R1 Lec-1 : 07/01/2015

Set : A well defined collection of distinct objects.
Sets are ~~dots~~ denoted with capital letters.

well defined : There is a rule that enables us to determine whether a given object is an element of the s

Distinct : Different.

$C = \{A, B, C, D, \ldots, Z\}$ ⇒ People in the classroom.

C ← Set of students in a classroom. A ← element

Null set $= \phi = \{\ \}$; Universal set $= U$

$E = \{0, 2, 4, 6, 8, 10\}$ ⇒ Set of even-digits.

Roster method :

$O = \{x \mid x \text{ is an odd digit}\} = \{1, 3, 5, 7, 9\}$

Correct listings : $\{10, 20, 30\}$, $\{30, 20, 10\}$, $\{20, 30, 10\}$
Incorrect listings : $\{10, 20, 30, 10\}$, $\{20, 30, 30, 20\}$

Equal set : $A = B$ if say $A = \{10, 20, 30\}$ and $B = \{30, 20, 10\}$

Sub-set : Say $C = \{1, 2, 3, 4, 5, 6\}$ and $D = \{1, 2, 3\}$
$E = \{4, 5, 6\}$.

Then $D \subseteq C$
$E \subseteq C$

Intersection : $C \cap D = \{1, 2, 3\}$, $D \cap E = \phi$
$C \cap E = \{4, 5, 6\}$

Union : $C \cup D = \{1, 2, 3, 4, 5, 6\}$, $D \cup E = \{1, 2, 3, 4, 5, 6\}$
$C \cup E = \{1, 2, 3, 4, 5, 6\}$

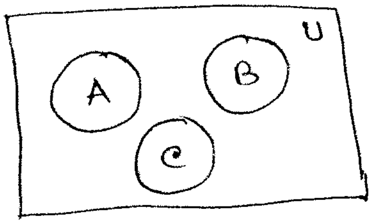
Complement : If $U = \{1, 2, 3, 4, 5, 6\}$ and $D = \{1, 2, 3\}$

Then $\bar{D} = \{4, 5, 6\}$, i.e. $D \cup \bar{D} = U$ and $D \cap \bar{D} = \phi$

Venn diagram :

1. $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,2,3\}$, $B = \{4,5,6\}$
$C = \{7,8,9,10\}$

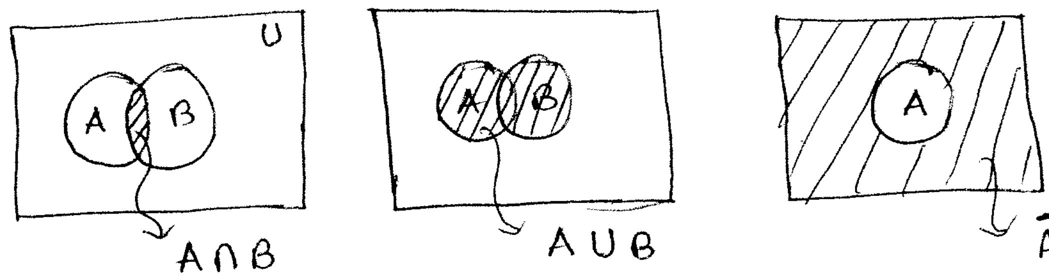
Then,



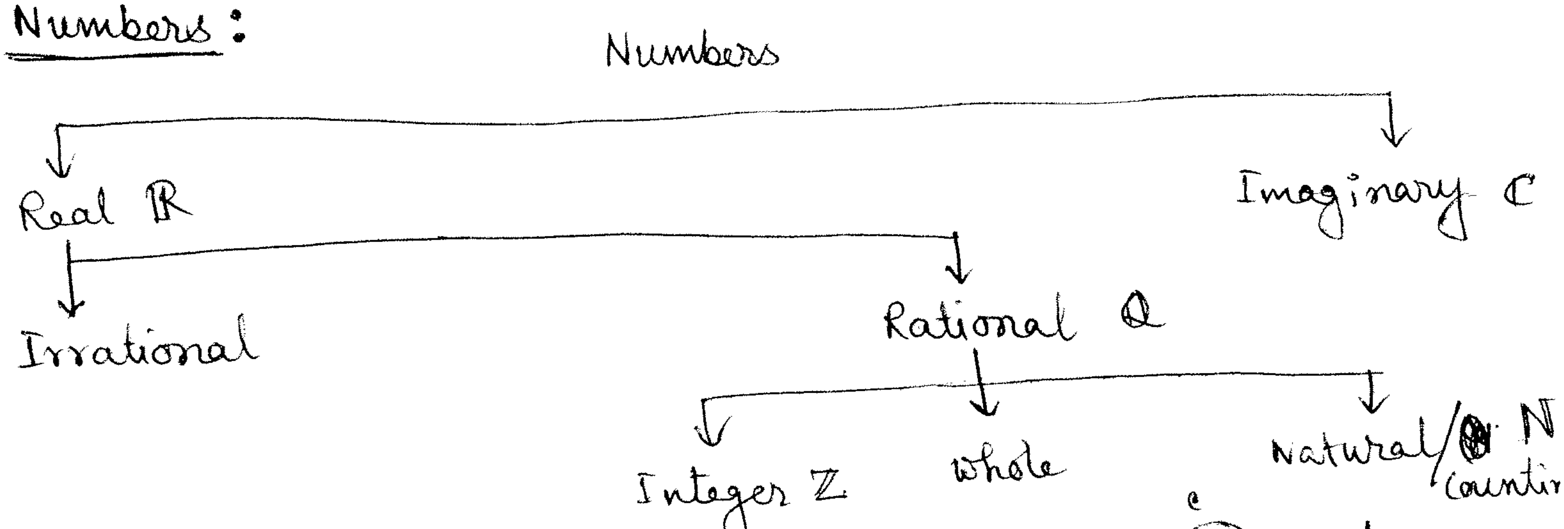
$A \cap B = \phi$, $A \cup U = U$, $A \cup B = \{1,2,3,4,$
$A \cap C = \phi$ $B \cup U = U$, $A \subseteq U$
$B \cap C = \phi$ $C \cup U = U$ $B \subseteq U$
$C \subseteq U$

2. $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,2,3\}$, $B = \{3,4,5\}$
$\bar{A} = \{4,5,6,7,8,9,10\}$

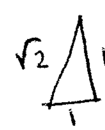
∴



Numbers :



$\{-5, \frac{4}{3}, 10, 0{\cdot}56, \sqrt{2}, \pi, 1{\cdot}121212\ldots, 0\}$; $\pi = \frac{c}{d}$ 

, $\sqrt{2}$ 

Decimals : $\frac{4}{3}$, $0{\cdot}56$, $1{\cdot}121212\cdots$ (3 dots called an ellipsis)

Integers : $-5, 10$

Rational : $0{\cdot}56$, $1{\cdot}121212\cdots$ (either terminating or repeting decimals)

Irrational : $\sqrt{2}$, π (other decimals which does not terminate or repeat)

Natural : 10 (which are used to count things)

whole : 0, 10 (Natural no. ~~with~~ which includes '0')

Real : All numbers listed.

Approximations :

Decimal numbers may be represented by a real no (either rational or irrational)

e.g. $\sqrt{2} \approx 1.4142$, $\pi \approx 3.1416$

Round-off (Rounding) : Identify the specified final digit in the decimal. If the next digit is 5 or more, add 1 to the final digit. If the digit is 4 or less, leave the final digit as it is. Then truncate the final digit

Trancation : Drop all the digits that follow the specified final digit in the decimal.

$50.52469876 \Rightarrow 50.525 \Rightarrow$ Rounding off in 3rd decim

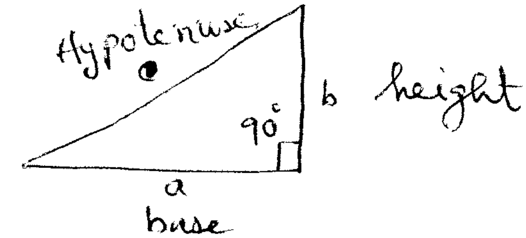
$\Rightarrow 50.524 \Rightarrow$ Trancation in 3rd decimal

Properties of real numbers :

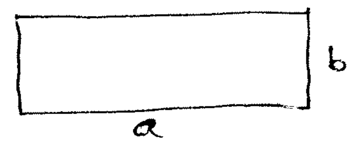
1. Reflexive : A number always equals to itself, i.e. $a = a$
2. Symmetric : If $a = b$ then $b = a$
3. Transitive : If $a = b$ and $b = c$ then $a = c$
4. Commutative :
$$A + B = B + A$$
$$A \cdot B = B \cdot A$$
5. Associative :
$$A + (B + C) = (A + B) + C = A + B + C$$
$$A \cdot (B \cdot C) = (A \cdot B) \cdot C = A \cdot B \cdot C$$
6. Distributive :
$$A \cdot (B + C) = A \cdot B + A \cdot C$$
$$(A + B) \cdot C = A \cdot C + B \cdot C$$
7. Identity :
$$A + 0 = A = 0 + A$$
$$A \cdot 1 = 1 \cdot A = A$$
8. Additive inverse : $A + (-A) = -A + A = 0$
9. Multiplicative inverse : $A \cdot \frac{1}{A} = \frac{1}{A} \cdot A = 1$ if $A \neq 0$

R3 : Geometry essentials

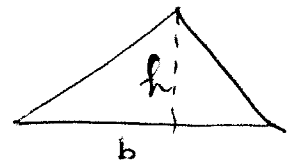
1. Pythagorean theorem : $c^2 = a^2 + b^2$
   for right angled triangle



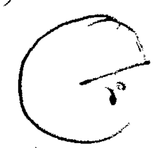
2. Area of a rectangle : $A = a \cdot b$
   Perimeter $= 2(a+b)$



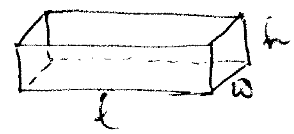
3. Area of a triangle : $A = \frac{1}{2}$ base $\times$ height



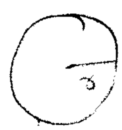
4. Area of a circle : $A = \pi \times (\text{radius})^2$



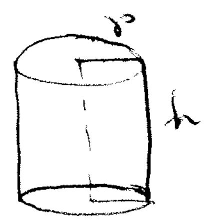
5. Volume of a closed rectangular box : lwh
   Surface area : $2lh + 2wh + 2lw$



6. Volume of a sphere : $\frac{4}{3}\pi r^3$
   Surface area : $4\pi r^2$



7. Volume of a right circular cylinder : $\pi r^2 h$
   Surface Area $= 2\pi r^2 + 2\pi r h$



R4 : Polynomials

Monomial : $a x^K$, a = constant
K = non-negative integer variable
$K \geq 0$

a is called coefficient

If $a \neq 0$, then K is the degree of the monomial.

examples : $6x^2$, $-2\sqrt{2}\,x^3$, 3, $-5x$, x^4

- $3x^{1/2}$ and $4x^{-3}$ are not monomials as the powers $\frac{1}{2}$ and -3 are not integer and positive.

Two monomials of <u>same degree</u> and <u>same variables</u> are called <u>like terms</u>. Monomials of like terms can be added while others can not. e.g.

$$5x^4 + 9x^4 - 2x^4 = 12x^4$$

$$5x^4 + 3x^3 = 5x^4 + 3x^3$$

The sum or difference of two monomials having different degrees is called a <u>binomial</u>.

$x^2 - 2 \rightarrow$ Binomial

$x^3 - 3x + 5 \rightarrow$ Trinomial

} Remember the variable x is similar for all the terms.

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0 \rightarrow \text{Polynomial.}$$

$a_n, a_{n-1}, \ldots, a_1, a_0$ are constants $\Rightarrow$ Coefficients.

$n \geq 0$ is an integer

x is variable.

If $a_n \neq 0$ then a_n is called leading coefficient and the term $a_n x^n$ is leading term and therefore n is the degree of the polynomial.

<u>Dividing two polynomials :</u>

95. $4x^5 - 3x^2 + x + 1$ divided by $2x^3 - 1$

$2x^2$ ← Quotient

Divisor → $2x^3 - 1 \;\big)\; 4x^5 - 3x^2 + x + 1$ ← Dividend

$\quad \underset{(-)}{}4x^5 \underset{(+)}{-} 2x^2$

$\quad -x^2 + x + 1$ ← Remainder

99. $-4x^3 + x^2 - 4$ divided by $x - 1$

$-4x^2 + 5x$

$x - 1 \;\big)\; -4x^3 + x^2 - 4$

$\quad \underset{(+)}{-}4x^3 \underset{(+)}{-} 4x^2$

$\quad +5x^2 - 4$

$\quad \underset{(-)}{+}5x^2 \quad \underset{(+)}{\mp}5x$

$\quad -5x - 4$ ← Remainder

R5 : Factoring a polynomial

$$2x^2 - 5x - 12 = \underbrace{(2x+3)(x-4)}_{\text{Factors}}$$

i.e. Finding the factors of a polynomial is called factoring.
If a polynomial cannot be written as the product of two other polynomials (excluding 1 and -1), then the polynomial is said to be prime.

- $a^2 - b^2 = (a+b)(a-b)$
- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $(a+b)(a^2 - ab + b^2) = (a^3 + b^3)$
- $(a-b)(a^2 + ab + b^2) = a^3 - b^3$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
-

To factor a second degree polynomial

$$Ax^2 + Bx + C$$

find integers whose product is AC and whose sum is B.
i.e. if there are numbers a, b, where $ab = AC$ and $a+b = B$
then $Ax^2 + Bx + C = (x+a)(x+b)$

e.g. $x^2 + 7x + 10$, consider $a = 5$, $b = 2$
then $ab = 10$ and $a+b = 7$

$\therefore$ The factor is
$(x+5)(x+2)$

R6 : Synthetic Division

A shortened version of long division is called Synthetic division

$$
\begin{array}{r l}
 & 2x^2 + 5x + 15 \leftarrow \text{Quotient} \\
x-3 & \overline{)\,2x^3 - x^2 \qquad + 3} \\
 & \underline{-(2x^3 - 6x^2)} \\
 & 5x^2 \\
 & \underline{-(5x^2 - 15x)} \\
 & 15x + 3 \\
 & \underline{-(15x - 45)} \\
 & 48 \\
 & \uparrow \\
 & \text{Remainder}
\end{array}
$$

$\Rightarrow$ Remove x and write in compact form

$$
\begin{array}{r l l l l l}
 & 2x^2 + 5x + 15 & & & & \text{Row 1} \\
x-3 & \overline{)\,2} & -1 & 0 & 3 & \text{Row 2} \\
 & & -6 & -15 & -45 & \text{Row 3} \\
 & & (+) & (+) & (+) & \\
\hline
 & 0 & 5 & 15 & 48 & \text{Row 4}
\end{array}
$$

Then one does not need to write Row 1.

Row 3 $\underbrace{2x^2 + 5x + 15}_{\text{Quotient}}$ $\quad \underset{\text{Remainder}}{\underset{\uparrow}{48}}$

Problems: (R4)

40. $8(1-y^3) + 4(1+y+y^2+y^3)$

$= 8 - 8y^3 + 4 + 4y + 4y^2 + 4y^3$

$= -4y^3 + 4y^2 + 4y + 12$

$= 4(-y^3 + y^2 + y + 3)$

46. $(2x-3)(x^2+x+1)$

$= 2x(x^2+x+1) - 3(x^2+x+1)$

$= 2x^3 + 2x^2 + 2x - 3x^2 - 3x - 3$

$= 2x^3 - x^2 - x - 3$

90.

$3x^2 - 7x + 15 \leftarrow$ Quotient

$x+2 \,\overline{)\, 3x^3 - x^2 + x - 2} \leftarrow$ Divident

Divisor

$\cancel{3x^3} + 6x^2$ $\quad (-)\ (-)$

$\cancel{-7x^2} + x - 2$

$\cancel{-7x^2} - 14x$ $\quad (+)\ (+)$

$\cancel{15x} - 2$

$\cancel{15x} + 30$ $\quad (-)\ (-)$

$-32 \leftarrow$ Remainder

104.

$x^4 + ax^3 + a^2x^2 + a^3x + a^4$

$x-a \,\overline{)\, x^5 - a^5}$

$\cancel{x^5} - ax^4$ $\quad (-)\ (+)$

$-a^5 + \cancel{ax^4}$

$+\cancel{ax^4} - a^2x^3$ $\quad (-)\ (+)$

$-a^5 + \cancel{a^2x^3}$

$+\cancel{a^2x^3} - a^3x^2$ $\quad (-)\ (+)$

$-a^5 + a^3x^2 \rightarrow$

$-a^5 + \cancel{a^3x^2}$

$+\cancel{a^3x^2} - a^4x$ $\quad (-)\ (+)$

$-a^5 + \cancel{a^4x}$

$-a^5 + \cancel{a^4x}$ $\quad (+)\ (-)$

$0 \leftarrow$ Remainder

R5

20. $x^2 - 25$

$= (x)^2 - (5)^2$

$= (x+5)(x-5)$

$a^2 - b^2 = (a+b)(a-b)$

29. $4x^2 + 4x + 1$

$= (2x)^2 + 2 \cdot 2x \cdot 1 + 1$

$= (2x+1)^2$

$= (2x+1)(2x+1)$

$(a+b)^2 = a^2 + 2ab + b^2$.

33. $x^3 - 27$

$= (x)^3 - (3)^3$

$= (x-3)(x^2 + 3x + 9)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

41. $x^2 + 7x + 6$

$= x^2 + x + 6x + 6$

$= x(x+1) + 6(x+1)$

$= (x+1)(x+6)$

$A = 1$, $B = 7$, $C = 6$.

$AC = 6$, $A + C = B$.

Consider $a = 1$ and $b = 6$.

so that $ab = 6$ and $a + b = B$.

Then split the middle term as

$Ax^2 + Bx + C$

$= Ax^2 + ax + bx + C$

52.

$3x^2 - 3x + 2x - 2$

$= 3x(x-1) + 2(x-1)$

$= (x-1)(3x+2)$

69. x^2+10x

$= x^2+10x+25$ $\Rightarrow$ 25 needs to be added to complete the expression.

$= x^2+5x+5x+25$

$= x(x+5)+5(x+5)$

$= (x+5)(x+5)$

R7

9. $\dfrac{24x^2}{12x^2-6x} = \dfrac{\cancel{6x}\cdot 4x}{\cancel{6x}(2x-1)} = \dfrac{4x}{(2x-1)}$

16. $\dfrac{2x^2+5x-3}{1-2x} = \dfrac{2x^2+6x-x-3}{1-2x}$

$= \dfrac{2x(x+3)-1(x+3)}{1-2x}$

$= \dfrac{(x+3)(2x-1)}{(1-2x)}$

$= \dfrac{-(x+3)\cancel{(1-2x)}}{\cancel{(1-2x)}} = -(x+3)$

23. $\dfrac{x^2-3x-10}{x^2+2x-35}\cdot\dfrac{x^2+4x-21}{x^2+9x+14}$

$= \dfrac{x^2-5x+2x-10}{x^2+7x-5x-35}\cdot\dfrac{x^2+7x-3x-21}{x^2+7x+2x+14}$ [Each term can be factored]

$= \dfrac{x(x-5)+2(x-5)}{x(x+7)-5(x+7)}\cdot\dfrac{x(x+7)-3(x+7)}{x(x+7)+2(x+7)}$

$= \dfrac{\cancel{(x-5)}\cancel{(x+2)}}{(x+7)\cancel{(x-5)}}\cdot\dfrac{\cancel{(x+7)}(x-3)}{(x+7)\cancel{(x+2)}} = \dfrac{x-3}{x+7}$

33.

$$\frac{\dfrac{2x^2-x-28}{3x^2-x-2}}{\dfrac{4x^2+16x+7}{3x^2+11x+6}}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}$$

$$= \frac{2x^2-x-28}{3x^2-x-2} \cdot \frac{3x^2+11x+6}{4x^2+16x+7}$$

$$= \frac{2x^2-8x+7x-28}{3x^2-3x+2x-2} \cdot \frac{3x^2+9x+2x+6}{4x^2+14x+2x+7}$$

$$= \frac{2x(x-4)+7(x-4)}{3x(x-1)+2(x-1)} \cdot \frac{3x(x+3)+2(x+3)}{2x(2x+7)+1(2x+7)}$$

$$= \frac{(x-4)\cancel{(2x+7)}}{\cancel{(3x+2)}(x-1)} \cdot \frac{(x+3)\cancel{(3x+2)}}{(2x+1)\cancel{(2x+7)}}$$

$$= \frac{(x-4)(x+3)}{(x-1)(2x+1)}$$

$2x^2-x-28 \Rightarrow AC = -56$

$A=2,\ B=-1,\ C=-28$

$\therefore$ Let us factor AC, to see the ~~factors~~:

$$\begin{array}{r|l} 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline & 7 \end{array}$$

$\therefore 56 = 2 \times 2 \times 2 \times 7$

Now, this becomes easy to select a, b.

Here $a = -7$ and $b = +2\times2\times2$

so that $ab = -56$

$a+b = -1$

~~Ex~~. 52.

$$\frac{x-1}{x^3} + \frac{x}{x^2+1}$$

$$= \frac{(x-1)(x^2+1) + x \cdot x^3}{x^3(x^2+1)}$$

$$= \frac{x^3+x-x^2-1+x^4}{x^3(x^2+1)}$$

$$= \frac{x^4+x^3-x^2+x-1}{x^3(x^2+1)}$$

L.C.M. of x^3 and x^2+1 is :

$$\left.\begin{aligned} x^3 &= x\cdot x\cdot x \\ (x^2+1) &= (x^2+1) \end{aligned}\right\} \text{No common factor.}$$

$\therefore$ LCM $= x^3 \cdot (x^2+1)$

57. $\cancel{4x^3-4x^2+x}$

$\cancel{= 4x^3 \ldots\ x(4x^2+4x+1)}$

Find LCM

$$4x^3-4x^2+x = x\,(4x^2+4x+1)$$

$$2x^3-x^2 = x \cdot x\,(2x-1)$$

$$x^3 = x \cdot x \cdot x$$

Let us first pick up the common factors :

$$x \cdot x$$

$\therefore$ L.C.M. = Common factors × other factors remaining.

$$= \underbrace{x \cdot x}_{\text{common factors.}} \cdot \underbrace{x\,(2x-1)\,(4x^2+4x+1)}_{\text{Remaining factors.}}$$

62. $\dfrac{x}{x-3} - \dfrac{x+1}{x^2+5x-24}$

$$= \frac{x}{x-3} - \frac{x+1}{x^2+8x-3x-24}$$

$$= \frac{x}{x-3} - \frac{x+1}{x(x+8)-3(x+8)}$$

$$= \frac{x}{x-3} - \frac{x+1}{(x+8)(x-3)}$$

LCM of : $(x+8)(x-3)$ and $(x-3)$ is $= (x+8)(x-3)$

$$= \frac{x(x+8) - (x+1)(1)}{(x-3)(x+8)}$$

$$= \frac{x^2+8x-x-1}{(x-3)(x+8)}$$

$$= \frac{x^2+7x-1}{(x-3)(x+8)}$$

82.

$$1-\frac{1}{1-\frac{1}{1-x}} = 1-\frac{1}{\frac{\cancel{1}-x-\cancel{1}}{1-x}} = 1-\frac{1}{\frac{-x}{1-x}}$$

$$= 1-\frac{1}{1}\times\frac{1-x}{-x} = 1+\frac{1-x}{x} = \frac{\cancel{x}+1-\cancel{x}}{x} = \frac{1}{x}$$