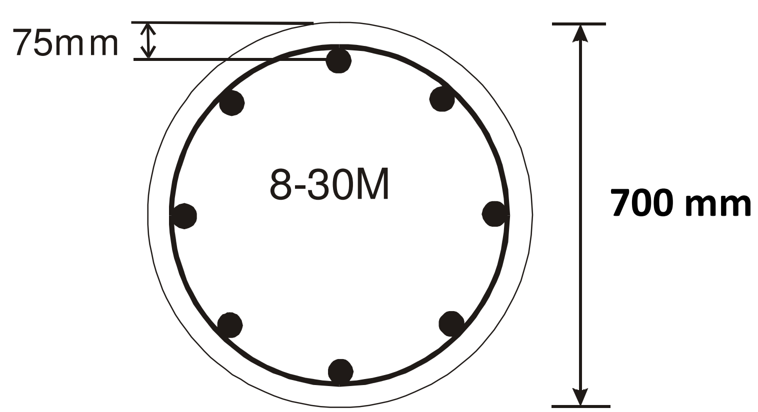
The University of Ottawa
Department of Civil Engineering
CVG 3148 – Reinforced Concrete Design I

<u>Assignment #4</u>

Plot the nominal moment-axial force interaction diagram (P_n vs. M_n) for a circular column section shown below. Assume that the rectangular stress block is applicable, and compute the values for the following four points and connect them to obtain the diagram.

i) Concentric compression, P_o (when moment is zero).
ii) When extreme fiber strain at one end is zero.
iii) At balanced section.
iv) When axial force is zero (pure bending – beam behaviour).

$f'_c = 35$ MPa; $f_y = 400$ MPa



Column Section

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- Assume rectangular stress block applies.

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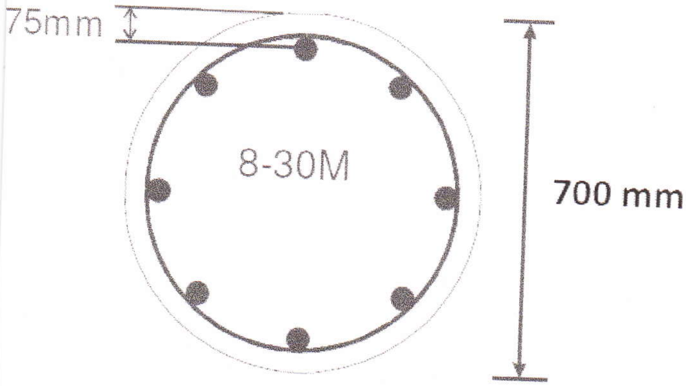
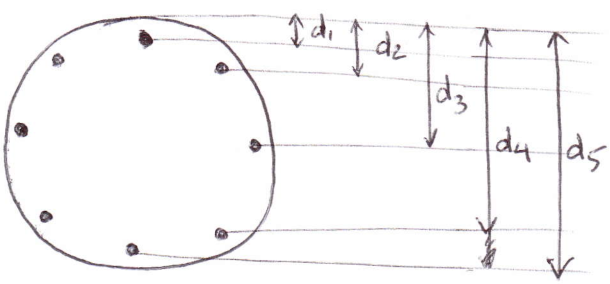


Column Section

$\alpha_1 = 0.85 - 0.0015\, f'_c = 0.85 - 0.0015 \times 35 = 0.798$

$\beta_1 = 0.97 - 0.0025\, f'_c = 0.97 - 0.0025 \times 35 = 0.883$



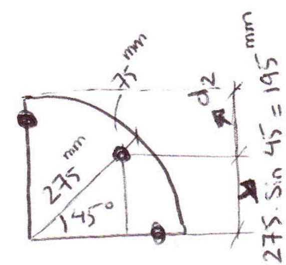
$d_1 = 75$ mm

$d_2 = 350 - 195 = 155$ mm

$d_3 = 350$ mm

$d_4 = 350 + 195 = 545$ mm

$d_5 = 700 - 75 = 625$ mm



① Concrete column capacity under concentric loading (zero eccentricity)

$$P_o = \alpha_1 . f'_c . (A_g - A_{st}) + f_y . A_{st}$$

$$P_o = 0.798 \times 35 \times \left[\left(\frac{\pi . 700^2}{4}\right) - 8 \times 700\right] + 400 \times 8 \times 700 = 12{,}832 \text{ kN}$$

$$P_{r,max} = 0.85 \times P_o = 0.85 \times 12{,}832 = 10{,}907 \text{ kN}$$

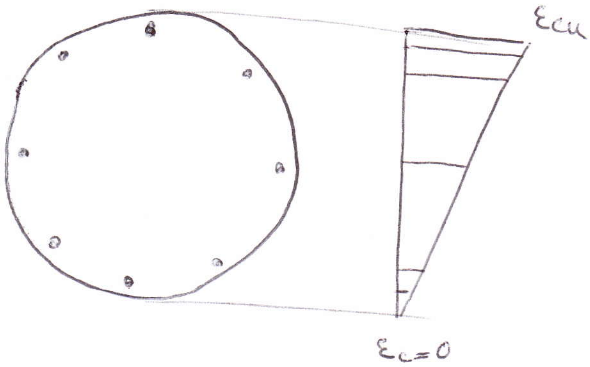
* Point 1: (cap value) $\begin{cases} P_n = 10{,}907 \text{ kN} \\ M_n = 0 \text{ kN.m} \end{cases}$

Point 1 (on the P-M): $\begin{cases} P_n = 12{,}832 \text{ kN} \\ M_n = 0 \text{ kN.m} \end{cases}$

(ii) One extreme fibre @ zero strain. ②

$\varepsilon_c \rightarrow \begin{cases} \varepsilon_{cu} = 0.0035 \\ \text{in compression zone} \end{cases}$

$\varepsilon_c = 0 \rightarrow$ in tension zone



$C = 700^{mm}$

$a = \beta_1 . C = 0.883 \times 700 = 618^{mm}$

- layer 1: $\varepsilon_{s1} = \left(\frac{700 - 75}{700}\right) \times 0.0035 = 0.00313 \quad > 0.002 \rightarrow$ yields.

$f_{s1} = f_y = 400$ MPa

$F_{s1} = A_{st} . f_{s1} = 700 \times 400 \times 10^{-3} = 280^{kN}$

$d_1 < a \rightarrow$ Steel layer 1 is in the compression zone.

- layer 2: $\varepsilon_{s2} = \left(\frac{700 - 151}{700}\right) \times 0.0035 = 0.00275 \quad > 0.002 \rightarrow$ yields.

$f_{s2} = f_y = 400$ MPa

$F_{s2} = A_{st} . f_{s2} = 2 \times 700 \times 400 \times 10^{-3} = 560^{kN}$

$d_2 < a \rightarrow$ Steel layer 2 is in the compression zone.

- layer 3: $\varepsilon_{s3} = \left(\frac{700 - 350}{700}\right) \times 0.0035 = 0.00175 \quad < 0.002$

$f_{s3} = \varepsilon_{s3} . E_s = 0.00175 \times 200{,}000 = 350$ MPa

$F_{s3} = A_{st} . f_{s3} = 2 \times 700 \times 350 \times 10^{-3} = 490$ kN

$d_3 < a \rightarrow$ steel layer 3 is in the compression zone.

- layer 4: $\varepsilon_{s4} = \left(\frac{700-545}{700}\right) \times 0.0035 = 0.000775 \quad < 0.002$

$f_{s4} = \varepsilon_{s4} \cdot E_s = 0.000775 \times 200{,}000 = 155$ MPa

$F_{s4} = A_{st} \cdot f_{s4} = 2 \times 700 \times 155 \times 10^{-3} = 217$ kN

$d_4 < a \longrightarrow$ steel layer 4 is in compression zone.

- layer 5: $\left(\frac{700-625}{700}\right) \times 0.0035 = 0.000375 \quad < 0.002$

$f_{s5} = \varepsilon_{s5} \cdot E_s = 0.000375 \times 200{,}000 = 75$ MPa

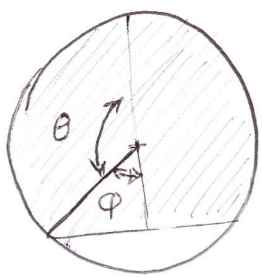
$F_{s5} = A_{st} \cdot f_{s5} = 1 \times 700 \times 75 \times 10^{-3} = 52.5$ kN

$d_5 < a \longrightarrow$ steel layer 5 is in the compression zone.

* Concrete Force: $C_{nc} = \alpha_1 \cdot f'_c \cdot (A - A_{st})$

$\hookrightarrow$ the area of steel in compression zone.

$\downarrow$ Total area (conc + steel) of the compression zone.



$\varphi = \cos^{-1}\left(\frac{2 \times 618}{700} - 1\right) = 40^\circ$

$\theta = 180 - \varphi = 180 - 40 = 140^\circ \quad , \theta = \frac{142^\circ}{180^\circ} \times \pi = 2.48$ rad

$A = h^2 \left(\frac{\theta - \sin\theta \cdot \cos\theta}{4}\right)$

$= 700^2 \left(\frac{2.48 - \sin(2.48)\cos(2.48)}{4}\right) = 364{,}120 \text{ mm}^2$

$C_{nc} = 0.798 \times 35 \times (364{,}120 - 8 \times 700) \times 10^{-3} = 10{,}013$ kN

* Nominal axial force resistance: $P_n = C_{nc} + F_{s1} + F_{s2} + F_{s3} + F_{s4} + F_{s5}$

$P_n = 10{,}013 + 280 + 560 + 490 + 217 + 52.5 = 11612.5$ kN

* Nominal Moment Capacity:

$$M_n = C_{nc} \cdot \bar{y} + \sum_{i=1}^{5} F_{si} \left(\frac{h}{2} - d_i\right)$$

$$A\bar{y} = h^3 \left(\frac{\sin^3\theta}{12}\right) \longrightarrow 364{,}120 \times \bar{y} = 700^3 \cdot \left(\frac{\sin^3 2.48}{12}\right) \longrightarrow \bar{y} = 20.8 \text{ mm}$$

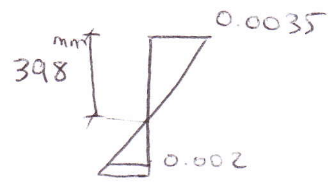
$$M_n = 10{,}013 \times 20.8 + 280\ (350-75) + 560\ (350-155) + 490\ \cancel{(350-350)}^{\,0} + 217\ (350-545) + 52.5\ (350-625)$$

$M_n = 337.7$ kN.m $\simeq 308$ kN.m

$\Rightarrow$ Point 2: $\begin{cases} P_n = 11{,}612 \text{ kN} \\ M_n = 308 \text{ kN.m} \end{cases}$

(on the P-M curve)

(iii) Balanced Condition : $\varepsilon_{cu} = 0.0035$, $\varepsilon_y = 0.002$



$$c = \frac{0.0035}{0.0035 + 0.002} \times 625 = 398 \text{ mm}$$

$a = \beta_1 c = 0.883 \times 398 = 351$ mm $\simeq 350$ mm

- layer 1 : $\varepsilon_{s1} = \left(\frac{398-75}{398}\right) \times 0.0035 = 0.0028 > 0.002 \longrightarrow$ yields. $f_{s1} = 400$ MPa

  $F_{s1} = 700 \times 400 \times 10^{-3} = 280$ kN , $d_1 < a \rightarrow$ compression

- layer 2 : $\varepsilon_{s2} = \left(\frac{398-155}{398}\right) \times 0.0035 = 0.0021 > 0.002 \longrightarrow$ steel yields $\longrightarrow f_{s2} = 400$ MPa

  $F_{s2} = 2 \times 700 \times 400 \times 10^{-3} = 560$ kN , $d_2 < a \rightarrow$ compression

layer 3 , $\varepsilon_{s3} = \left(\frac{398-350}{398}\right) \times 0.0035 = 4.22 \times 10^{-4} \longrightarrow f_{s3} = 84.4$ MPa

$F_{s4} = 2 \times 700 \times 84.4 \times 10^{-3} = 118$ kN , $d_3 = a$

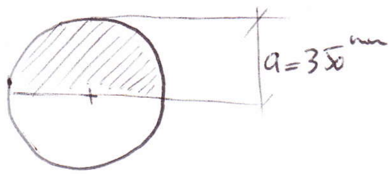
- layer 4: $\varepsilon_{s4} = \left(\frac{398-545}{398}\right) \times 0.0035 = -0.0013 > -0.002 \rightarrow f_{s2} = -258$ MPa (tension) (-)

$F_{s4} = f_s . A_s = -258 \times 2 \times 700 \times 10^{-3} = -361$ kN and $d_4 > a$

- layer 5: $\varepsilon_{s5} = \left(\frac{398-625}{398}\right) 0.0035 = -0.002 \rightarrow f_{s5} = 0.002$ (as assumed)

$F_{s5} = 400 \times 700 \times 10^{-3} = -280$ kN and $d_5 > a$

- Concrete force: $C_{nc} = \alpha_1 . f'_c . (A - A_{st})$



$$A = \frac{1}{2} \times \frac{\pi d^2}{4} = \frac{1}{2} \times \pi \times \frac{700^2}{4} = 192{,}422 \text{ mm}^2$$

$$A_{st} = \frac{1}{2} \times 8 \times 700 = 2800 \text{ mm}^2$$

$$C_{nc} = 0.798 \times 35 \, (192{,}422 - 2800) \times 10^{-3} = 5296 \text{ kN}$$

* Nominal axial force resistance, $P_n = 5296 + 280 + 560 + 118 - 361 - 280 = 5613$ kN

* Nominal moment capacity: $A\bar{y} = h^3 . \frac{\sin^3\theta}{12} \longrightarrow \bar{y} = 149$ mm

$M_n = 5296 \times 149 + 280 \times (350 - 75) + 560\,(350 - 155) + 118\,\cancel{(350 - 350)}^{0} - 361\,(350 - 545)$

$- 280\,(350 - 625)$

$M_n = 1123$ kN.m

$\Rightarrow$ Point 3: $\begin{cases} P_n = 5613 \text{ kN} \\ M_n = 1123 \text{ kN.m} \end{cases}$

(iv) Axial force is zero $\longrightarrow$ Pure bending (similar to a beam)

Try & error $\longrightarrow$ let $c = 140$ mm , $a = \beta_1 c = 0.883 \times 140 = 123.6$ mm

$- \varepsilon_{s1} = \frac{c-d_1}{c} \times \varepsilon_{cu} = \frac{140-75}{140} \times 0.0035 = 0.001625 \quad < 0.002$

$f_{s1} = E_s . \varepsilon_{s1} = 200{,}000 \times 0.001625 = 325 \text{ MPa}$

$F_{s1} = A_s . f_{s1} = 700 \times 325 \times 10^{-3} = 227.5 \text{ kN} \qquad , d_1 < a$

$- \varepsilon_{s2} = -0.00038 < 0.002 \longrightarrow f_{s2} = -75^{\text{MPa}} \longrightarrow F_{s2} = -105^{\text{kN}} \qquad d_2 > a$

$- \varepsilon_{s3} = -0.00525 > 0.002 \longrightarrow f_{s3} = 400^{\text{MPa}} \longrightarrow F_{s3} = -560^{\text{kN}} \qquad d_3 > a$

$- \varepsilon_{s4} = -0.0101 > 0.002 \longrightarrow f_{s4} = -400^{\text{MPa}} \longrightarrow F_{s4} = -560^{\text{kN}} \qquad d_4 > a$

$- \varepsilon_{s5} = -0.0121 > 0.002 \longrightarrow f_{s5} = -400^{\text{MPa}} \longrightarrow F_{s5} = -280^{\text{kN}} \qquad d_5 > a$

$F_{s,total} = 227.5 - 105 - 560 - 560 - 280 = -1277.5^{\text{kN}} \qquad (I)$

$* C_{nc} = \alpha_1 . f'_c . (A - A_{st})$

$\theta = \cos^{-1}\left(1 - \frac{2a}{h}\right) = \cos^{-1}\left(1 - \frac{2 \times 123.6}{700}\right) = 49.7^\circ \longrightarrow \theta = 0.867^{\text{rad}}$

$A = \frac{h^2}{4}(\theta - \sin\theta . \cos\theta) = \frac{700^2}{4}\left(0.867 - \sin(0.867) . \cos(0.867)\right) = 45832^{\text{mm}^2}$

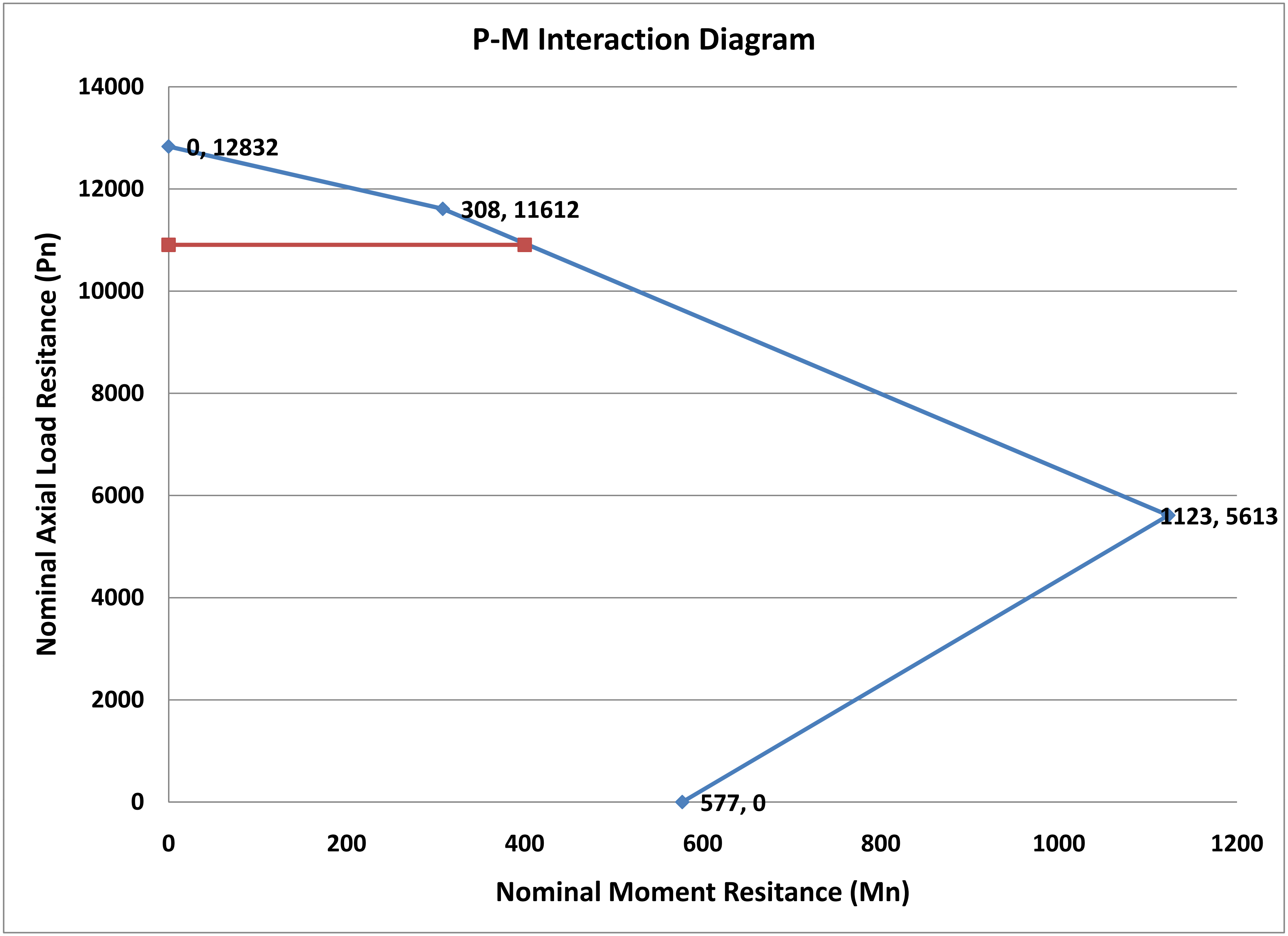
$C_{nc} = 0.798 \times 35 \times (45832 - 700) \times 10^{-3} = 1260.5^{\text{kN}} \qquad (II)$

$(I) \simeq (II) \longrightarrow c = 140 \quad \checkmark \text{ok.}$

$A\bar{y} = h^3 . \left(\frac{\sin^3\theta}{12}\right) \longrightarrow \bar{y} = 277^{\text{mm}}$

$M_n = 1260.5 \times 277 + 227(350-75) - 105(350-155) - 560\,\cancel{(350-350)}^{0} - 560(350-545) - 280(350-625)$

$M_n = 577^{\text{kN.m}} \qquad \longrightarrow \text{Point } 4 \; \Big| \; P_n = 0, \; M_n = 577^{\text{kN.m}}$


P-M Interaction Diagram
Nominal Axial Load Resitance (Pn)
Nominal Moment Resitance (Mn)
14000
12000
10000
8000
6000
4000
2000
0
0
200
400
600
800
1000
1200
0, 12832
308, 11612
1123, 5613
577, 0