) The heights of 1000 students are approximately normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Suppose random samples of size 25 are drawn from this population. Determine:
   (a) the mean and standard deviation of the sampling distribution of $\overline{X}$;
   (b) the probability that the sample mean falls between 172.5 and 175.8 centimeters inclusive.
   (c) If we repeated this sampling 200 times (each sample with 25 students), for how many samples would we expect the sample mean to be below 172.0 centimeters? (I.e. What is the mathematical expectation for this number?)

3. (4.26) Two alloys A and B are being used to manufacture a certain steel product. An experiment needs to be designed to compare the two in terms of maximum load capacity in tons (the maximum weight that can be tolerated without breaking). It is known that the two standard deviations in load capacity are equal at 5 tons each. An experiment is conducted in which 30 specimens of each alloy (A and B) are tested and the results recorded as follows:

$$\bar{x}_A = 49.5, \quad \bar{x}_B = 45.5; \quad \bar{x}_A - \bar{x}_B = 4.$$

   The manufacturers of alloy A are convinced that this evidence shows conclusively that $\mu_A > \mu_B$ and strongly supports the claim that their alloy is superior. Manufacturers of alloy B claim that the experiment could easily have given $\bar{x}_A - \bar{x}_B = 4$ even if the two population means are equal. In other words, "the results are inconclusive!"
   (a) Make an argument that manufacturers of alloy B are wrong. Do it by computing

$$P(\bar{x}_A - \bar{x}_B \geq 4 \mid \mu_A > \mu_B).$$

   (b) Do you think these data strongly support alloy A?

4. The amount of cereal packed into a cereal box is supposed to be 500g. Either too high or too low a population mean is considered bad (either the company is wasting money by giving away free product, or customers will complain about being shortchanged), so the t-value is required to be in the middle 95%. A sample of 20 boxes is taken, and the sample mean is 480 and sample standard deviation 2.23.
   (a) What is the t-value for our sample?
   (b) What is the range of t-values that the company considers acceptable?
   (c) Will the company find this t-value acceptable?

5. (R problem) (a) Input data into R using the command
   ```
   x<-c(49, 38, 87, 21, 9, 20, 96, 79, 36, 84, 89, 9, 74, 55, 89, 48, 56, 22,
   39, 99, 71, 41, 28, 70, 54, 56, 56, 33, 57, 54, 32, 30, 46, 35, 25, 45, 19,
   143, 89, 73, 25, 55, 36, 60, 71, 43, 33, 57, 33, 2, 72, 62, 87, 20, 38, 35,
   73, 59, 54, 91, 130, 26)
   ```
   (b) Use the summary, mean, and sd functions to determine the quartiles, mean and standard deviation of this sample. Write down the results.
   (c) Use hist to produce a histogram of x, and boxplot to produce a box plot of x. Print out the result.
   (d) Use qqnorm to produce a QQ plot of x. Print out the result. Does the plot suggest that x is normal? Why or why not?