The University of Ottawa

Department of Civil Engineering

CVG 3148 REINFORCED CONCRETE DESIGN I

Assignment 2

Part 1) Design the simply supported beam shown below for flexure and indicate the bar cut-off locations. Design the beam at two sections along the length and show all the relevant requirements of CSA A23.3-2005, including bar spacing, crack control and minimum reinforcement requirements.

f'_c = 30 MPa; f_y = 400 MPa; Maximum Aggregate Size: 20 mm; Interior exposure

Unfactored Uniformly Distributed Dead Load (including member weight): 25 kN/m

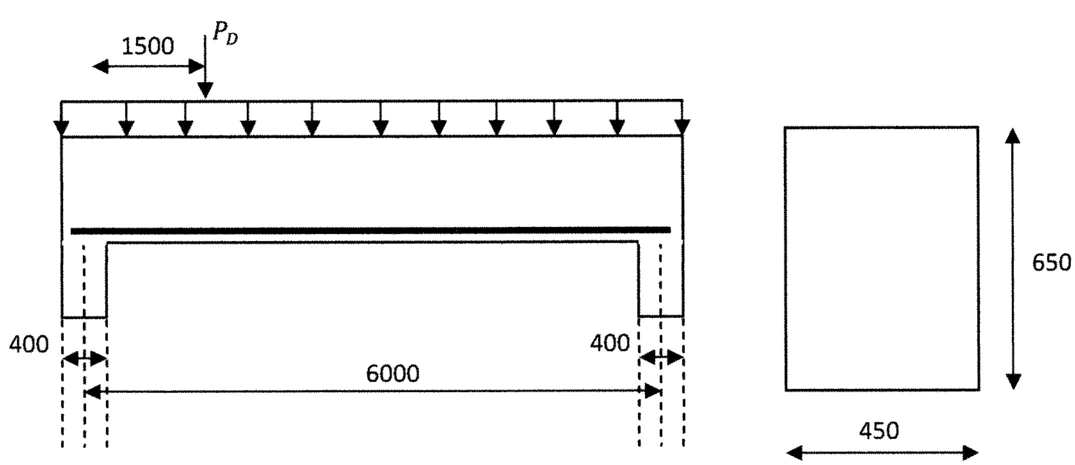
Unfactored Uniformly Distributed Live Load: 35 kN/m

Unfactored Concentric Dead Load: 100 kN

Low density concrete; Uncoated bars.

Shear reinforcement is in excess of the CSA 23.3 minimum requirement.

Part 2) Design the beam for shear. Use vertical stirrups as shear reinforcement and show the stirrup arrangement along the length of the beam. Identify regions where minimum reinforcement is placed and where no shear reinforcement is required.



Assignment #2: Solution

(1)

Part 1) Flexure Design:

$f'_c = 30$ MPa

$f_y = 400$ MPa

$a_{max} = 20$ mm

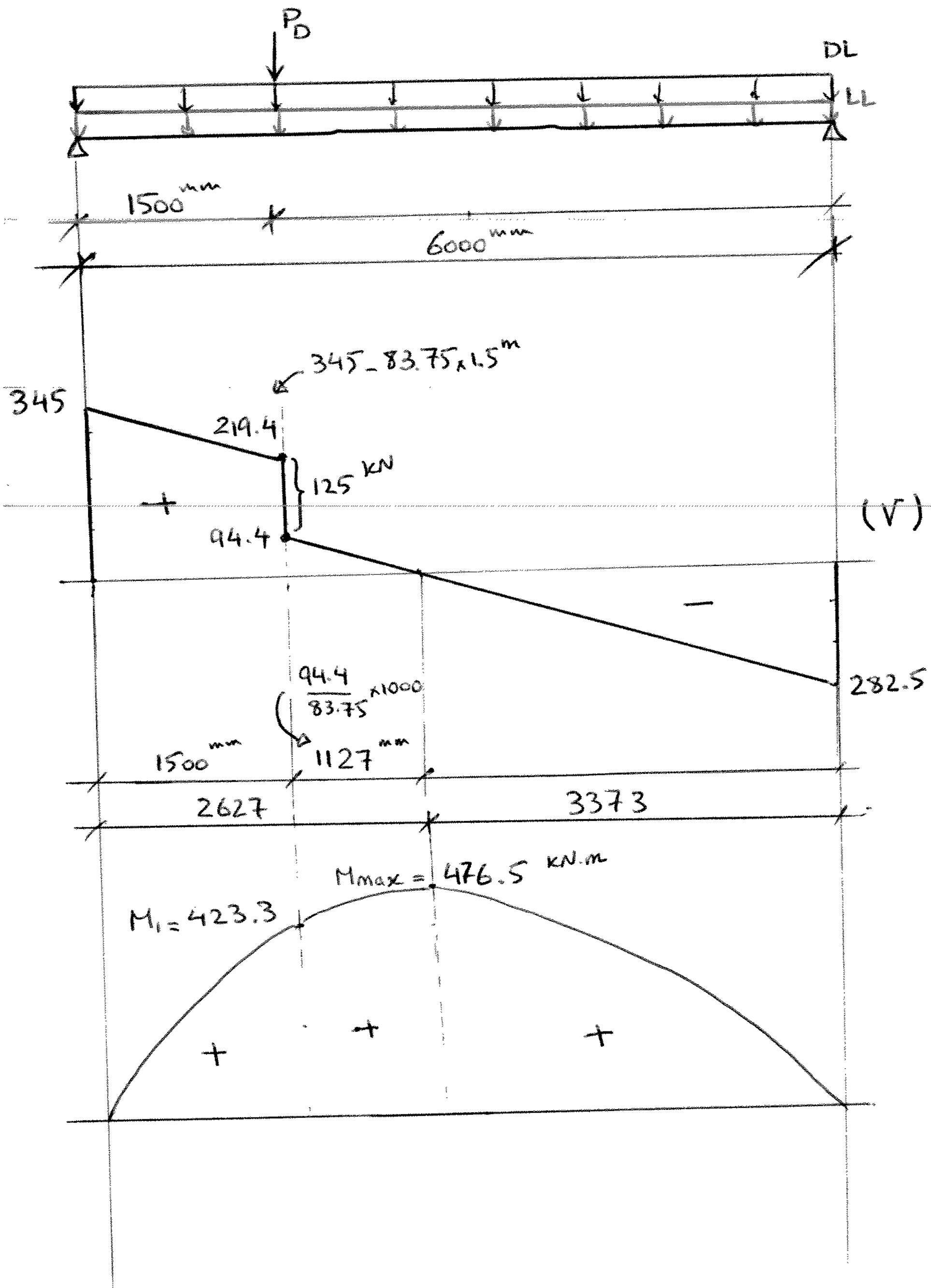
DL = 25 kN/m

LL = 35 kN/m

$P_D = 100$ kN

Low density concrete

Uncoated bars.



$M_f = 476.5$ kN.m

* ~~Governing load~~ combination:

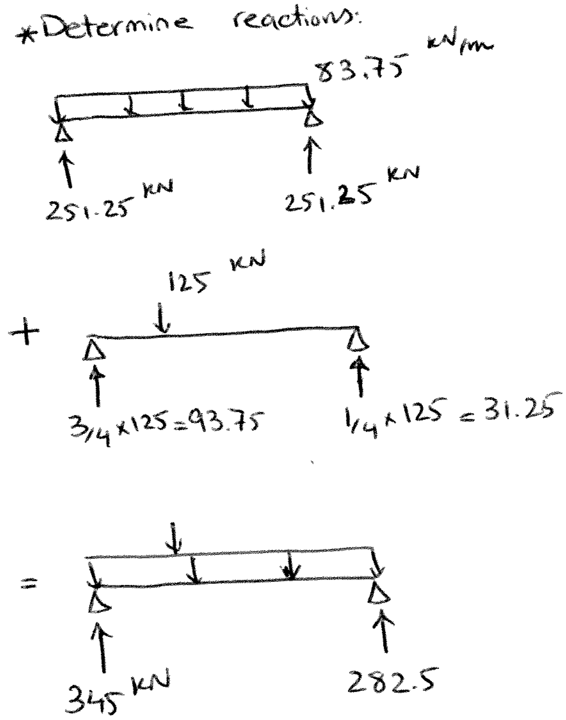
1.25 DL + 1.5 LL

including uniform and point-load.

$w_f = 1.25(25) + 1.5(35) = 83.75$ kN/m

$P_f = 1.25 \times 100 = 125$ kN

*Determine reactions:



$* M_1 = \frac{1}{2}(345 + 219.4) \times 1.5 = 423.3$

$* M_{max} = 423.3 + \frac{1}{2} \times 94.4 \times 1.127$

$= 476.5$ kN.m

Required Tension Reinforcement

use No.25 rebars.

$d = h - cover - d_s - \frac{1}{2} d_b$

$d = 650 - 40 - 10 - \frac{1}{2} \times 25 = 587.5 \simeq 585$ mm

finding A_s using direct method:

$$A_s = \frac{\alpha_1 \cdot \phi_c \cdot f'_c \cdot b}{\phi_s \cdot f_y} \left(d - \sqrt{d^2 - \frac{2 M_r}{\alpha_1 \cdot \phi_c \cdot f'_c \cdot b}} \right)$$

$\alpha_1 = 0.85 - 0.0015 f'_c$

$\alpha_1 = 0.85 - 0.0015 \times 30$

$\alpha_1 = 0.805$

$b = 450$ mm

$$A_s = \frac{0.805 \times 0.65 \times 30 \times 450}{0.85 \times 400} \left(585 - \sqrt{585^2 - \frac{2 \times 476.5 \times 10^6}{0.805 \times 0.65 \times 30 \times 450}} \right)$$

$A_s = 2694\ mm^2$ $\longrightarrow$ use No.25 $\longrightarrow$ 6 No.25 rebars are needed.

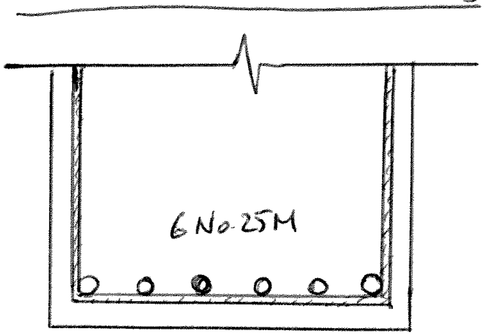
$\hookrightarrow A_s = 3000\ mm^2$

Check if $f_s = f_y$?

$$a = \frac{\phi_s \cdot A_s \cdot f_y}{\phi_c \cdot \alpha_1 \cdot f'_c \cdot b} = \frac{0.85 \times 3000 \times 400}{0.65 \times 0.805 \times 30 \times 450} = 137 \text{ mm}$$

$$\frac{a}{d} = \frac{137}{585} = 0.234 < 0.5$$ $\longrightarrow$ Steel yields ✓ OK.

Check for bar spacing



$S \geq \begin{cases} 1.4\, d_b = 1.4 \times 25 = 35 \text{ mm} & \leftarrow \text{govern.} \\ 1.4\, a_{max} = 1.4 \times 20 = 28 \text{ mm} \\ 30 \text{ mm} \end{cases}$

$b_{min} = 2 \times cover + 2 \times d_s + 6 \times d_b + 5 \times S \Rightarrow$

$b_{min} = 2 \times 40 + 2 \times 10 + 6 \times 25 + 5 \times 35 = 425 \text{ mm} < 450$ mm

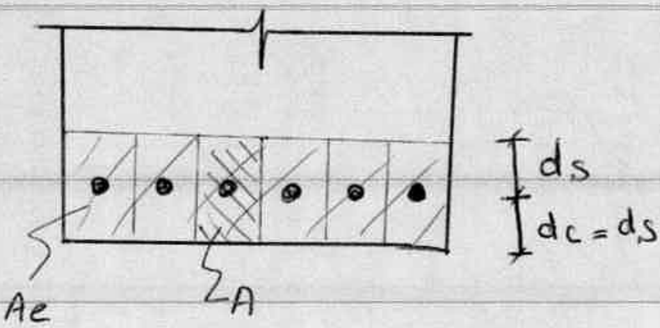
Can fit in one layer.

Check minimum reinforcement requirement

$$A_{s,min} = \frac{0.2\sqrt{f'_c}}{f_y} \times bh = \frac{0.2\sqrt{30}}{400} \times 450 \times 650 = 801 \, mm^2 \quad < A_s = 3000 \, mm^2$$

✓ OK

Check for crack control (Z)



$d_c = \text{cover} + d_s + \frac{1}{2} d_b \rightarrow$

$d_c = 40 + 10 + \frac{25}{2} = 62.5 \, mm$

$A_e = b(2d_s) = 450 \times 2 \times 62.5 = 56{,}250 \, mm^2$

$A = \frac{A_e}{N} = \frac{56{,}250}{6} = 9375 \, mm^2$

$f_s = 0.6 f_y = 0.6 \times 400 = 240 \, MPa$

$$Z = f_s \cdot \sqrt[3]{d_c \cdot A} = 240 \times \sqrt[3]{62.5 \times 9375} = 20{,}082 \, N/mm \quad < 30{,}000 \, N/mm$$

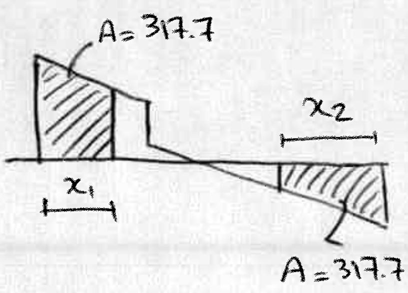
↳ interior exposure

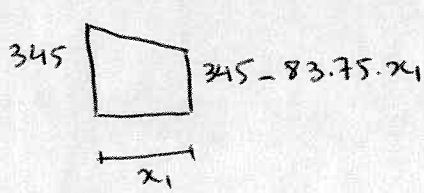
$h < 750 \longrightarrow$ No skin reinforcement ~~is~~ required.

Bar Cut-offs (2 No.25 bars will be cut-off)

Theoretical cut-off point: 6 No.25M $\longrightarrow M_{f(6)} = 476.5 \, kN \cdot m$

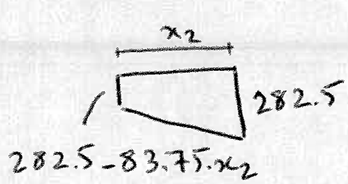
4 No.25M $\longrightarrow M_{f(4)} = 4/6 \times 476.5 = 317.7 \, kN \cdot m$





$\frac{1}{2}(345 + 345 - 83.75 \cdot x_1) x_1 = 317.7$

$x_1 = 1.05 \, m$



$\frac{1}{2}(282.5 + 282.5 - x_2 \times 83.75) x_2 = 317.7$

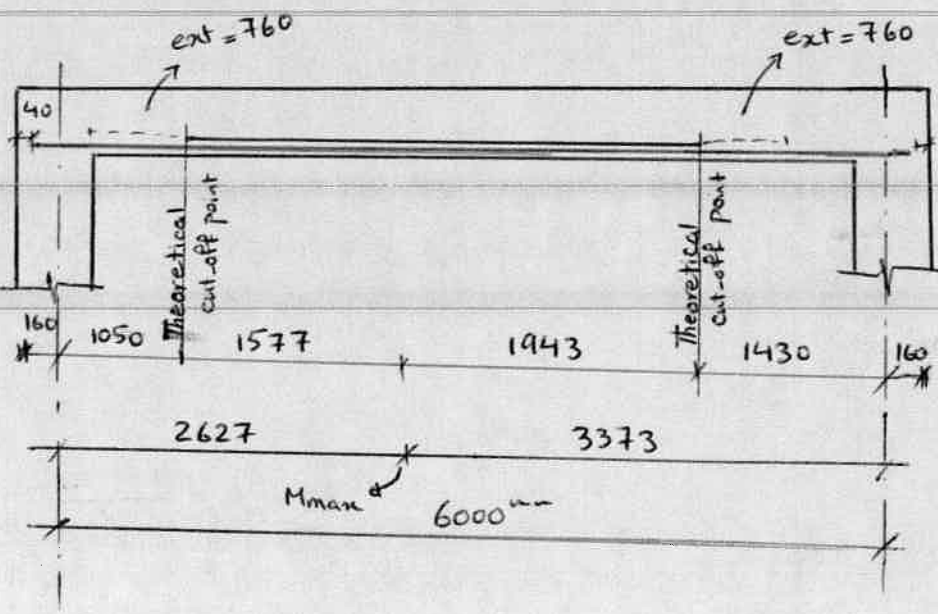
$x_2 = 1.43 \, m$

$$\text{Extension} = d_v \cdot \cot\theta \rightarrow \text{Ext} \geq \begin{cases} 1.3d = 1.3 \times 585 = 760^{mm} \checkmark \text{ governs.} \\ h = 650^{mm} \end{cases}$$

$$l_d = 0.45 \cdot k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot \frac{f_y}{\sqrt{f'_c}} \cdot d_b = 0.45 \times 1.0 \times 1.0 \times 1.3 \times 1.0 \times \frac{400}{\sqrt{30}} \times 25$$

top bars / uncoated bars / low density concrete / No.25M

$$= 1068^{mm}$$



$L_{actual} \rightarrow$ left side $= 1577 + 760 = 2337^{mm}$ from the critical point @M_{max}

$L_{actual} \rightarrow$ right side $= 1943 + 760 = 2703^{mm}$ "

L_{actual} for left and right $> l_d = 1068^{mm}$ ✓ ok.

The length of continuing bars from theoretical cut-off location:

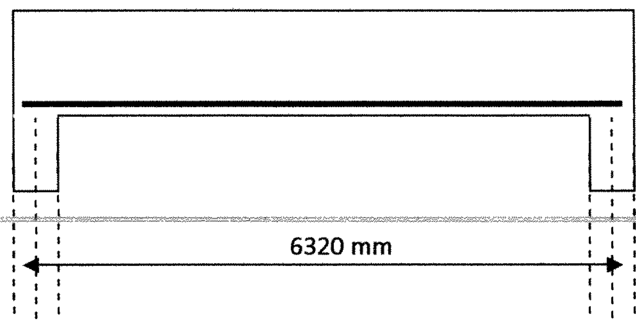
Left side: $1050 + 160 = 1210^{mm}$

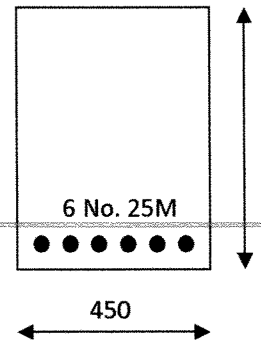
Right side: $1430 + 160 = 1590^{mm}$

$$\geq \begin{cases} l_d + d = 1068 + 585 = 1653^{mm} \leftarrow \text{governs} \\ l_d + 12d_b = 1068 + 12 \times 25 = 1368 \end{cases}$$

* Both rebars are too close to the end → No bar cut-offs allowed.

⑤





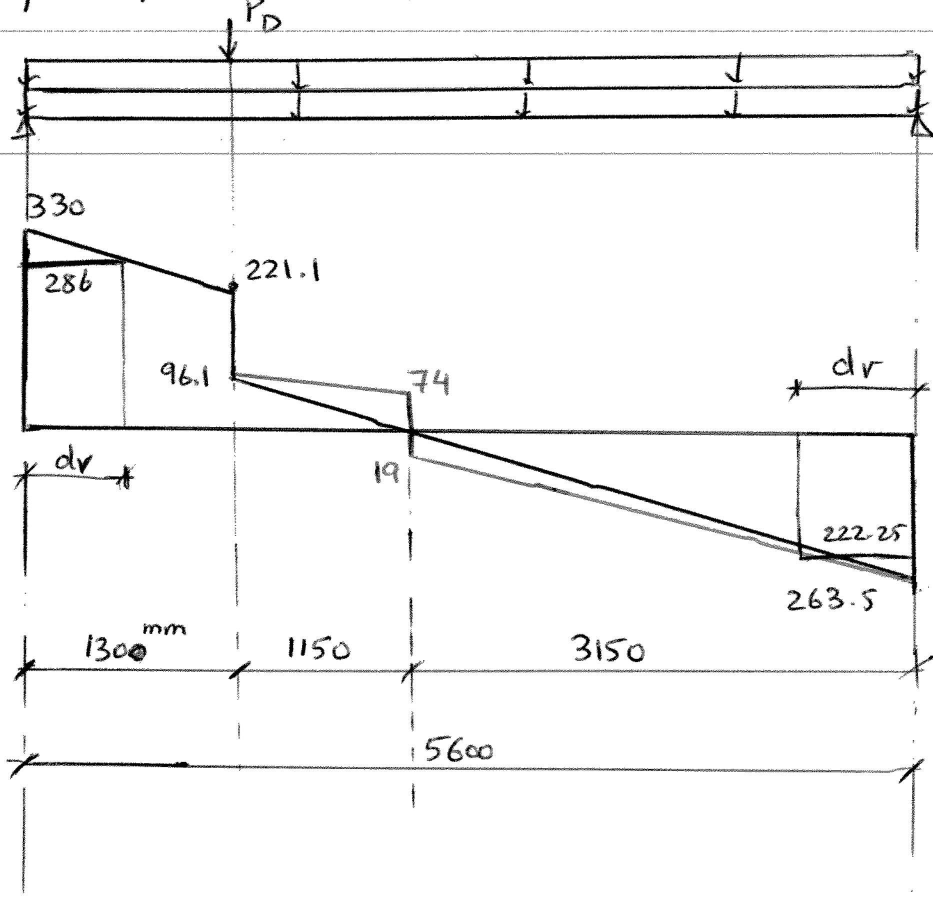
Part 2) Shear Design

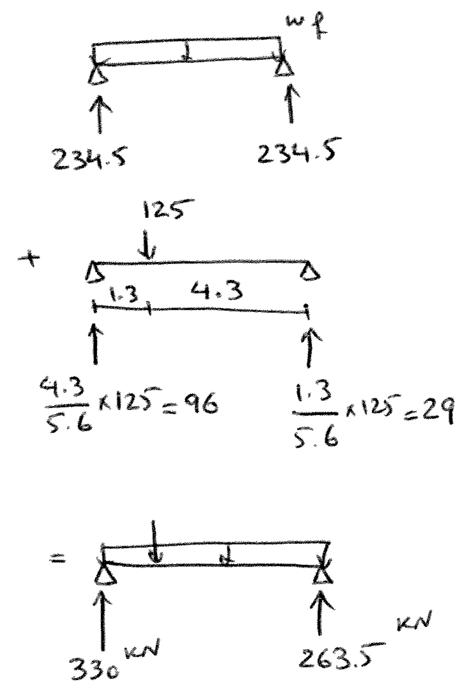
* The shear diagram for the applied loads from inside face of the support to the othe inside face. Then, the shear force envelope should be found to determine the highest shear demand on the beam at all locations. This can be done by varying the live load location on the beam, specially around the point where maximum moment occurs.

(shear design is considered for)

$w_f = 83.75$ kN/m

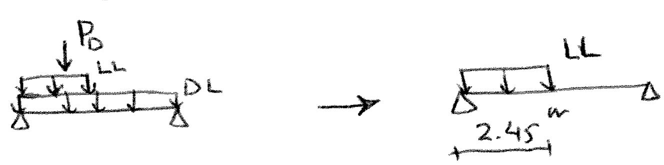
$l = 5.6^m$



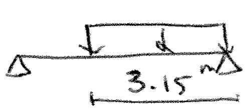


* Determine the jump in shear diagram @ zero shear.

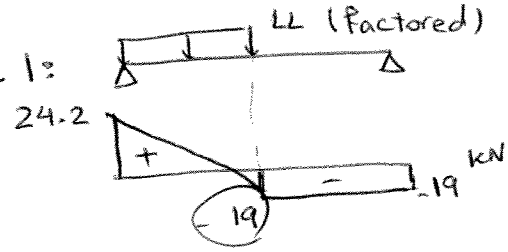
* Case 1:



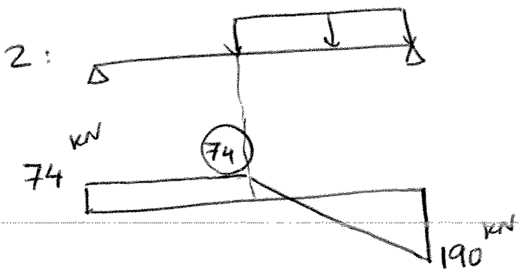
* Case 2:



* Case 1:



* Case 2:



Adjust the shear diagram in the previous page based on the new shear values.

* reduce the shear demand at the regions around the support (length = d_v)

$$d_v \geq \begin{cases} 0.9d = 0.9 \times 585 = 526.5 \checkmark \\ 0.72h = 0.72 \times 650 = 468 \end{cases} \longrightarrow d_v = 525 \text{ mm}$$

$V_{f@d_{v-\ell}} = 330 - 83.75 \times 0.525 = 286$ kN (Maximum shear demand @ left side)

$V_{f@d_{v-r}} = 263.5 - \dfrac{263.5 - 19}{3.15} \times 0.525 = 222.25$ kN (Maximum shear demand @ right side)

Concrete Shear Resistance (V_c)

$V_c = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f'_c} \cdot b_w \cdot d_v \Rightarrow$

$\lambda = 0.75$

low-density concrete

$\beta = 0.18 \longrightarrow$ Region 1 and Region 2

cl. 8.6.5

$\beta = \dfrac{230}{1000 + d_v} = \dfrac{230}{1000 + 525} = 0.151 \longrightarrow$ Region Ø

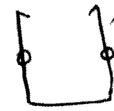
$V_c = 0.65 \times 0.75 \times 0.18 \times \sqrt{30} \times 450 \times 525 \times 10^{-3} = 113.5$ kN $\longrightarrow$ Region 1 and 2

$V_c = 0.65 \times 0.75 \times 0.151 \times \sqrt{30} \times 450 \times 525 \times 10^{-3} = 95.3$ kN $\longrightarrow$ Region Ø

Regions: Region 0: No shear reinf. is required $V_f < 95.3$

Region 1: Minimum shear reinf. is provided.

$$S = \frac{A_v \cdot f_y}{0.06 \times \sqrt{f'_c} \cdot b_w} = \frac{200 \times 400}{0.06 \times \sqrt{30} \times 450} = 541 \ mm$$



if $V_f \overset{?}{<} 0.125 \cdot \lambda \cdot f_c \cdot b_w \cdot d_v \cdot \phi_c \longrightarrow S_{max} \leqslant \begin{cases} 600 \\ 0.7 d_v = 368 \ mm \end{cases}$ governs

$V_{f,max} = 286 < 0.125 \times 0.75 \times 30 \times 450 \times 525 \times 0.65 = 432 \ kN$ ✓

$S_1 = 350 \ mm$

select $S_{max} = 350 \ mm$

$$V_s = \frac{\phi_s \cdot A_v \cdot f_y \cdot d_v \cot\theta}{S} = \frac{0.85 \times 200 \times 400 \times 525 \times \cot 35^\circ}{350}$$

$V_s = 146 \ kN$

$V_r = V_c + V_s = 113.5 + 146 = 259.5 \ kN$

$V_{f,max} = 286$ @ left ← almost covered by region 1

222.25 @ right ← already covered with region 1

Region 2: $259.5 < V_f < 286 \ kN$

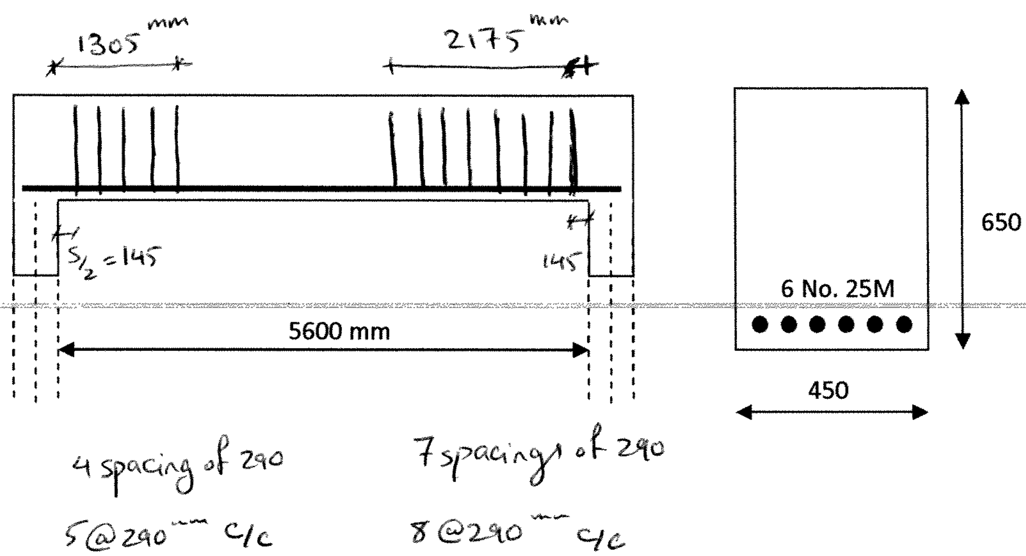
$V_s = V_f - V_c = 286 - 113.5 = 172.5 \ kN$

$$S_2 = \frac{\phi_s \ A_v \cdot f_y \ d_v \cdot \cot\theta}{V_s} = \frac{0.85 \times 200 \times 400 \times 525 \times \cot 35^\circ}{172500} \Rightarrow S = 296 \ mm$$

$S_2 = 290 \ mm$

(9)

* In order to facilitate the assembly process, a uniform "s" value of s = 290 for both regions 1 and 2 is used.



Maximum allowable shear resistance

$$V_{r,max} = 0.25 \cdot \varphi_c \cdot f'_c \cdot b_w \cdot d_v \qquad \text{cl. } 11.3.3$$

$$V_{r,max} = 0.25 \times 0.65 \times 30 \times 450 \times 525 \times 10^{-3} = 1152 \text{ kN} > 286 \text{ kN} \qquad \text{OK.}$$