

COMP 1805 — Test 1

Student Name:

Student Number:

25 marks total

Instructions: This is a closed book test. No calculators are allowed. All questions should be answered in the space provided. You have 45 minutes.

Question 1: (3 marks) Use a truth table to show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

Question 2: (4 marks) Use logical equivalences to show that $p \rightarrow (\neg q \wedge r)$ and $\neg p \vee \neg(r \rightarrow q)$ are logically equivalent.

Question 3: (4 marks) Let $P(x, y)$ be the predicate " $x^2 = y + 17$ " and let the universe of discourse be the set of real numbers. Determine the truth value of the following proposition, and justify your answer:

$$\neg \exists x \forall y : \neg P(x, y).$$

Question 4: (4 marks) Give a proof by contradiction of the following statement: If n is an integer and n^2 is odd, then n is odd.

Question 5: (4 marks) Let $A = \{x, y\}$ and $B = \{x, \{x\}\}$. For each of the following four statements, say whether it is true or false:

- $x \subseteq B$.
- $\emptyset \in P(B)$.
- $\{x\} \subseteq A - B$.
- $|P(A)| = 4$.

Question 6: (6 marks) Let N be the set of all possible novels written in French. Show that the set N is countable.