PART A (50 marks) (2 marks for each problem)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET. FOR SAFETY, ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

2 marks A1. Evaluate $\arcsin(\sin \frac{4\pi}{3})$.

$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

so $\arcsin(\sin 4\pi/3) = \arcsin(-\sqrt{3}/2)$

$= -\pi/3$

A: $\frac{4\pi}{3}$	B: $\frac{\pi}{3}$	C: $\frac{2\pi}{3}$	D: $-\frac{\pi}{3}$	E: $-\frac{2\pi}{3}$

2 marks A2. Determine $\lim_{x\to\infty} \ln(\sin \frac{1}{x})$.

Let $t = \frac{1}{x}$. As $x \to \infty$, $t \to 0^+$

Let $u = \sin\frac{1}{x}, = \sin t$. As $t \to 0^+$, $u \to 0^+$

Thus, $\lim_{x\to\infty} \ln(\sin\frac{1}{x}) = \lim_{u\to 0^+} \ln u = -\infty$

A: 0	B: $-\infty$	C: 1	D: e	E: ∞

2 marks A3. If $x + \arctan(xy) = 1$, use implicit differentiation to find y' at the point $(1, 0)$.

Differentiate both sides with respect to x to get

$1 + \frac{1}{1+(xy)^2}\left(x\frac{dy}{dx} + y\right) = 0$

At $(1,0)$ we get $1 + \frac{1}{1+0^2}\left(\frac{dy}{dx} + 0\right) = 0$

i.e., $1 + \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -1$

A: 0	B: 1	C: 2	D: -1	E: -2

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2 marks A4. Determine $\lim_{x\to\infty} \frac{\ln x}{e^x}$.

$$\lim_{x\to\infty} \frac{\ln x}{e^x} \quad \left[\frac{\infty}{\infty}\right]$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{1/x}{e^x}$$

$$= \lim_{x\to\infty} \frac{1}{xe^x} = 0$$

A: 0	B: 1	C: $\frac{1}{e}$	D: ∞	E: $-\infty$

2 marks A5. Determine $\lim_{x\to 0} \frac{e^x - 1}{\arcsin x}$.

$$\lim_{x\to 0} \frac{e^x-1}{\arcsin x} \quad \left[\frac{0}{0}\right]$$

$$\overset{H}{=} \lim_{x\to 0} \frac{e^x}{\sqrt{1-x^2}} = \frac{e^0}{\sqrt{1-0^2}} = \frac{1}{1} = 1$$

A: ∞	B: $-\infty$	C: 1	D: -1	E: 0

2 marks A6. Which of the following integrals equals

$$\lim_{n\to\infty} \left(\frac{1}{n}\sum_{i=1}^{n} \sqrt{\cos\left(\frac{i}{n}\right)}\right) ?$$

Compare with $\lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f\left(a + i\,\frac{b-a}{n}\right)$

to see that $b-a=1$, $f\left(a+\frac{i}{n}\right) = \sqrt{\cos\frac{i}{n}}$

so $a=0$, $b=1$ and $f(x) = \sqrt{\cos x}$

A: $\int_0^{\pi/2} \sqrt{\cos x}\,dx$	B: $\int_0^{1} \sqrt{\cos x}\,dx$	C: $\int_1^{2} \sqrt{\cos x}\,dx$	D: $\int_1^{n} \sqrt{\cos x}\,dx$	E: $\frac{1}{2}\int_0^{2} \sqrt{\cos x}\,dx$

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2 marks A7. If $f(x) = \int_{\pi/2}^{x} \sqrt{1+\sin t}\, dt$, find $f'(\frac{\pi}{2})$.

$f'(x) = \sqrt{1+\sin x}$ by Fundamental Theorem of Calculus I

so $f'(\pi/2) = \sqrt{1+\sin \pi/2}$
$= \sqrt{1+1}$
$= \sqrt{2}$

A: $\sqrt{2}$	B: $-\sqrt{2}$	C: 0	D: $\sqrt{\frac{\pi}{2}+1}$	E: 1

2 marks A8. Evaluate $\int_{-2}^{1} |x|dx$.

$\int_{-2}^{1} |x| dx = \int_{-2}^{0} |x| dx + \int_{0}^{1} |x| dx$ | Recall $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$= \int_{-2}^{0} -x\, dx + \int_{0}^{1} x\, dx$

$= -\frac{x^2}{2}\Big|_{0}^{2} + \frac{x^2}{2}\Big|_{0}^{1}$

$= 0 - (-4/2) + (\frac{1}{2} - 0) = 2 + 1/2 = 5/2$

A: $\frac{1}{2}$	B: $\frac{5}{2}$	C: 1	D: $\frac{3}{2}$	E: 0

2 marks A9. A particle is moving with the given data. The velocity $v(t) = \frac{t^2}{2} - 2t + 1$ and $s(0) = 3$, where $s(t)$ is the position (displacement) of the particle. Find a formula for $s(t)$.

$s(t) = \int v(t)dt = \int (\frac{t^2}{2} - 2t + 1) dt$

$= \frac{t^3}{6} - t^2 + t + c$

$s(0) = 0 - 0 + 0 + C$ and $s(0) = 3$ so $c = 3$

Thus, $s(t) = \frac{t^3}{6} - t^2 + t + 3$

A: $\frac{t^3}{6} - t^2 + t + 3$	B: $2t - 2$	C: $2t + 1$	D: $t^3 - t^2 + t + 3$	E: $4t + 1$

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2 marks A10. Evaluate $\int \frac{\sin\sqrt{x}}{2\sqrt{x}}\,dx$.

Let $u = \sqrt{x} = x^{1/2}$

Then $du = \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2\sqrt{x}}\,dx$

Thus, $\int \frac{\sin\sqrt{x}}{2\sqrt{x}}\,dx$

$= \int \sin u\,du$

$= -\cos u + C$

$= -\cos\sqrt{x} + C$

A: $\frac{\cos\sqrt{x}}{3x^{3/2}} + C$	B: $-\frac{\cos\sqrt{x}}{3x^{3/2}} + C$	C: $-\cos\sqrt{x} + C$
D: $\cos\sqrt{x} + C$	E: $-4\frac{\cos\sqrt{x}}{3x^{3/2}} + C$	

2 marks A11. If $x^2 - y^2 = 1$, find $= \frac{d^2y}{dx^2}$ at the point $(2, \sqrt{3})$.

Differentiate implicitly to get

$2x - 2y\frac{dy}{dx} = 0$ so $\frac{dy}{dx} = \frac{2x}{2y} = \frac{x}{y}$.

Thus, $\frac{d^2y}{dx^2} = \frac{y \cdot 1 - x\,dy/dx}{y^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2}$.

Hence $\left.\frac{d^2y}{dx^2}\right|_{(2,\sqrt{3})} = \frac{\sqrt{3} - 2^2/\sqrt{3}}{3} = \frac{\sqrt{3} - 4/\sqrt{3}}{3} = \frac{3-4}{3\sqrt{3}} = \frac{-1}{3\sqrt{3}}$

A: 1	B: $\frac{2}{\sqrt{3}}$	C: $\frac{\sqrt{3}}{2}$	D: $\frac{1}{3\sqrt{3}}$	E: $-\frac{1}{3\sqrt{3}}$

2 marks A12.

Evaluate $\int_0^1 \frac{4}{1+x^2}\,dx$.

$\int_0^1 \frac{4}{1+x^2}\,dx = 4\int_0^1 \frac{1}{1+x^2}\,dx$

$= 4\tan^{-1}x \Big|_0^1$

$= 4(\tan^{-1}1 - \tan^{-1}0)$

$= 4(\pi/4 - 0)$

$= \pi$

A: π	B: 2π	C: -2π	D: 0	E: 4

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2 marks A13. Find the indefinite integral (the most general antiderivative) of $\sec x \tan x + \frac{1}{x}$ for $0 < x < \frac{\pi}{2}$.

$\int \left(\sec x \tan x + \frac{1}{x}\right) dx$

$= \sec x + \ln x + C$

A: $\sec x - \frac{1}{x^2} + C$	B: $\sec x + \ln\lvert x\rvert + C$	C: $\tan x + \ln\lvert x\rvert + C$
D: $\tan x - \frac{1}{x^2} + C$	E: $\sec^3 x + \sec x \tan^2 x - \frac{1}{x^2} + C$	

2 marks A14. If $f'(x) = \sec^2 x$ for $0 < x < \frac{\pi}{2}$ and $f(\frac{\pi}{3}) = 1$, find $f(x)$.

$f(x) = \tan x + c$

$f(\pi/3) = \tan \pi/3 + c$

$= \sqrt{3} + c$

and $f(\pi/3) = 1$ } so $\sqrt{3} + c = 1$ so $c = 1 - \sqrt{3}$

Hence, $f(x) = \tan x + 1 - \sqrt{3}$

A: $\frac{\sec^3 x}{3} - \frac{8}{3} + 1$	B: $\csc^2 x + 1$	C: $\frac{\sec^3 x}{3} - 1 + \frac{8}{3}$
D: $\tan x + 1 - \sqrt{3}$	E: $\tan x - 1 + \sqrt{3}$	

2 marks A15. If $y = \log_3(\arctan x)$, find $\frac{dy}{dx}$.

Let $u = \arctan x$. Then $y = \log_3 u$.

$\frac{du}{dx} = \frac{1}{1+x^2}$ $\quad \frac{dy}{du} = \frac{1}{u \ln 3}$

so $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{u \ln 3}\left(\frac{1}{1+x^2}\right)$

$= \frac{1}{\ln 3 (\arctan x)(1+x^2)}$

A: $\frac{1}{(\ln 3)(1+x^2)\arctan x}$	B: $\frac{\ln 3}{(1+x^2)\arctan x}$	C: $\frac{(\tan x)\ln 3}{1+x^2}$
D: $\frac{1}{(\ln 3)\sqrt{1-x^2}\arctan x}$	E: $\frac{\ln 3}{\sqrt{1-x^2}\arctan x}$	

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2 marks A16. Evaluate $\int (\cos x)e^{\sin x}dx$.

Let $u = \sin x$

$du = \cos x \, dx$

$\int (\cos x) e^{\sin x} dx = \int e^u du$

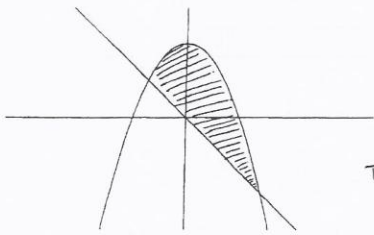
$= e^u + C$

$= e^{\sin x} + C$

A: $e^{\sin x} + C$	B: $e^{\cos x} + C$	C: $-(\sin x)e^{\sin x} + (\cos^2 x)e^{\sin x} + C$
D: $(\cos x)e^{\cos x} + C$	E: $(\sin x)e^{\sin x} + C$	

2 marks A17. Let R be the region, shown below, bounded by the parabola $y = 2 - x^2$ and the line $y = -x$. Choose the integral the value of which is the area of R.

To find the points of intersection, solve



$2 - x^2 = -x$

$\Rightarrow \quad 0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

so $x = -1, 2$

The limits of integration are -1 and 2

The upper curve is $y = 2 - x^2$

The lower curve is $y = -x$

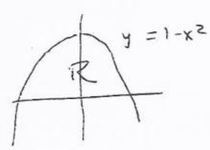
The area is given by $\int_{-1}^{2} [(2 - x^2) - (-x)] dx$

A: $\int_{-1}^{2}[(2-x^2)-(-x)]dx$	B: $\int_{-2}^{1}[(2-x^2)-(-x)]dx$	C: $\int_{-1}^{2}[-x-(2-x^2)]dx$
D: $\int_{-2}^{1}[-x-(2-x^2)]dx$	E: $\int_{-1}^{2}(x-2+x^2)dx$	

2 marks A18. Let R be the region bounded by $y = 1 - x^2$ and the x-axis. Find the area of R.

To get the points of intersection, solve

$1 - x^2 = 0 \Rightarrow x = \pm 1$



Thus the area is $\int_{-1}^{1} (1 - x^2) - 0 \,) dx$

$= x - \frac{x^3}{3} \Big|_{-1}^{1} = (1 - \frac{1}{3}) - (-1 + \frac{1}{3})$

$= 2 - 2/3 = 4/3$

A: $\frac{4}{3}$	B: $\frac{2}{3}$	C: $-\frac{4}{3}$	D: 0	E: $-\frac{2}{3}$

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2 marks A19. A wasp population starts with 3 wasps and increases at a rate of $n'(t)$ wasps per day. What does $3 + \int_0^{10} n'(t)dt$ represent ?

A: The population of wasps after 3 days
→ B: The population of wasps after 10 days
C: The rate at which the population of wasps increases after 3 days
D: The rate at which the population of wasps increases after 10 days

2 marks A20. Find the number n such that $\sum_{i=1}^{n} i = 15$.

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = 15$ so $n(n+1) = 30 \Rightarrow n^2 + n - 30 = 0$

$\Rightarrow (n-5)(n+6) = 0$ so $n = 5$

A: 15	B: 10	C: 5	D: 3	E: 4

2 marks A21. Suppose $\sum_{i=1}^{20} a_i = 5$ and $\sum_{i=1}^{20} b_i = 7$. Find the value of $\sum_{i=1}^{20}(3a_i - 2b_i)$.

$\sum_{i=1}^{20}(3a_i - 2b_i) = \sum_{i=1}^{20} 3a_i - \sum_{i=1}^{20} 2b_i = 3\sum_{i=1}^{20} a_i - 2\sum_{i=1}^{20} b_i$

$= 3(5) - 2(7) = 15 - 14 = 1$

A: 1	B: 2	C: 3	D: 7	E: 5

2 marks A22. If $f(x) = x^{\sin x}$, find $f'(\frac{\pi}{2})$.

Let $y = x^{\sin x}$

Then $\ln y = \ln(x^{\sin x}) = \sin x \ln x$

A: 0	B: 1	C: $\frac{\pi}{2}$	D: $\frac{2}{\pi}$	E: $\ln 2$

Thus, $\frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \cos x \ln x$

so $\frac{dy}{dx} = y\left(\frac{\sin x}{x} + \cos x \ln x\right) = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \ln x\right)$

so $\left.\frac{dy}{dx}\right|_{x=\pi/2} = \left(\frac{\pi}{2}\right)^1\left(\frac{1}{\pi/2} + \cos\frac{\pi}{2}\ln\frac{\pi}{2}\right) = \frac{\pi}{2}\left(\frac{1}{\pi/2} + 0\right)$

$= 1$

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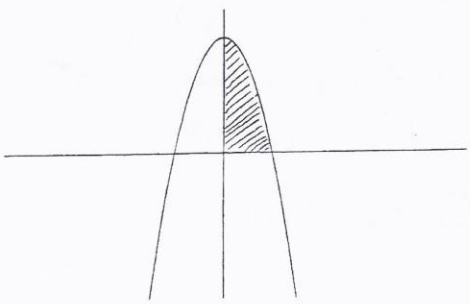
2 marks A23. Evaluate $\int_{-5}^{5} x^3\sqrt{25-x^2}\,dx$. Answer 0, since

$f(x) = x^3\sqrt{25-x^2}$ is odd and the limits of integration are -5 and 5

A: 25	B: 125	C: 0	D: -25	E: -125

For A24 and A25, let R be the region shown below, **bounded by** $y = 4 - 4x^2$, $x = 0$ **and** $y = 0$, **for** $x \geq 0$.



\# 24 use the disc method

$4 - 4x^2 = 0 \Rightarrow x = \pm 1 \Rightarrow x = 1$

Thickness Δx

radius $4 - 4x^2$

volume of the disc

$\pi(4-4x^2)^2 \Delta x$

\#25 use the shell method

Thickness Δx

radius x

height $4 - 4x^2$

volume of the shell

$2\pi x(4-4x^2)\Delta x$

2 marks A24. Choose the integral which gives the volume of the solid generated by rotating the region R about the x-axis.

A: $\int_0^1 \pi(4-4x^2)dx$	B: $\int_0^1 \pi(4-4x^2)^2 dx$	C: $\int_0^4 2\pi x(4-4x^2)dx$
D: $\int_0^4 2\pi(1-\frac{y}{4})^2 dy$	E: $\int_0^4 \pi y(1-\frac{y}{4})dy$	

2 marks A25. Choose the integral which gives the volume of the solid generated by rotating the region R about the y-axis.

A: $\int_0^1 2\pi x(4-4x^2)dx$	B: $\int_0^1 \pi(4-4x^2)^2 dx$	C: $\int_0^4 2\pi(1-\frac{y}{4})dy$
D: $\int_0^4 \pi y(1-\frac{y}{4})dy$	E: $\int_0^4 2\pi(1-\frac{y}{4})^2 dy$	

PART B (50 marks)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B26.

Find $\frac{dy}{dx}$ if

$$y = \int_{e^x}^{e^{\sqrt{x}}} \ln(t^2+1)\,dt.$$

Use the generalized version of the Fundamental Theorem of the calculus I which is

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\,h'(x) - f(g(x))\,g'(x)$$

to get (using $f(t) = \ln(t^2+1)$, $g(x) = e^x$ and $h(x) = e^{\sqrt{x}}$)

$$\frac{dy}{dx} = \ln\left(e^{2\sqrt{x}}+1\right)\frac{e^{\sqrt{x}}}{2\sqrt{x}} - \ln\left(e^{2x}+1\right)(e^x)$$

$$= \frac{\ln(e^{2\sqrt{x}}+1)e^{\sqrt{x}}}{2\sqrt{x}} - e^x\ln\left(e^{2x}+1\right)$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 *marks* B27. Determine
$$\lim_{x\to 0+} (\cos\sqrt{x})^{1/x}. \qquad [1^\infty]$$

Let $y = (\cos\sqrt{x})^{1/x}$

$\ln y = \ln\left[(\cos\sqrt{x})^{1/x}\right]$

$= \frac{1}{x}\ln(\cos\sqrt{x})$

$= \frac{\ln(\cos\sqrt{x})}{x}$

$\lim_{x\to 0} \ln y = \lim_{x\to 0^+} \frac{\ln(\cos\sqrt{x})}{x} \qquad \left[\frac{0}{0}\right]$

$\overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{\cos\sqrt{x}}(-\sin\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)}{1}$

$= \lim_{x\to 0^+} \frac{-\sin\sqrt{x}}{2\sqrt{x}\cos\sqrt{x}} \qquad \left[\frac{0}{0}\right] \qquad (*)$

$\overset{H}{=} \lim_{x\to 0^+} \frac{-\cos\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)}{2\sqrt{x}(-\sin\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right) + 2\frac{1}{2\sqrt{x}}\cos\sqrt{x}}$

$= \lim_{x\to 0^+} \frac{-\cos\sqrt{x}}{-2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x}}$

$= \frac{-1}{0+2}$

$= -1/2$

(*) OR $= \lim_{x\to 0^+} -\frac{1}{2}\left(\frac{\sin\sqrt{x}}{\sqrt{x}}\right)\left(\frac{1}{\cos\sqrt{x}}\right)$

$= -\frac{1}{2}(1)\left(\frac{1}{1}\right)$

$= -1/2$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

7 marks B28. Find the area of the region R, shown below, bounded by $x = -y^2$ and $x = 2 - 3y^2$.

To find the points of intersection

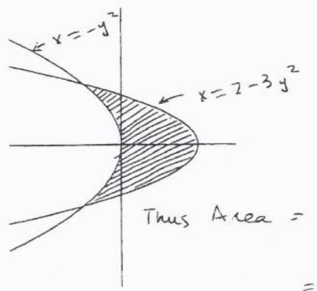
solve $-y^2 = 2 - 3y^2$

$\Rightarrow 3y^2 - y^2 = 2$

$\Rightarrow 2y^2 = 2$

$y^2 = 1$

$y = \pm 1$



Thus Area $= \int_{-1}^{1} (2 - 3y^2) - (-y^2)\,dy$

$= \int_{-1}^{1} (2 - 2y^2)\,dy$

$= 2\int_0^1 (2 - 2y^2)\,dy$ by symmetry

$= 2\left(2y - \frac{2y^3}{3}\right)\Big|_0^1$

$= 2\left[(2 - 2/3) - 0\right]$

$= 2(4/3)$

$= 8/3$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B29.

If $y = (\ln x)^x$, find $\frac{dy}{dx}$.

Use logarithmic differentiation to get

$$\ln y = \ln\left[(\ln x)^x\right]$$

$$= x \ln(\ln x)$$

Then $\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{\ln x}\right)\left(\frac{1}{x}\right) + \ln(\ln x)$

$$= \frac{1}{\ln x} + \ln(\ln x)$$

Thus, $\frac{dy}{dx} = y\left[\frac{1}{\ln x} + \ln(\ln x)\right]$

$$= (\ln x)^x\left[\frac{1}{\ln x} + \ln(\ln x)\right]$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B30. Evaluate

$$\int_1^2 \frac{2x}{(x^2+1)[\ln(x^2+1)]^3}\,dx.$$

Let $u = \ln(x^2+1)$

Then $\frac{du}{dx} = \frac{1}{x^2+1}(2x+0)$

so $du = \frac{2x}{x^2+1}\,dx$

when $x=1$, $u = \ln(1^2+1) = \ln 2$

when $x=2$, $u = \ln 5$

Thus, $\int_1^2 \frac{2x}{(x^2+1)[\ln(x^2+1)]^3}\,dx$

$= \int_{\ln 2}^{\ln 5} \frac{1}{u^3}\,du = \int_{\ln 2}^{\ln 5} u^{-3}\,du$

$= \left.\frac{u^{-2}}{-2}\right|_{\ln 2}^{\ln 5} = \left.-\frac{1}{2u^2}\right|_{\ln 2}^{\ln 5}$

$= -\frac{1}{2(\ln 5)^2} - \left(-\frac{1}{2(\ln 2)^2}\right)$

$= \frac{1}{2(\ln 2)^2} - \frac{1}{2(\ln 5)^2}$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

4 marks **B31.**

Evaluate
$$\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx.$$

Let $u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx$$

$$= \int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \sin^{-1} u + C$$

$$= \sin^{-1}(\ln x) + C$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

7 marks B32. Let S be the region in the first quadrant, bounded by the curve $y = (R^2 - x^2)^{1/2}$, the x-axis, and the line $x = r$, with $R > r$. Find the volume of the solid generated by rotating S about the y-axis.

Note $r \geq 0$ since S is in the first quadrant

Use the shell method

thickness Δx

radius $x - 0 = x$

height $y = \sqrt{R^2 - x^2}$

volume of the shell

$2\pi x \sqrt{R^2 - x^2}\, \Delta x$

Volume of the solid $=$

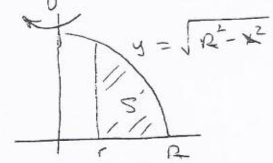
$$\int_r^R 2\pi x \sqrt{R^2 - x^2}\, dx$$

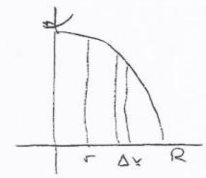
$$= \int_{R^2 - r^2}^{0} \pi \sqrt{u}\, (-du)$$

$$= \pi \int_0^{R^2 - r^2} u^{1/2}\, du$$

$$= \pi \left. \frac{u^{3/2}}{3/2} \right|_0^{R^2 - r^2}$$

$$= \frac{2\pi}{3} (R^2 - r^2)^{3/2}$$





Let $u = R^2 - x^2$

$du = -2x\, dx$

$-du = 2x\, dx$

when $x = r$, $u = R^2 - r^2$

when $x = R$, $u = 0$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 *marks* B33. Evaluate

$$\int_0^6 |x^2 - 9|dx.$$

$$|x^2-9| = \begin{cases} x^2-9 & \text{if } x^2-9 \geq 0 \\ -(x^2-9) & \text{if } x^2-9 < 0 \end{cases}$$

$$= \begin{cases} x^2-9 & \text{if } x \geq 3 \text{ or } x \leq -3 \\ 9-x^2 & \text{if } -3 \leq x \leq 3 \end{cases}$$

Thus, $\int_0^6 |x^2-9|\,dx$

$$= \int_0^3 |x^2-9|\,dx + \int_3^6 |x^2-9|\,dx$$

$$= \int_0^3 (9-x^2)\,dx + \int_3^6 (x^2-9)\,dx$$

$$= \left[9x - \frac{x^3}{3}\right|_0^3 + \left[\frac{x^3}{3} - 9x\right]_3^6$$

$$= [(27-9)-0] + [(\tfrac{216}{3} - 54) - (9-27)]$$

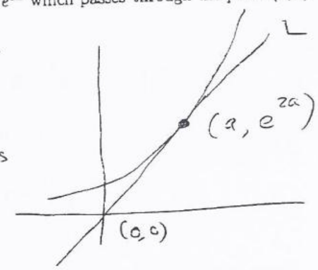
$$= 18 + (72-54) - (-18)$$

$$= 90 - 54 + 18$$

$$= 54$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

4 *marks* B34. Find an equation of the tangent line to $y = e^{2x}$ which passes through the point $(0,0)$.



Let (a, e^{2a}) be the point on $y = e^{2x}$ at which the tangent line passes through $(0,0)$ and L be the tangent line. Then the slope of L is

$$\frac{e^{2a} - 0}{a - 0} = \frac{e^{2a}}{a}$$

and for $y = e^{2x}$, $\frac{dy}{dx} = e^{2x}(2) = 2e^{2x}$

so at (a, e^{2a}), $m = \left.\frac{dy}{dx}\right|_{x=a} = 2e^{2a}$

Hence $\frac{e^{2a}}{a} = 2e^{2a} \Rightarrow \frac{1}{a} = 2$

so that $a = \frac{1}{2}$

The slope of L is $2e^{2(1/2)} = 2e$

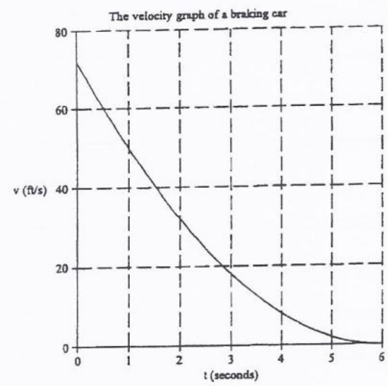
An equation of L is $y - 0 = 2e(x - 0)$

or $y = 2ex$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

3 marks B35.

The velocity of a braking car is given by $v(t) = 2(t-6)^2$ (see the velocity graph below). Find the distance traveled by the car while the brakes are applied.



Distance $= s = \int_0^6 v(t)\,dt$

$= \int_0^6 2(t-6)^2\,dt$

$= \int_0^6 2(t^2 - 12t + 36)\,dt$

$= 2\left(\frac{t^3}{3} - \frac{12t^2}{2} + 36t\right)\Big|_0^6$

$= 2[(72 - 6\cdot 36 + 36\cdot 6) - 0]$

$= 144$

Answer 144 feet.