MAT 2377, Probability and statistics for engineers

Assignment 1 - Solutions

[5] 1. Let E be the event of needing engine repairs while under the warranty and D be the event of needing repairs to the drive train while under the warranty. We are given : $P(E) = 0.85$, $P(D) = 0.3$ and $P(E \cup D) = 0.98$.

(a) We want $P(E \cup D) = 0.98$.
(b) We want $P(E \cap D) = P(E) + P(D) - P(E \cap D) = 0.85 + 0.3 - 0.98 = 0.17$.
(c) We want $P(E' \cap D') = 1 - P(E \cup D) = 1 - 0.98 = 0.02$.
(d) We want $P(E \cap D') = P(E) - P(E \cap D) = 0.85 - 0.17 = 0.68$.
(e) We want $P(E' \cup D') = 1 - P(E \cap D) = 1 - 0.17 = 0.83$.

[3] 2. Let A be the event that at least one defective card is in the sample. Its complement A' is the event that there are no defective cards in the sample. We have

$$P(A') = \frac{\binom{40}{10}}{\binom{45}{10}} = 0.26571.$$

Thus, $P(A) = 1 - P(A') = 1 - 0.26571 = 0.73429$.

[3] 3. The first number is one of 60 possible numbers. There are 57 possible choices for the second and also for the third number. So by the multiplication principle, there are $60 \times 57 \times 57 = 194,940$ possible arrangements.

[3] 4. There are ${}_5P_5 = 5! = 120$ different arrangements of the five machining operations and ${}_5P_5 = 5! = 120$ different arrangements of the five assembly operations. By the multiplication principle, there are $(120)(120) = 14,400$ different possible production sequences.

[6] 5. Let D_i be the event that the ith device is functional, for $i = 1, 2, \ldots, 9$. We will define the sub circuit 10 as the devices 3, 6, and 9, which are put in parallel. The probability that this sub circuit is operational is

$$\begin{aligned} P(D_{10}) = P[D_3 \cup D_6 \cup D_9] &= 1 - P(D_3')P(D_6')P(D_9') \\ &= 1 - (0.05)^3 = 0.999875. \end{aligned}$$

We will define the sub circuit 11 as the devices 1 and 2, which are put in series. The probability that this sub circuit is operational is

$$P(D_{11}) = P(D_1 \cap D_2) = P(D_1)P(D_2) = (0.99)^2 = 0.9801.$$

Similarly, we can define sub circuit 12 as the devices 4 and 5 and the sub circuit 13 as the devices 7 and 8. We have $P(D_{12}) = P(D_4)P(D_5) = 0.9801$ and $P(D_{13}) = P(D_7)P(D_8) = 0.98^2 = 0.9604$. We will denote the sub circuits 11, 12 and 13 as the larger sub circuit 14. Since 11, 12 and 13 are assembled in parallel, then

$$\begin{aligned} P(D_{14}) = P[D_{11} \cup D_{12} \cup D_{13}] &= 1 - P(D_{11}')P(D_{12}')P(D_{13}') \\ &= 1 - (1 - 0.9801)(1 - 0.9801)(1 - 0.9604) \\ &= 0.9999843. \end{aligned}$$

The circuit will be operational if and only if 14 and 1 are functional, thus the probability that the circuit is operational is

$$P(D_{14} \cap D_1) = P(D_{15})P(D_1) = (0.9999843)\,(0.999875) = 0.9998593.$$

[4] 6. (a) We want $P(A) = N(A)/N(S) = 94/104 = 0.9038$.

(b) We want $P(A|B) = P(A \cap B)/P(B) = (6/104)/(10/104) = 0.6$.

(c) Since $P(A|B) \neq P(A)$, then the events A and B are not independent.

(d) Since A and B are not disjoint, that is $A \cap B \neq \emptyset$, then A and B are not mutually exclusive.

[/**24**]