MAT 2377, Probability and statistics for engineers

Assignment 3 - solutions

Solve the following exercises with a TI-30, TI-34, Casio FX-260 or Casio FX-300 calculator.

[5] 1.

(a) The expected lifetime of a component is

$$\begin{aligned} E[X] &= \int_{-\infty}^{\infty} x\, f_X(x)\, dx = \int_0^{500} \frac{5\,x^5}{(500)^5}\, dx \\ &= \left[\frac{5\,x^6}{6(500)^5}\right]_0^{500} = 416.6667 \quad \text{days.} \end{aligned}$$

(b) The variance is

$$\sigma_X^2 = E[X^2] - \mu_X^2 = \left(\int_0^{500} \frac{5\,x^6}{(500)^5}\, dx\right) - (416.6667)^2 = 4960.2897.$$

The standard deviation of the lifetime of a component is $\sigma_X = \sqrt{4960.2897} = 70.4293$ days.

(c) We want

$$P(X < 365) = \int_0^{365} \frac{5\,x^4}{(500)^5}\, dx = 0.2073.$$

(d) The cumulative distribution function for Y (for $y > 0$) is

$$\begin{aligned} F_Y(y) &= P(Y \le y) = P(X_1 \le y)P(X_2 \le y)P(X_3 \le y) \text{ by independence} \\ &= \left[\int_0^{y} \frac{5\,x^4}{(500)^5}\, dx\right]^3 = \frac{y^{15}}{500^{15}}. \end{aligned}$$

Differentiating with respect to y gives the probability density function:

$$f_Y(y) = \frac{15\,y^{14}}{500^{15}}, \quad y > 0.$$

(e) We want

$$P(Y < 365) = F_Y(365) = \frac{(365)^{15}}{500^{15}} = 0.009.$$

The chances of failing in less than 365 days has been reduced from about 21% to 0.9%.

[5] 2. Let $N(t)$ be the number of flaws in t square feet. $N(t)$ has a Poisson distribution with mean $\mu = \lambda\, t = 0.057\, t$ flaws.

(a) W has an exponential distribution with rate λ=0.057 flaw. Its mean is $E[W] = 1/\lambda = 1/0.057 = 17.5438$ square feet and its standard deviation is $\sigma_W = 1/\lambda = 1/0.057 = 17.5438$ square feet.

(b) The probability that there are no surface flaws in an automobile's interior is

$$P(N(10) = 0) = e^{-(0.057)(10)} \frac{[(0.057)(10)]^0}{0!} = 0.5655.$$

(c) Let X be the number of inspected car required to observe the r = 3rd car with at least two flaws. X has a negative binomial distribution with $r = 3$ and

$$\begin{aligned} p &= P[N(10) \geq 2] = 1 - P[N(10) \leq 1] \\ &= 1 - \left[e^{-(0.057)(10)} \frac{[(0.057)(10)]^0}{0!} + e^{-(0.057)(10)} \frac{[(0.057)(10)]^1}{1!} \right] \\ &= 0.43447. \end{aligned}$$

We want

$$P(X = 10) = \binom{10-1}{3-1} p^3 (1-p)^7 = 0.0546.$$

[5] 3.

Let T be the time in minutes to form a packet. T has an Erlang distribution with r=3 messages and rate parameter $\lambda = 36$ messages per minute. Let $N(t)$ be the number of messages that arrive in an interval of t minutes. $N(t)$ has a Poisson distribution with mean $\mu = \lambda\, t = 36\, t$ packets.

(a) The mean waiting time for a packet is $E[T] = r/\lambda = 3/36 = 0.083333$ minutes.

(b) The standard deviation of the waiting time for a packet is $\sigma_T = \sqrt{r/\lambda^2} = \sqrt{3/36^2} = 0.04811$ minutes.

(c) Note that 15 seconds is 0.25 minute. We want

$$\begin{aligned} P(T > 15) &= P[N(0.25) \leq 2] \\ &= e^{36(0.25)} \left[\frac{(36(0.25))^0}{0!} + \frac{(36(0.25))^1}{1!} + \frac{(36(0.25))^2}{2!} \right] \\ &= 0.00623. \end{aligned}$$

(d) Note that 10 seconds is 0.16667 minute. We want

$$\begin{aligned} & P(T < 10) \\ = \ & P[N(0.16667) \geq 3] \\ = \ & 1 - P[N(0.16667) \leq 2] \\ = \ & 1 - e^{36(0.16667)} \left[\frac{(36(0.16667))^0}{0!} + \frac{(36(0.16667))^1}{1!} + \frac{(36(0.16667))^2}{2!} \right] \\ = \ & 0.9380. \end{aligned}$$

[5] 4. Let X be the strength (in GPa) of an aluminum alloy. X has a normal distribution with $\mu = 10$ and $\sigma = 1.4$.

(a) We want

$$\begin{aligned} P(X > 12) &= 1 - P\left(Z < \frac{12 - 10}{1.4}\right) \\ &= 1 - P(Z > 1.43) = 1 - 0.9236 = 0.0764. \end{aligned}$$

(b) We want x such that

$$0.25 = P(X < x) = P\left(Z < \frac{x - 10}{1.4}\right).$$

Thus,

$$-0.67 = \frac{x - 10}{1.4} \Rightarrow x = (-1.67)(1.4) + 10 = 7.662.$$

(c) The 90th percentile of the strengths of this alloy is a value x such that

$$0.90 = P(X < x) = P\left(Z < \frac{x - 10}{1.4}\right).$$

Thus,

$$1.28 = \frac{x - 10}{1.4} \Rightarrow x = (1.28)(1.4) + 10 = 11.792.$$

[5] 5. Let X be the thickness of a film in microns. It has a normal distribution with $\mu = 110$ microns and $\sigma = 10$ microns.

(a) We want

$$P(X < 90) = P\left(Z < \frac{90 - 110}{10}\right) = P(Z < -2) = 0.0228.$$

(b) We want μ such that

$$0.01 = P(X < 90) = P\left(Z < \frac{90 - \mu}{10}\right).$$

Thus,

$$-2.33 = \frac{90 - \mu}{10} \Rightarrow \mu = -(-2.33)(10) + 90 = 113.3.$$

(c) We want σ such that

$$0.01 = P(X < 90) = P\left(Z < \frac{90 - 110}{\sigma}\right).$$

Thus,

$$-2.33 = \frac{90 - 110}{\sigma} \Rightarrow \sigma = \frac{90 - 110}{2.33} = 8.5837.$$

[/**25**]