Ryerson University
Department of Mechanical and Industrial Engineering
MEC613 – Machine Design I
Winter 2013 Midterm Examination

Instructor: D. Oguamanam

Duration: **110 mins**

Instructions

(1) *This examination is OPEN BOOK, but only the course textbook, notes, and tutorial problems are allowed.*
(2) *Communicable and/or programmable calculators are not permitted.*
(3) *Cellphones and smartphones are not permitted.*
(4) ***Photocopied Textbooks** are not permitted.*
(5) *Materials that are deemed to infringe copyright laws are not permitted*
(6) *Please read the questions carefully.*
(7) *To maximize your chances of doing well, please **show all your work** in the space below each question, **clearly** stating the formulae employed, and any assumptions.*

Good luck!!!

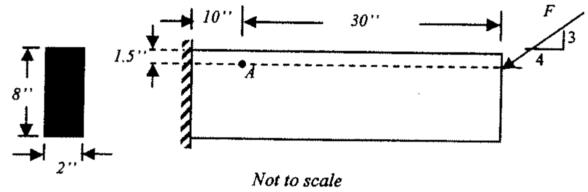
NAME:	Solution
STUDENT NUMBER:	
SECTION:	

Name: ______________________

Student Id: ______________________

Question 2: (25 marks)

The figure is a schematic of a rectangular cross section shaft that is loaded by a concentrated force F lbf which produces only axial and bending deformation (i.e. no torsion).

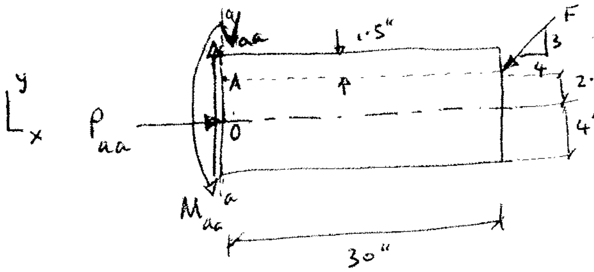


(a) Determine the stresses at point A.

(b) Sketch the state of stress on an infinitesimal element located at point A.

(b) Determine the von Mises stress at point A.

(a) $I = \frac{bh^3}{12} = \frac{2(8)^3}{12} = \frac{256}{3}\ in^4 \approx 85.3333\ in^4$ $\quad A = 2(8) = 16\ in^2$



$+\uparrow \sum F_y = 0: \quad V_{aa} - \frac{3}{5}F = 0 \Rightarrow V_{aa} = \frac{3}{5}F$ lbf $\uparrow$

$\overset{+}{\rightarrow} \sum F_x = 0: \quad P_{aa} - \frac{4}{5}F = 0 \Rightarrow P_{aa} = \frac{4}{5}F$ lbf $\rightarrow$

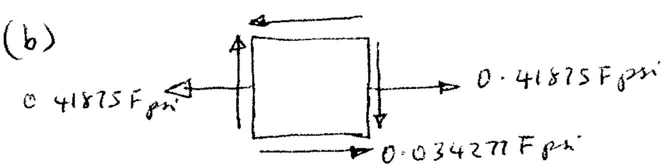
$(+\sum M_{o} = 0: \quad M_{aa} - \frac{3}{5}F(30) + \frac{4}{5}F\left(\frac{5}{2}\right) = 0$

$\therefore M_{aa} = 16F$ lbf-in.

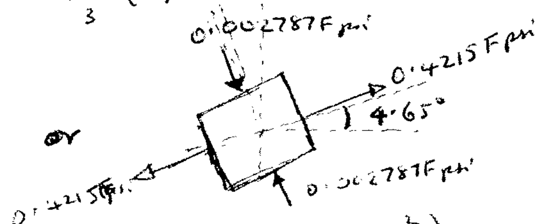
The normal stress @ A $\sigma_A = \frac{M_{aa}(5/2)}{I} - \frac{P_{aa}}{A} = \frac{16F(5/2)}{\frac{256}{3}} - \frac{\frac{4}{5}(F)}{16} = \frac{67}{160}F$ psi $\approx 0.41875F$ psi

The shear stress @ A $\tau_A = \frac{V_{aa}Q}{Ib} = \frac{\frac{3}{5}F\left[2\left(\frac{3}{2}\right)\left(\frac{3}{4}+\frac{5}{2}\right)\right]}{\frac{256}{3}(2)} = \frac{351}{10240}F \approx 0.034277F$ psi

(b)



or



(c) $2\sigma_V^2 = (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)$

Let $\sigma_x = \sigma_A$ and $\tau_{xy} = \tau_A$ $\Rightarrow \sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$

$\therefore 2\sigma_V^2 = 2\sigma_A^2 + 6\tau_A^2 \Rightarrow \sigma_V^2 = \sigma_A^2 + 3\tau_A^2$

$= \left(\frac{67}{160}F\right)^2 + 3\left(\frac{351}{10240}F\right)^2$

$\sigma_V = \frac{F}{10240}\left[(64\times 67)^2 + 3(351)^2\right]^{1/2} = 0.4229F$ psi

or $2\sigma_V^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$

$= 2(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$

$\therefore \sigma_V^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$

$= (0.4215F)^2 - (0.4215F)(-0.002787F) + (0.002787F)^2$

$= 178.8447\times 10^{-3}F^2$

$\therefore \sigma_V = 0.4229F$ psi

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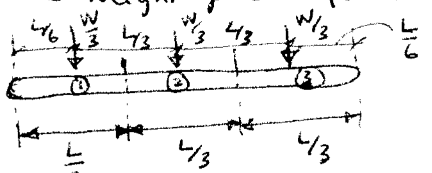
Question 3: (25 marks)

A uniform diameter shaft is 750 mm long between bearings, and it is rotating at a speed of 3000 rpm. The specific weight of the steel material $\gamma = 80$ kN/m^3 and its Young's modulus of elasticity $E = 200$ GPa. Take acceleration due to gravity to be 9.81 m/s^2.

(a) Derive an expression for the critical speed of the shaft using the Rayleigh method with the shaft segmented into three elements.

(b) Use the expression determined in (a) to estimate the minimum diameter of the shaft.

The weight of the shaft $W = \gamma A L = 80\times10^3\left(\frac{\pi d^2}{4}\right)0.75 = 47{,}123.9\, d^2$ N



rotating speed $n = 3000$ rpm $\equiv \frac{2\pi}{60}(3000) = 100\pi$ rad/s

$I = \frac{\pi d^4}{64}$ m^4 $\quad W_1 = W_2 = W_3 = \frac{W}{3} = 15{,}708.0\, d^2$ N

(a) Influence coefficients:

$$\delta_{11} = \delta_{33} = \frac{\left(\frac{L}{6}\right)\left(\frac{5L}{6}\right)}{6EIL}\left[L^2 - \left(\frac{L}{6}\right)^2 - \left(\frac{5L}{6}\right)^2\right] = \frac{5L^2}{6^3 EIL}\left(\frac{10L^2}{6^2}\right) = \frac{50L^3}{6^5 EI} \approx \frac{276.310655\times10^{-15}}{d^4}\ \frac{m}{N}$$

$$\delta_{12} = \delta_{32} = \delta_{21} = \delta_{23} = \frac{\left(\frac{L}{6}\right)\left(\frac{L}{2}\right)}{6EIL}\left[L^2 - \left(\frac{L}{6}\right)^2 - \left(\frac{L}{2}\right)^2\right] = \frac{L^2}{2(6^2)EI}\left(\frac{26L^2}{36}\right) = \frac{13L^3}{6^4 EI} \approx \frac{431.044638\times10^{-15}}{d^4}\ \frac{m}{N}$$

$$\delta_{13} = \delta_{31} = \frac{\left(\frac{L}{6}\right)\left(\frac{L}{6}\right)}{6EIL}\left[L^2 - \left(\frac{L}{6}\right)^2 - \left(\frac{L}{6}\right)^2\right] = \frac{L^2}{6^3 EIL}\left(\frac{34L^2}{36}\right) = \frac{34L^3}{6^5 EI} \approx \frac{187.891252\times10^{-15}}{d^4}\ \frac{m}{N}$$

$$\delta_{22} = \frac{\left(\frac{L}{2}\right)\left(\frac{L}{2}\right)}{6EIL}\left[L^2 - \left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^2\right] = \frac{L^2}{4(6)EIL}\left(\frac{2L^2}{4}\right) = \frac{L^3}{48EI} \approx \frac{895.246555\times10^{-15}}{d^4}\ \frac{m}{N}$$

Deflections:

$$y_1 = y_3 = W_1\delta_{11} + W_2\delta_{12} + W_3\delta_{13} = \frac{W}{3}\left(\frac{50L^3}{6^5EI} + \frac{13L^3}{6^4EI} + \frac{34L^3}{6^5EI}\right) = \frac{54WL^3}{6^5EI} \approx \frac{14.0625\times10^{-9}}{d^2}\ m$$

$$y_2 = W_1\delta_{21} + W_2\delta_{22} + W_3\delta_{23} = \frac{W}{3}\left(\frac{13L^3}{6^4EI} + \frac{L^3}{48EI} + \frac{13L^3}{6^4EI}\right) = \frac{106WL^3}{6^5EI} \approx \frac{27.6041667\times10^{-9}}{d^2}\ m$$

$$\omega = \sqrt{\frac{g\sum W_i y_i}{\sum W_i y_i^2}} \Rightarrow \omega^2 = \frac{g\sum y_i}{\sum y_i^2} = \frac{g\left[2\left(\frac{54}{6^5}\right) + \frac{106}{6^5}\right]\frac{WL^3}{EI}}{\left[2\left(\frac{54}{6^5}\right)^2 + \left(\frac{106}{6^5}\right)^2\right]\left(\frac{WL^3}{EI}\right)^2}$$

$$= \frac{g\,6^5\,214}{17068}\frac{EI}{WL^3} \equiv \frac{g\,6^5\,214}{17068}\left(\frac{Ed^2}{16L^4\gamma}\right) = \frac{26001g}{4267}\left(\frac{Ed^2}{\gamma L^4}\right)$$

$\therefore\ \omega = 21732.8\, d$ rad/s

(b) The guideline is that $\omega_{crit} = \omega_1 \geq 2\times$ operating speed

$\Rightarrow 21732.8d \geq 200\pi$

$\therefore\ d \geq 28.9111\times10^{-3}$ m

Hence the estimated minimum shaft diameter $d_{min} \approx 30$ mm