Final
EXAMINATION
FALL 2012

DURATION: 3 HOURS **No. Of Students: about 325 expected**

Department Name & Course Number: ECOR 2606 A, B & C

Course Instructor (s): John Bryant, Bruce Wallace

AUTHORIZED MEMORANDA
Approved calculator and instructor supplied crib sheet

Students MUST count the number of pages in this examination question paper before beginning to write, and report any discrepancy to a proctor. This question paper has ____11____pages + cover page = _12__ pages in all.

This examination question paper may not be taken from the examination room.

In addition to this question paper, students require: an examination booklet no
Scantron Sheet no

Name: ____

Student Number: ____

There are 6 questions worth a total of 100 marks. Attempt all of them.

Answer all questions on the question paper. An extra page has been provided at the end. If you need extra space for any question make a note ("See extra page") and complete your answer on the extra page.

Do NOT write in the boxes below.

Q1	Q2	Q3	Q4	Q5	Q6	Total

Question 1 (18 Marks)

Suppose that we would like solve the following series of equations:

$$20x_1 + 16x_2 + 12x_3 = 56$$
$$5x_1 + 5x_2 - 2x_3 = -3$$
$$10x_1 + 6x_2 + 8x_3 = 30$$

i) Demonstrate your understanding of Gaussian elimination by reducing the augmented matrix to upper triangular form. Use row pivotting as required and apply the algorithm as taught (do NOT use the "student" variation).

ii) Demonstrate your understanding of back substitution by using it to find x_1, x_2, and x_3.

iii) Suppose that we would like to use an iterative approach to solve the following system of equations.

$$\begin{aligned} 5x_1 + x_2 - 2x_3 &= 40 \\ x_1 + 4x_2 - 3x_3 &= -8 \\ 4x_1 - 2x_2 + 8x_3 &= 24 \end{aligned}$$

Assuming no rearrangement of rows, what would matrix *C* and vector *d* look like?

iv) Assuming the Gauss-Seidel technique (reminder: this is that one that updates as it goes along) and that $\mathbf{x}_0$ = [0 0 0]', what would $\mathbf{x}_1$ look like

$\mathbf{x}_0$
0
0
0

$\mathbf{x}$(1) updated
0
0

$\mathbf{x}$(2) updated
0

$\mathbf{x}_1$

v) Would you expect the Gauss-Seidel technique to converge in this case? Explain why (there are no marks for just "yes" or "no").

Question 2 (18 marks)

(i) The curves defined by $f(x) = e^x$ and $g(x) = 3x$ intersect at some point between x = 1 and x = 2. Give all of the Matlab code necessary to locate the point of intersection and output its x and y coordinates.

(ii) How well does the curve $y = x^2 + 3x$ fit the following set of data points? Calculate the sum of the squares of the errors and the correlation coefficient. The extra rows in the table are for your convenience. You may use them as you see fit. The average of the *y* values is 8.0.

x	0	1	2	3
y	0.2	3.6	9.9	18.3

Sum of squares of errors: ________________________

Correlation coefficient: _________________________

iii) Write a Matlab function that fits a curve of the form $y = Ax^B$ to a set of data points. Your function should accept a vector of x values and a vector of y values. It should return the values of A and B. The sample code below illustrates how your function might be used. Your function must be compatible with this example.

```
x = [ .........];  % actual values omitted
y = [...........]; % actual values omitted
[A, B] = powerFit (x, y);
fprintf ('The best fit curve is %f * x ^ %f.\n',  A, B);
```

iv) It has been suggested that the relationship between the ion product of water (K_w) and absolute temperature (T_a) can be modelled using an equation of the form

$$-log_{10}\, K_w = \frac{A}{T_a} + B log_{10} T_a + C T_a + D$$

where A, B, C, and D are constants. Assume a set of data values stored in two vectors (called Ta and Kw). The best fit values for A, B, C, and D can be computed using a technique taught in this course and a bit of ingenuity. Give all of the Matlab code necessary to compute and output the values.

Question 3 (12 Marks)

Assume that the data in the following table has been placed in two Matlab vectors as shown.

Temperature (˚C)	10	20	30	40	50	60	70	80
Pressure (kPa)	1.2281	2.3392	4.2469	7.3851	12.352	19.947	31.202	47.416

T = 10:10:80;   P = [1.2281 2.3392 4.2469 7.3851 12.352 19.947 31.202 47.416];

(i) Give Matlab code that plots pressure versus temperature for temperatures from 10 to 80 ˚C. Two curves should be plotted on the same graph. The first curve (to be shown in black) should be based on the following assumptions

- the data is experimental (probably contains errors)
- theory suggests that $P = aT^3 + bT^2 + cT + d$, where a, b, c, and d are constants

The second curve (to be shown in red) should be based on the assumption that the data is precise (that are no errors).

The curves should be smooth. Use 100 temperatures values to in creating each of them. You do NOT have to give the graph a title and label the axes.

(ii) Briefly explain the difference between regression and interpolation. Which of the two did you use for your red curve and for your black curve? If you used interpolation in either case, justify your choice of interpolation method.

Question 4 (16 marks)

Assume that we have some tabulated values for an unknown function $y = f(x)$ and that these data points are stored in two Matlab vectors as shown below.

```
x = [ 0    3    7    10 ];          y = [ 7   -32   84   507 ];
```

i) Suppose that we would like to estimate the first derivate of the function at $x = 6$. One approach is to use all of the points to generate an interpolating polynomial, differentiate this polynomial, and then evaluate the derivative at $x = 6$. Write Matlab code that implements this approach. Have your code output both the coefficients of the interpolating polynomial (in the form of a self explanatory message) and the final answer.

ii) Assume the following tabulated values for some unknown function $y = f(x)$.

x	2	5	8	11	14
y	-14	25	118	265	466

Use difference formulas to estimate the first derivate of the function at $x = 2$ and $x = 8$. You need not employ Romberg extrapolation and only the formulas on the reference sheet should be considered, but within this framework you should come up with the best possible estimates.

iii) In theory students in this course should all be able to generate interpolating polynomials manually. Demonstrate that you can do this by finding the polynomial in *x* that passes through all of the following points:

x	-1	0	1	2
y	2	6	16	44

You can use whichever method you prefer but must show all of your work. You need not simplify your polynomial but must evaluate it at $x = 1.5$.

Value of polynomial at $x = 1.5$: ______________________________

Question 5 (18 marks)

Suppose that we would like to evaluate $\int_4^8 e^x/_{x^3}\, dx$

To save you some time some values of $e^x/_{x^3}$ are tabulated below. You may need to calculate the value of the function at other points. Show all answers with five decimal places.

x	4	5	6	7	8
$e^x/_{x^3}$	0.85310	1.18731	1.86773	3.19718	5.82218

i) Manually evaluate the integral using trapezoidal integration and all of the tabulated data points.

ii) Trapezoidal integration with $h = 4$ gives a result of 13.3506 and trapezoidal integration with $h = 2$ gives a result of 10.4107. Based on these results and you answer to part (i), what is the best possible estimate of the integral?

iii) Estimate the relative error in your answer to part (ii).

(iv) Manually estimate the integral using Simpson's 1/3 Rule and all of the data points.

(v) Manually estimate the integral using three point Gaussian quadrature.

(vi) Suppose that you had access to Matlab and would like the best possible estimate of the integral. How would you go about evaluating the integral? Give all of the necessary commands.

Question 6 (18 marks)

i) The relationship between some quantity y and time t is defined by

$$\frac{dy}{dt} = -2y + 4t$$

Demonstrate your understanding of Euler's method by using it to complete the following table.

i	t_i	y_i	φ (slope)
0	0	20	
1	0.5		
2	1.0		

ii) What would y_1 be if the midpoint method was used with a step size of 1 second?

iii) What would y_1 be if the Heun's method (without iteration) was used with a step size of 2 seconds?

iv) Give all of the Matlab code required to produce a table showing t (first column) and y (second column) for t from 0 to 4 seconds in steps of 0.2 seconds. Use function *ode45*. Display times with one decimal place and y values with four. The columns should of course line up nicely.

Extra space (can be used for any question).