MAT 2377, Probability and statistics for engineers

Assignment 2

Deadline : Before 3 pm on Friday, February 13

Submit the assignment in the drop box at 585 King Edward

Solve the following exercises with a TI-30, TI-34, Casio FX-260 or Casio FX-300 calculator.

1. A ball bearing factory produces its product on three machines, A, B and C. Machine A produces 30% of the ball bearings, machine B produces 50% and machine C produces the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 2% from machine B and 4% from machine C.
   (a) What fraction of total production is defective?
   (b) A ball bearing is chosen at random from the production line and found to be defective. What is the probability that it came from
   (i) machine A, (ii) machine B, (iii) machine C?

2. Suppose that the probability mass function $f(x)$ for the discrete random variable X is given by the following table:

x	4	6	8	10
$f(x)$	0.4	0.3	0.2	0.1

   (a) Find $P(X = 6)$.
   (b) Find $P(X \leq 7)$.
   (c) Find the mean of X.
   (d) Find the standard devation of X.
   (e) What is the expected value of $7 + 10X$?

3. The waiting time, in minutes, between successive speeders spotted by a radar unit is a continuous random variable X with probability density function

$$f(x) = \begin{cases} 0 & \text{if } x < 3, \\ \frac{81}{x^4} & \text{if } x \geq 3. \end{cases}$$

   (a) Find the cumulative distribution function of X. Hint: it is 0 for $x < 3$.
   (b) Find the probability of waiting more than 5 minutes between successive speeders
      (i) using the cumulative distribution function of X,
      (ii) using the probability density function of X.
   (c) Find the expected waiting time between successive speeders.
   (d) Find the standard deviation of X.

4. Suppose that the joint probability distribution of X and Y is given by

$$f(x, y) = \frac{x + y}{30}, \quad \text{for } x = 0,\ 1,\ 2,\ 3,\ y = 0,\ 1,\ 2.$$

   (a) Find the marginal distribution for X.
   (b) Find the marginal distribution for Y.
   (c) Find $P(X = 1, Y = 1)$, $P(X = 1)$, and $P(X = 1|Y = 1)$.
   (d) Use the result from part (c) to determine whether X and Y are statistically independent. Explain your reasoning in at most one line.
   (e) Find the covariance of X and Y.

5. Suppose that the engines on an airplane fail independently, each with probability 0.3. Assume that a plane makes a safe flight if at least half its engines do not fail.
   (a) What is the probability of a successful flight of a 2-engine plane?
   (b) What is the probability of a successful flight of a 4-engine plane?

6. Suppose that a student has a 25% probability of getting a perfect score on any assignment, and that the scores on separate assignments are independent.
   (a) What is the probability that the second perfect score will occur on the sixth assignment?
   (b) What is the expected number of assignments the student would write to get her first perfect score?

7. (**Bonus**)A large company has an inspection system for the batches of small compressors purchased from vendors. A batch typically contains 15 compressors. In the inspection system, a random sample of 5 is selected and all are tested. Suppose there are 3 faulty compressors in the batch of 15. What is the probability that a given sample will have exactly 2 faulty compressors?