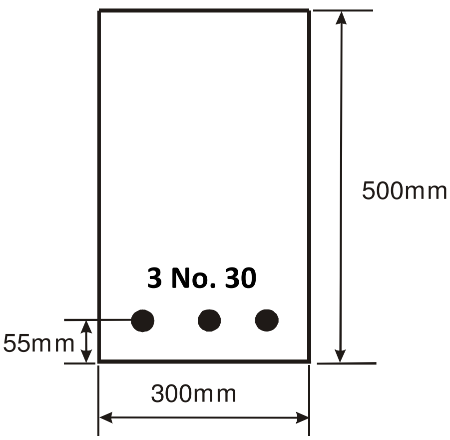
Assignment #1 – Solution CVG 3148

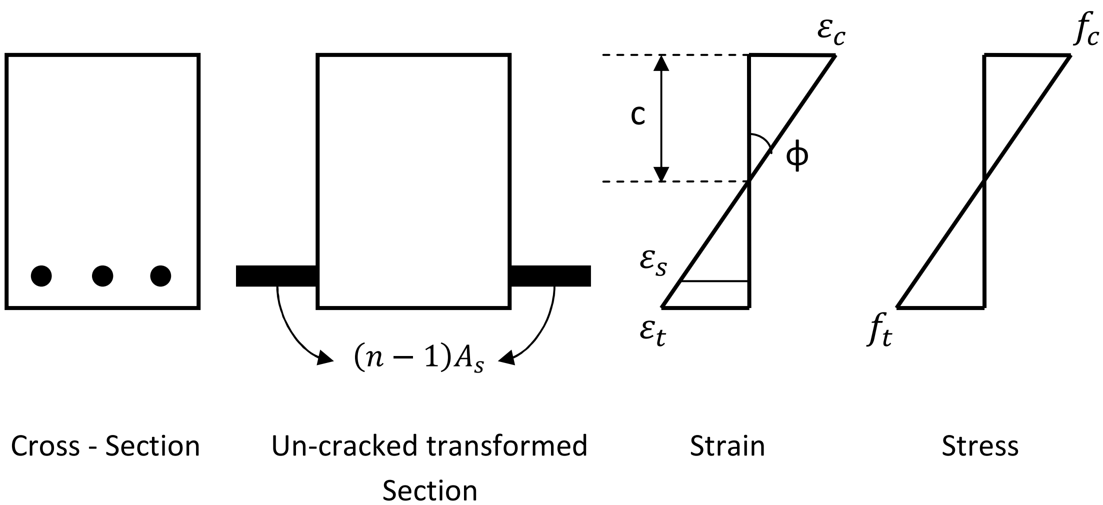
Establish the moment-curvature relationship for the beam shown below.



$f_c' = 35\ MPa$
$f_y = 400\ MPa$
$\varepsilon_0 = 0.002$
$d = 445\ mm$

i) ***Before cracking***

Note: The concrete is close to cracking and works in both in tension and compression. The steel is placed in the tension zone. Since the section is not cracked and the load is small, tangential modulus of elasticity is used ($E_c = 5500\sqrt{f_c'}$). Transformed section is used to calculate the moment of inertia of the section.



$$n = \frac{E_s}{E_c} = \frac{200{,}000}{5500\sqrt{f_c'}} = \frac{200{,}000}{5500\sqrt{35}} = 6.15$$

$$(n-1)A_s = (6.15-1)(3 \times 700) = 10{,}815\ mm^2$$

- Determine the neutral axis of the section from the top fibre:

$$c = \frac{\sum A.\bar{y}}{\sum A}$$

$c = \frac{(300\times500)(250)+10{,}815\times445}{(300\times500)+10{,}815} = 263\ mm$ From the top of the beam

$I_{tr} = \sum I + \sum Ad^2 = \frac{bh^3}{12} + bh(263-250)^2 + (n-1)A_s(263-445)^2$

$I_{tr} = \frac{300\times500^3}{12} + (300\times500)(13)^2 + 10{,}815(182)^2 = 3.125\times10^9 + 2.535\times10^7 + 3.582\times10^8$

$I_{tr} = 3.51\times10^9\ mm^4$

Rupture stress for concrete in tension:

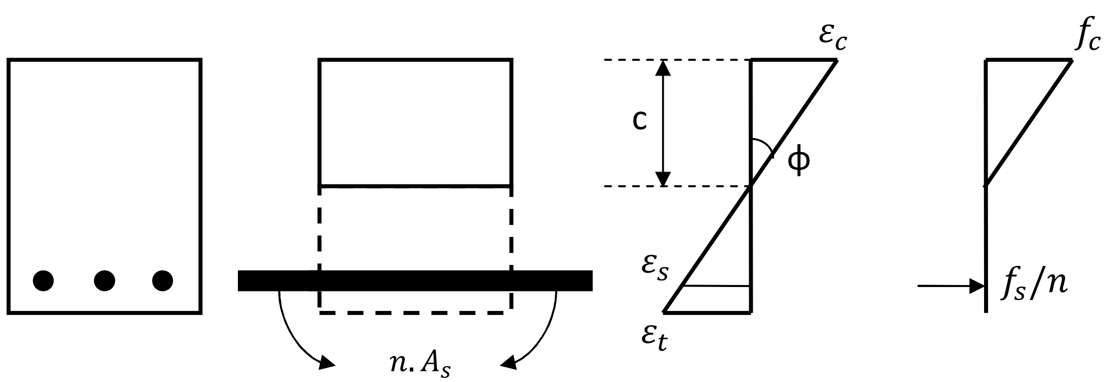
$f_t = f_r = 0.6\sqrt{f_c'} = 0.6\sqrt{35} = 3.55\ MPa$

$f_t = \frac{M_{cr}(h-c)}{I_{tr}} \Rightarrow M_{cr} = \frac{f_t.I_{tr}}{(h-c)} = \frac{3.55\times3.51\times10^9}{(500-263)}\times10^{-6} = 52.6\ kN.m$

$\emptyset_{cr} = \frac{\varepsilon_t}{h-c} = \frac{f_t/E_c}{h-c} = \frac{3.55/5500\sqrt{35}}{500-263} = 0.46\times10^{-6}\ \ rad/mm$

ii) ***<u>After cracking</u>***

<u>Note:</u> The concrete is just cracked and works only in compression and the steel works in tension. Since the load is small, tangential modulus of elasticity is used ($E_c = 5500\sqrt{f_c'}$). Cracked transformed section is used to calculate the moment of inertia of the section.



Cross - Section Cracked transformed Section Strain Stress

$f_t = f_r = 0.6\sqrt{f_c'} = 0.6\sqrt{35} = 3.55\ MPa$

$\varepsilon_t = \frac{f_t}{E_c} = \frac{3.55}{5500\sqrt{35}} = 0.0001$

The section cracks if the tensile strain exceeds the rupture strain. Hence, the strain level is selected slightly higher than the rupture strain. $\varepsilon_t = 0.00011$.

$T_s = C_c$

$T_s = A_s . f_s \qquad ; f_s = E_s . \varepsilon_s$

$C_c = \frac{1}{2} f_c . b . c \qquad ; f_c = E_c . \varepsilon_c$

$\frac{\varepsilon_s}{\varepsilon_t} = \frac{d-c}{h-c}$
$\frac{\varepsilon_c}{\varepsilon_t} = \frac{c}{h-c}$

$A_s . f_s = \frac{1}{2} f_c . b . c \;\Rightarrow\; A_s . E_s . \varepsilon_s = \frac{1}{2} E_c . \varepsilon_c . b . c \;\Rightarrow\; A_s . E_s . \varepsilon_t . \frac{d-c}{h-c} = \frac{1}{2} E_c . \varepsilon_t . \frac{c}{h-c} . b . c$

$3 \times 700 \times 200{,}000 \times 0.00011 \times \frac{445-c}{500-c} = \frac{1}{2} \times 5500\sqrt{35} \times 0.00011 \times \frac{c}{500-c} \times 300 \times c$

$c = 157\ mm$

$M = C_c(d - c/3) = T_s(d - c/3)$

$T_s = A_s . E_s . \varepsilon_t . \frac{d-c}{h-c} = 3 \times 700 \times 200{,}000 \times 0.00011 \times \frac{445-157}{500-157} \times 10^{-3} = 38.8\ kN$

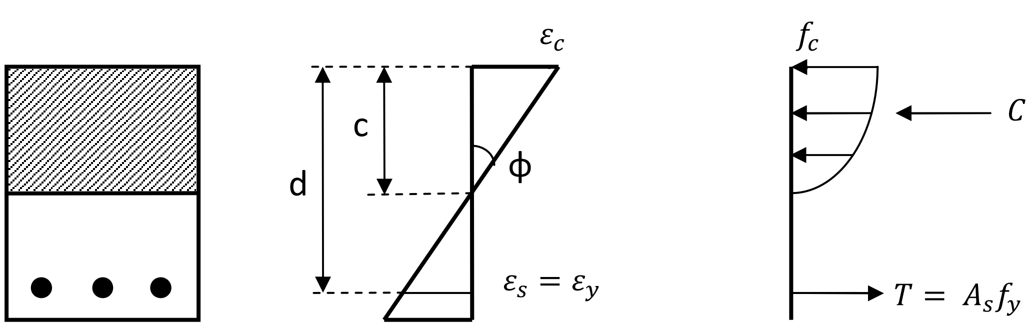
$M = 38.8(445 - 157/3) \times 10^{-3} = 15.2\ kN.m$

$\emptyset_{cr} = \frac{\varepsilon_t}{h-c} = \frac{0.00011}{500-157} = 0.32 \times 10^{-6}\ rad/mm$

Note: $I_{cr} = \frac{bc^3}{12} + n . A_s . (d-c)^2$

iii) Yielding of steel

Note: At this stage of loading concrete is substantially loaded and the Hognestad's model is used to simulate concrete behaviour at this stage. The Modulus of elasticity of concrete is reduced to $E_c = 4500\sqrt{f_c'}$. This can be explained by cracking of the concrete. A try and error process is needed to determine the correct "c" value to determine the depth of the compression zone. The moment capacity is calculated based on the balanced tension and compression forces.



Cross - Section Strain Stress, Force

Strain Compatibility: Select a “c” value and verify if C = T.

Let c =150 mm

$$\frac{\varepsilon_c}{\varepsilon_s} = \frac{c}{d-c} \Rightarrow \varepsilon_c = \varepsilon_s \frac{c}{d-c} = 0.002 \times \frac{150}{445-150} = 1.02 \times 10^{-3}$$

$$\varepsilon_c < \varepsilon_0 = 0.002 \Rightarrow \alpha = \frac{\varepsilon_c}{\varepsilon_0}\left[1 - \frac{\varepsilon_c}{3\varepsilon_0}\right] = \frac{1.02\times10^{-3}}{0.002}\left[1 - \frac{1.02\times10^{-3}}{3\times0.002}\right] = 0.423$$

$$C = \alpha.f_c'.b.c = 0.423 \times 35 \times 300 \times 150 = 666{,}225\ N$$

$$T = A_s.f_y = 3 \times 700 \times 400 = 840{,}000\ N$$

$C \neq T$: Therefore “c” value has to be revised. Since the tension force is more than compression, the depth of the compression zone should be increased.

Let c = 167 mm

$$\varepsilon_c = \varepsilon_s \frac{c}{d-c} = 0.002 \times \frac{167}{445-167} = 1.20 \times 10^{-3}$$

$$\varepsilon_c < \varepsilon_0 = 0.002 \Rightarrow \alpha = \frac{\varepsilon_c}{\varepsilon_0}\left[1 - \frac{\varepsilon_c}{3\varepsilon_0}\right] = \frac{1.20\times10^{-3}}{0.002}\left[1 - \frac{1.20\times10^{-3}}{3\times0.002}\right] = 0.480$$

$$C = \alpha.f_c'.b.c = 0.480 \times 35 \times 300 \times 167 = 841{,}680\ N$$

$$T = A_s.f_y = 3 \times 700 \times 400 = 840{,}000\ N$$

$$C \cong T \Rightarrow Ok.$$

$$\varepsilon_c < \varepsilon_0 = 0.002 \Rightarrow \gamma = 1 - \frac{\left[\frac{2}{3} - \frac{\varepsilon_c}{4.\varepsilon_0}\right]}{\left[1 - \frac{\varepsilon_c}{3.\varepsilon_0}\right]} = 1 - \frac{\left[\frac{2}{3} - \frac{1.20\times10^{-3}}{4\times0.002}\right]}{\left[1 - \frac{1.20\times10^{-3}}{3\times0.002}\right]} = 0.354$$

$$M_y = Moment\ @\ Yield = C(d - \gamma.c) = T(d - \gamma.c) = 840(445 - 0.354 \times 167) \times 10^{-3}$$

$$M_y = 324\ kN.m$$

$$\varphi_y = \frac{\varepsilon_s}{d-c} = \frac{0.002}{445-167} = 7.19 \times 10^{-6}\ rad/mm$$

iv) <u>Ultimate capacity</u>

Note: At the ultimate stage, the section is about to fail by crushing of the concrete in the compression zone. In a properly designed beam, such as the one above, the steel yields and shows significant deformation prior to failure of the section. This stage is the only time where the equivalent rectangular stress block proposed by the code is applicable. Furthermore, the Hognestad’s model is also still applicable. Choice of the method is left to the designer. Modulus of elasticity of the concrete is $E_c = 4500\sqrt{f_c'}$. Similar to previous section, try and error process is used to determine the “c” value.

Hognestad's model: $\varepsilon_{cu} = 0.0038$.

Let c = 101 mm

$$\varepsilon_s = \varepsilon_c \frac{d-c}{c} = 0.0038 \times \frac{445-101}{101} = 0.0129 > 0.002 \Rightarrow Steel\ Yields\ (f_s = f_y)$$

$$T = A_s . f_y = 3 \times 700 \times 400 = 840{,}000\ N$$

$$\varepsilon_c = \varepsilon_{cu} @Ultimate = 0.0038 > \ \varepsilon_0 = 0.002 \Rightarrow \alpha = 41.667 \frac{{\varepsilon_0}^2}{\varepsilon_c} - 0.5 \frac{\varepsilon_0}{\varepsilon_c} - 41.667\varepsilon_c + 1.167$$

$$\alpha = 0.789$$

$$\gamma = 1 - \frac{1}{\alpha}\left(27.78 \frac{{\varepsilon_0}^3}{{\varepsilon_c}^2} - 0.167 \frac{{\varepsilon_0}^2}{{\varepsilon_c}^2} - 27.78\varepsilon_c + 0.538\right)$$

$$\gamma = 0.491$$

$$C = \alpha . f_c' . b . c = 0.789 \times 35 \times 300 \times 101 = 863{,}735\ N$$

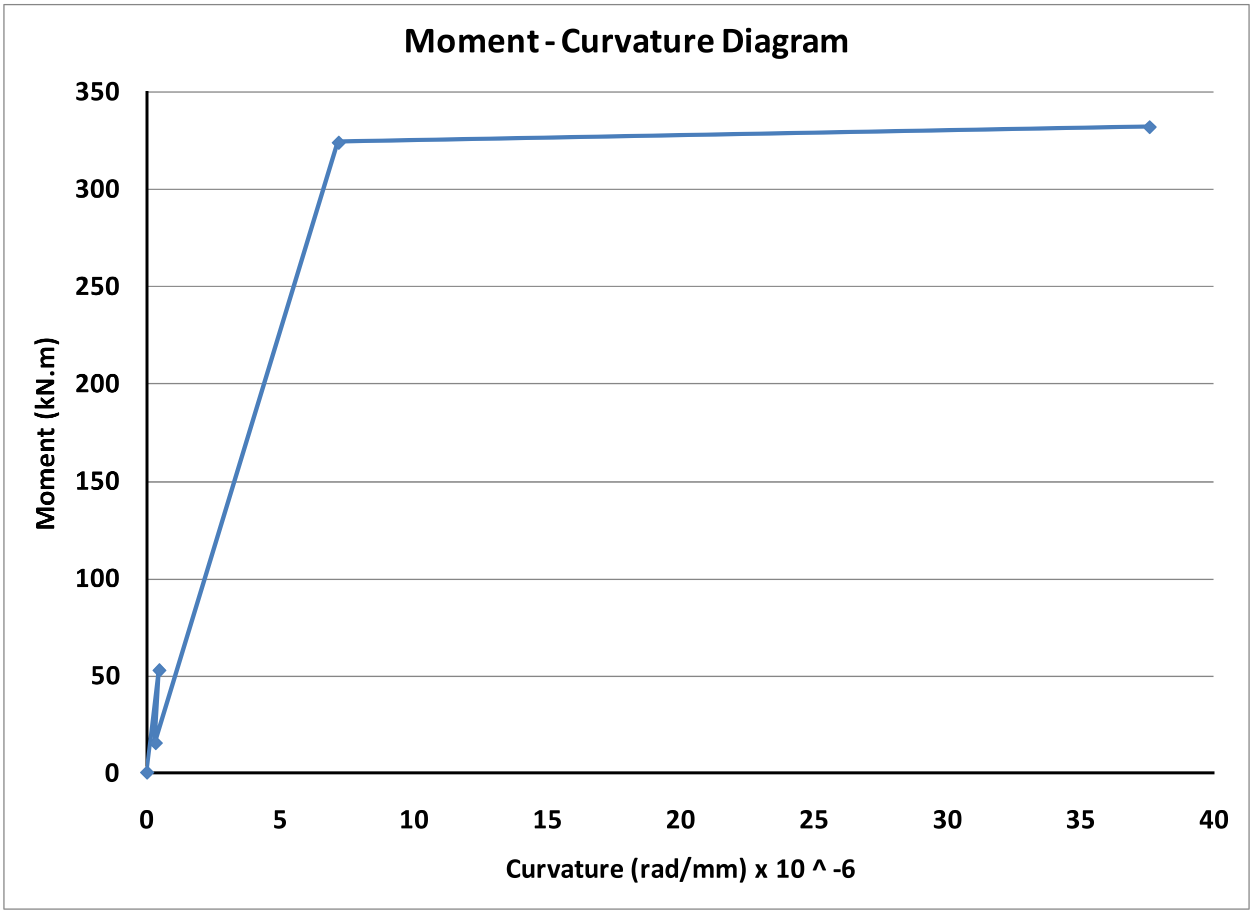
$$C \cong T \ \Rightarrow \ Ok.$$

$$M_u = T(d - \gamma . c) = 840(445 - 0.491 \times 101) \times 10^{-3}$$

$$M_u = 332\ kN.m$$

$$\varphi_y = \frac{\varepsilon_c}{c} = \frac{0.0038}{101} = 37.62 \times 10^{-6}\ rad/mm$$

Significant curvature at rupture point is observed.


Moment - Curvature Diagram
Moment (kN.m)
0
50
100
150
200
250
300
350
0
5
10
15
20
25
30
35
40
Curvature (rad/mm) x 10 ^ -6