MAT 2377, Probability and statistics for engineers

Assignment 3

Deadline : Before 3 pm on Friday, March 6

Submit the assignment in the drop box at 585 King Edward

There are a total of 5 questions.

Solve the following exercises with a TI-30, TI-34, Casio FX-260 or Casio FX-300 calculator.

1. Suppose that the probability density function (p.d.f.) of the lifetime (in days) of a particular type of component is

$$f_X(x) = \frac{5\,x^4}{(500)^5}, \quad 0 \le x < 500.$$

   (a) What is the expected lifetime of a component?

   (b) Compute the standard deviation of the lifetime of a component?

   (c) Compute the probability a component will fail in less than 365 days.

   (d) Redundancy is an engineering principle to increase the reliability of a system by assembling components in parallel. To decrease the probability in part (c), three independent components are placed in parallel. So the system will fail only when all three components fail. Let $Y = \max\{X_1, X_2, X_3\}$ denote the lifetime of such a system, where X_i denotes the lifetime of the ith component. Show that

$$f_Y(y) = 15\,\frac{y^{14}}{(500)^{15}}, \quad y > 0.$$

   Hint: First construct $F_Y(y) = P(Y \le y)$, by noticing that

$$\{Y \le y\} = \{X_1 \le y\} \cap \{X_2 \le y\} \cap \{X_3 \le y\}.$$

   (e) Determine $P(Y \le 365)$ and compare it to the answer in part (c).

2. The surface flaws in plastic panels used in the interior of automobiles can be modeled as a Poisson process with a rate of 0.057 flaws per square foot of plastic panel. Assume an automobile interior contains 10 square feet of plastic panel.

(a) Let W be the surface area in square feet required to observe a flaw. Give the mean and the standard deviation of W.

(b) What is the probability that there are no surface flaws in an automobile's interior?

(c) We inspect the cars one at a time for flaws. What is the probability that the 10th inspected car is the 3rd car containing at least 2 flaws?

3. In a communication system, many messages arriving at a node are grouped into a packet before being transmitted in the network. Suppose that the message arrive at the node according to a Poisson process at a rate of 36 messages per minute. A packet is a grouping of 3 messages.

   (a) A packet has been formed and transmitted. What is the mean waiting time (in minutes) until the next packet is formed and transmitted?

   (b) What is the standard deviation of the waiting time to form a packet?

   (c) Compute the probability of waiting more than 15 seconds to form a packet.

   (d) Compute the probability of waiting less than 10 seconds to form a packet.

4. The strength of an aluminum alloy is normally distributed with a mean of 10 gigapascals (GPa) and a standard deviation of 1.4 GPa.

   (a) What is the probability that a specimen of this alloy will have a strength greater than 12 GPa?

   (b) Twenty five percent of the aluminum alloys have a strength less than x. Find x. (Note: x is called the 25th percentile).

   (c) Find the 90th percentile of the strengths of this alloy.

5. A film coating process produces films whose thickness is normally distributed with a mean of 110 microns and a standard deviation of 10 microns. For a certain application, the minimum acceptable thickness is 90 microns.

   (a) What proportion of films are going to be too thin?

(b) Assuming the standard deviation remains at 10 microns, to what value should the mean be set so that only 1% of the films are too thin?

(c) If the mean remains at 110 microns, what must the standard deviation be so that only 1% of the films are going to be too thin?