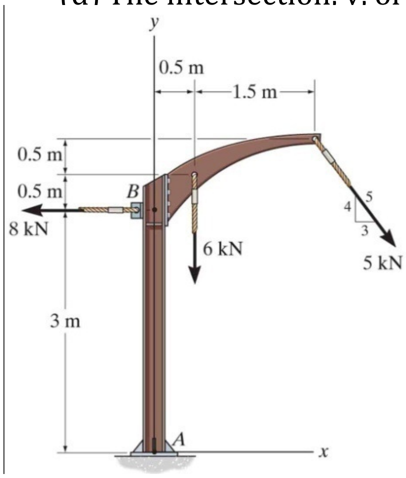
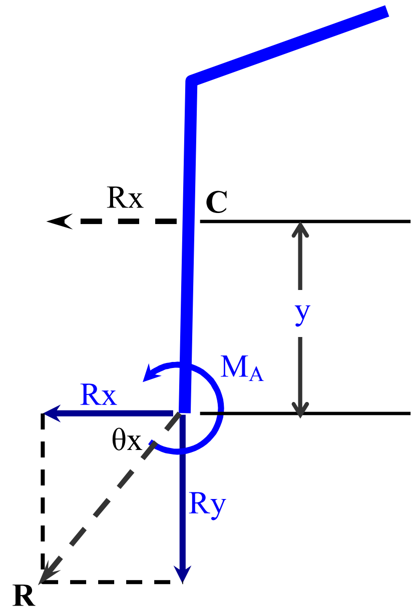
Problem 1 (15 Marks) F 4.34 (p 178)

For the system shown, determine

(a) The rectangular components of the resultant force, R.
(b) The magnitude of the resultant force, R.
(c) The orientation, θ_X, of the resultant force, with respect to the horizontal axis.
(d) The intersection, y, of the line of action of the resultant with the y-axis





Force Resultant at A

$$Rx = -8 + \left(\tfrac{3}{5}\right)5 = -5\text{ kN} \blacktriangleleft \text{(a)}$$

$$Ry = -6 - \left(\tfrac{4}{5}\right)5 = -10\text{ kN} \blacktriangleleft \text{(a)}$$

$$R = \sqrt{R_x^2 + R_y^2} = 11.2\text{ kN} \blacktriangleleft \text{(b)}$$

$$\theta_x = Tan^{-1}\left[\frac{Ry}{Rx}\right] = Tan^{-1}[2] = 63.4^{\circ} \blacktriangleleft \text{(c)}$$

Moment Resultant at A

$$M_A = 8(3) - 6(0.5) - \left(\tfrac{3}{5}\right)5(3 + 0.5 + 0.5) - \left(\tfrac{4}{5}\right)5(0.5 + 1.5)$$

$$M_A = 24 - 3 - 12 - 8 = 1\text{ kN.m}$$

Move resultant force from A to C

(1) Slide Ry upwards along its vertical line of action.

[This does not produce any moment]

(2) Slide Rx upwards, parallel to its line of action

[This " carries" a moment = Rx (y)]

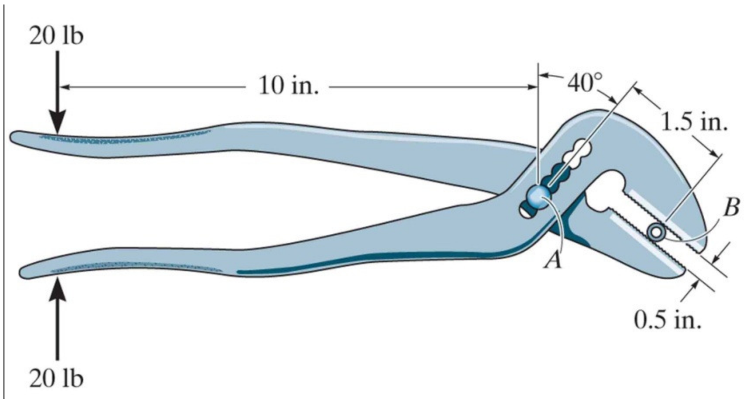
The resultant moment at C should be zero

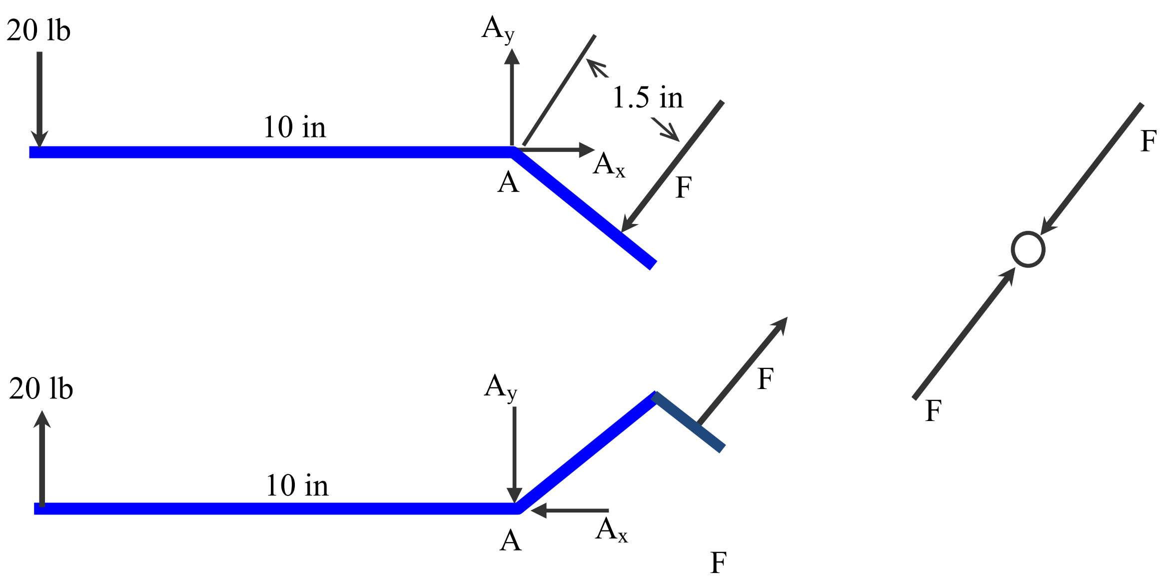
$$M_C = -Rx\,(y) + Ry(0) + M_A = 0$$

Solve for y

$$y = \frac{M_A}{Rx} = \frac{1\text{ kN.m}}{5\text{ kN}} = 0.200\text{ m} \qquad \text{(d)}$$

Problem 2(10Marks) 6.127 (p326)
For the pliers shown:
(a) Draw a free body diagram of the top handle
(b) Draw a free body diagram of the bottom handle
(c) Draw a free body diagram of the smooth pipe at B
(d) Determine the clamping force exerted on the smooth pipe at B if a force of 20 lb is applied to the handles of the pliers. The pliers are pinned at A.





From the FBD of the top handle:

$$\Sigma M_A = 0$$

$$20(10) - F(1.5) = 0$$

$$F = \frac{200}{1.5} = 133 \text{ lb}$$

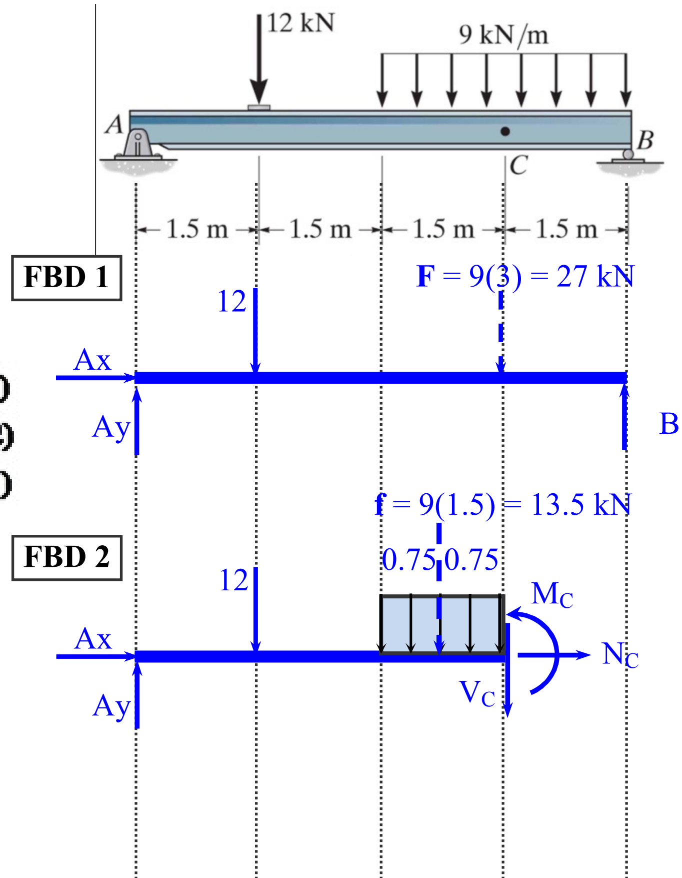
Problem 3(15Marks) F 7.4 (p337)

For the beam shown, determine:

(a) Reactions at A and B

(b) Internal forces at point C

- Axial Force, N_C
- Shear Force, V_C
- Bending Moment, M_C



(a) Reactions. From FBD 1

$\Sigma Fx = 0 \rightarrow \quad Ax = 0$ ____________________ (1)

$\Sigma Fy = 0 \rightarrow \quad Ay - 12 - F + B = 0$ ____________ (2)

$\Sigma M_B = 0 \rightarrow \; -Ay(6) + 12(4.5) + 27(1.5) = 0$ __ (3)

From (3) $\quad Ay = \dfrac{94.5}{6} = 15.75 = 15.8 \text{ kN}$

From (2) $\quad B = 27 + 12 - 15.75 = 23.25 \text{ kN}$

(b) Internal forces at C. From FBD 2

$\Sigma Fx = 0 \rightarrow \quad N_C = 0$

$\Sigma Fy = 0 \rightarrow \quad 15.75 - 12 - 13.5 - V_C = 0$

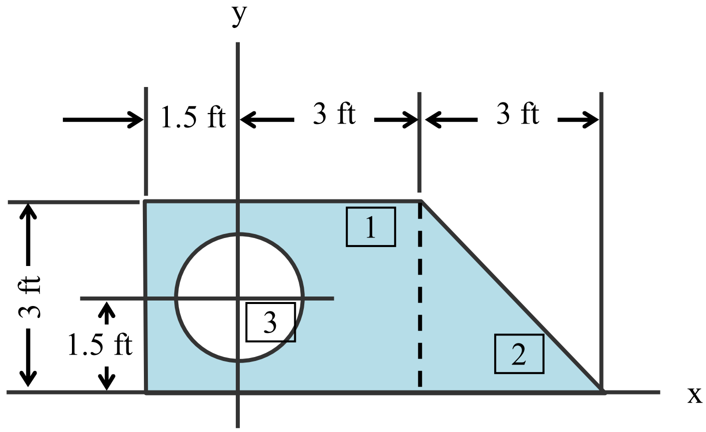
$V_C = -9.75 = 9.8 \text{ kN} \uparrow\downarrow$

$\Sigma M_C = 0 \rightarrow \; -15.75(4.5) + 12(3) + 13.5(0.75) + M_C = 0$

$M_C = +24.75 = 24.8 \text{ kN.m}$

Problem 4(20Marks) 9.60 (p 479)

A 2 ft diameter hole is punched out of the trapezoidal plate shown. Locate the coordinates $\bar{x}$ and $\bar{y}$ for the centre of gravity of this composite.

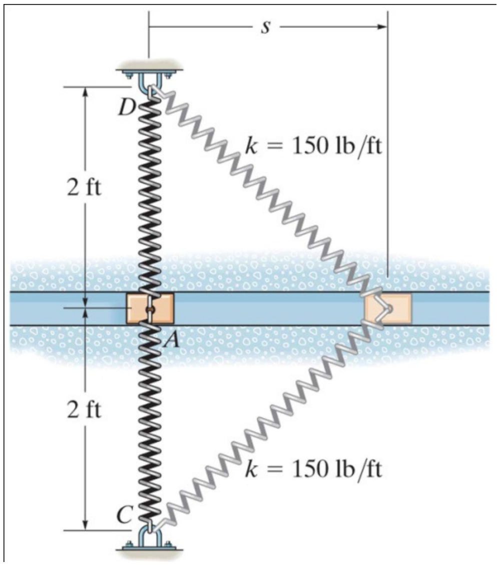


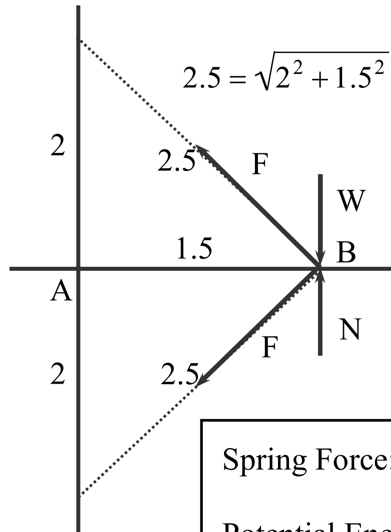
Component	A_i (ft^2)	x_i (ft)	y_i (ft)	$A_i x_i$ (ft^3)	$A_i y_i$ (ft^3)
Rectangle (1)	3(4.5)=13.5	0.5(4.5)-1.5=0.75	0.5(3)=1.5	(13.5)(0.75)	(13.5)(1.5)
Triangle (2)	0.5(3)(3)=4.5	3+3/3=4	3/3=1	(4.5)(4)	(4.5)(1)
Circle (3)	$-\Pi(1)^2=-\Pi$	0	1.5	0	-1.5 Π
	∑=14.858			∑=28.125	∑=20.038

Centre of Gravity:

x = 28.125 / 14.858 = 1.89 ft
y = 20.038 / 14.858 = 1.35 ft.

Problem 5(20Marks) 14.79 (p 211)
Block A has a weight of 1.5 lb and slides in the smooth horizontal slot. If the block is drawn back to s = 1.5 ft and released from rest, determine its speed at the instant s = 0. Each of the two springs has a stiffness of k = 150 lb/ft and an unstretched length of 0.5 ft.





Length units in ft.

Spring Force: F = k ΔL

Potential Energy of Spring: $V = ½\, k\, \Delta L^2$

Conservation of Energy → $T_B + V_B = T_A + V_A$

$$T_B = \tfrac{1}{2} m v_B^2 = \tfrac{1}{2} m\, 0^2 = 0$$

$$V_B = 2\left[\tfrac{1}{2} k \Delta L^2\right] = k \Delta L^2 \rightarrow \text{Two Springs}$$

$$V_B = 150(2.5 - 0.5)^2 = 600J$$

$$T_A = \tfrac{1}{2} m v_A^2 = \tfrac{1}{2}\left(\frac{W}{g}\right) v_A^2 = \tfrac{1}{2}\left(\frac{1.5}{32.2}\right) v_A^2 = \frac{v_A^2}{42.933}$$

$$V_A = k \Delta L^2 = 150(2 - 0.5)^2 = 337.5J$$

Conservation of Energy

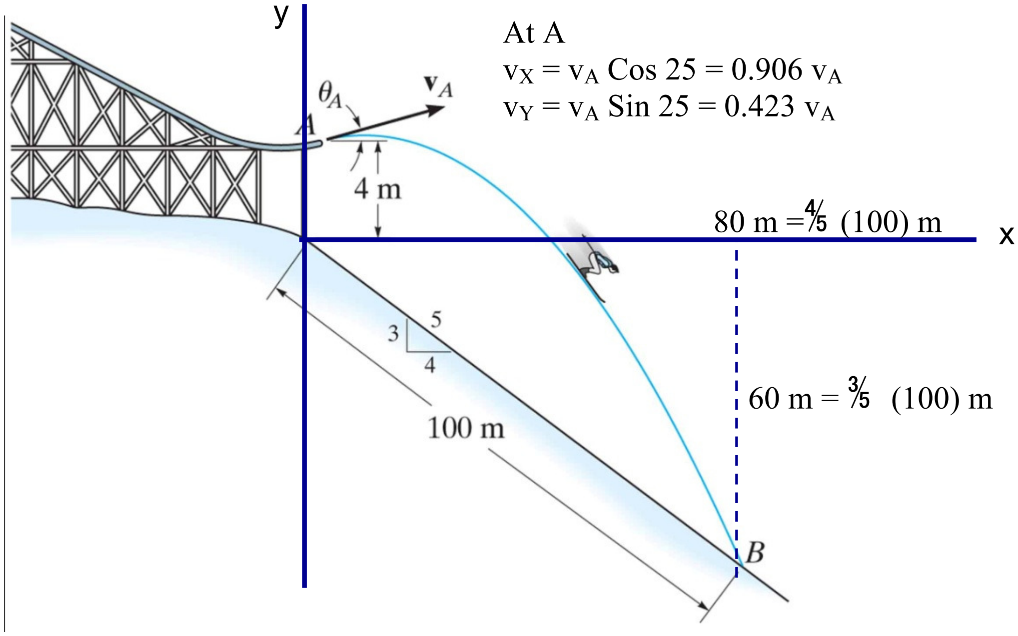
$$0 + 600 = \frac{v_A^2}{42.933} + 337.5$$

$$v_A^2 = (600 - 337.5)42.933 = 11269.9$$

$$v_A = 106.2 ft/s \quad \triangleleft$$

Problem 6(20Marks) 12.110 (p 52)

The skier leaves the ramp A at an angle $\theta_A = 25^o$ with the horizontal. If she strikes the ground at B, determine her initial speed v_A and the time of flight t_{AB}.



Horizontal Motion

$$x_B = v_x t_B$$

$$80 = 0.906 v_A t_B$$

$$t_B = \frac{88.3}{v_A}$$

Vertical Motion

$$y = y_0 + v_y t_B + \tfrac{1}{2} a_c t_B^2$$

$$-60 = 4 + 0.423 v_A t_B + \tfrac{1}{2}(-9.8) t_B^2$$

Substitute $t_B = \dfrac{88.3}{v_A}$

$$0 = 64 + 0.423 v_A \left(\frac{88.3}{v_A}\right) - 4.9\left(\frac{88.3}{v_A}\right)^2$$

Solving

$v_A = 19.4$ m/s ◁

$t_B = \dfrac{88.3}{v_A} = \dfrac{88.3}{19.4} = 4.6$ s ◁