MATH 208—SUMMER 2008

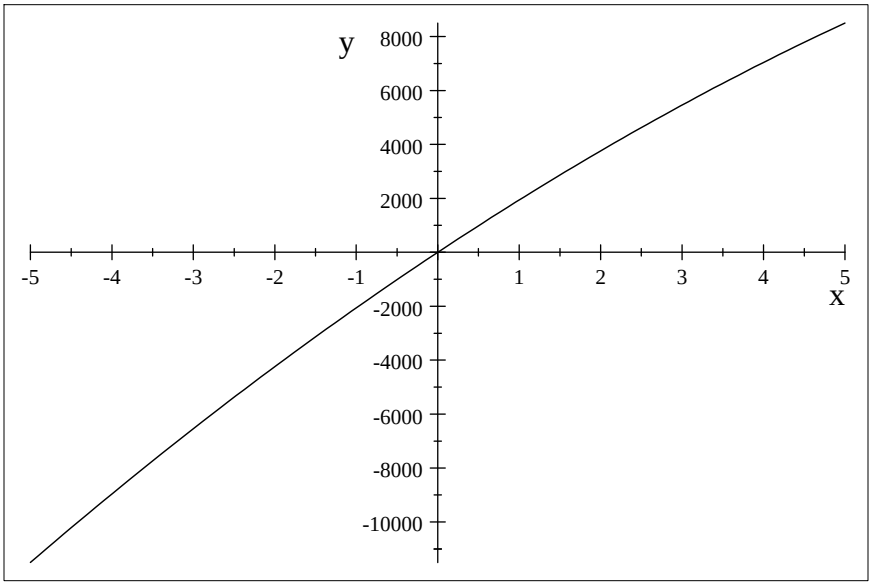
Solutions

Question 1

A)

$R(x) = x(2000 - 60x)$

$x(2000 - 60x)$



B)

$x_1 = 0, x_2 = \frac{2000}{60}$

$x_{vertex} = \frac{x_1+x_2}{2} = \frac{0+\frac{2000}{60}}{2} = \frac{50}{3} = 16.\overline{6} = 16667$ computers

$R(16667) = 16.667(2000 - 60 \times 16.667) = 16666.67\$$

C)

$p(x) = 2000 - 60x$

$p(16.667) = 2000 - 60 \times 16.667 = 999.98\$$

Question 2

A)

$$6^{329x+191} = 16^{204x-2059}$$

$329x + 191 = 204x - 2059$, Solution is: -18

B)

$$49^{2x} = 7^{x^2-12}$$

$$7^{4x} = 7^{x^2-12}$$

$4x = x^2 - 12$, Solutions are: $6, -2$

C)

$$\frac{3}{2}\log_b 4 - \frac{2}{3}\log_b 8 + \log_b 2 = \log_b x$$

$4^{\frac{3}{2}} \times 8^{-\frac{2}{3}} \times 2 = x$, Solution is: 4

D)

$$5\log_5(3x+8) - 5\log_5(2x-5) = 5$$

$$\log_5\left(\frac{(3x+8)^5}{(2x-5)^5}\right) = 5$$

$$\frac{(3x+8)^5}{(2x-5)^5} = 5^5$$

$\frac{3x+8}{2x-5} = 5$, Solution is: $\frac{33}{7}$

E)

$$\log_{10}(x+6) - \log_{10}(x-3) = 1$$

$$\log_{10}\left(\frac{x+6}{x-3}\right) = 1$$

$\frac{x+6}{x-3} = 10^1$, Solution is : $\{x = 4\}$

Question 3

A)

$a_1 = f(0) = -3, n = 50, a_n = a_{50} = f(49) = 2(49) - 3 = 95.0$
$S_n = \frac{n}{2}(a_1 + a_n) = \frac{50}{2}(-3 + 95) = 2300.0$
$\sum_0^{49}(2x-3) = 2300$

B)

$a_1 = g(0) = 3^{-3} = \frac{1}{27}, r = 3, n = 20$
$S_n = a_1\frac{r^n-1}{r-1} = \frac{1}{27}\frac{3^{20}-1}{3-1} = 6.457 \times 10^7$
$\sum_0^{19} 3^{x-3} = \frac{1743392200}{27} = 6.457 \times 10^7$

Question 4

A)

$FV = 100000\$, m = 24, t = 4, r = 0.072, i = \frac{r}{m} = \frac{0.072}{24}, n = mt = 96$
$PMT = FV\frac{i}{(1+i)^n-1} = 100000\frac{\frac{0.072}{24}}{(1+\frac{0.072}{24})^{96}-1} = 900.41$

B)

$100000\$ - PMTxn = 100000 - 900.41 \times 96 = 13561\$$

C)

$m = 24, n = mt = 24t, i = \frac{0.072}{24}, PMT = 800, FV = \$100000$

$FV = PMT\frac{(1+i)^{mt}-1}{i}$

$100000 = 800\frac{(1+\frac{0.072}{24})^{24t}-1}{\frac{0.072}{24}}$, Solution is : $\{t = 4.4296\}$

$4.4296 \times 24 = 106.31 = 107$ payments

Question 5

A)

$PV = 120000, t = 25, m = 12, n = mt = 300, r = 0.054, i = \frac{r}{m} = \frac{0.054}{12}$

$PMT = PV\frac{i}{1-(1+i)^{-n}} = 120000\frac{\frac{0.054}{12}}{1-(1+\frac{0.054}{12})^{-300}} = 729.76$

B)

$m = 12, t_r = 10, n_r = 120$

$PV_r = PMT\frac{1-(1+i)^{-nr}}{i} = 729.76\frac{1-(1+\frac{0.054}{12})^{-120}}{\frac{0.054}{12}} = 67551\$$

C)

Saving=would have paid-did pay: $PMT \times n_r - PV_r$

$729.76 \times 120 - 67551 = 20020.0\$$

Question 6

$\begin{bmatrix} 3 & 6 & 9 & 3 \\ 2 & 3 & 4 & 3 \\ 3 & 6 & 3 & 9 \end{bmatrix}$, row echelon form: $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$x_1 = 2, x_2 = 1, x_3 = -1$

Question 7

x_1=shipping output($), x_2=agriculture output($), x_3=mining output($)

inputs: $\begin{bmatrix} 0.30 & 0.20 & 0.40 \\ 0.30 & 0.20 & 0.20 \\ 0.20 & 0.30 & 0.30 \end{bmatrix}$, transpose: $\begin{bmatrix} .30 & .30 & .20 \\ .20 & .20 & .30 \\ .40 & .20 & .30 \end{bmatrix}$

A)

cost/technology matrix: $\begin{bmatrix} .30 & .30 & .20 \\ .20 & .20 & .30 \\ .40 & .20 & .30 \end{bmatrix}$

B)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} .30 & .30 & .20 \\ .20 & .20 & .30 \\ .40 & .20 & .30 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 180M\$ \\ 200M\$ \\ 300M\$ \end{bmatrix}$$

C)

$$\begin{cases} x_1 - 0.30x_1 - 0.30x_2 - 0.20x_3 = 180M\$ \\ x_2 - 0.20x_1 - 0.20x_2 - 0.30x_3 = 200M\$ \\ x_3 - 0.40x_1 - 0.20x_2 - 0.30x_3 = 300M\$ \end{cases}$$

$$\begin{cases} 0.70x_1 - 0.30x_2 - 0.20x_3 = 180M\$ \\ -0.20x_1 + 0.80x_2 - 0.30x_3 = 200M\$ \\ -0.40x_1 - 0.20x_2 + 0.70x_3 = 300M\$ \end{cases}$$

$$\begin{bmatrix} 7 & -3 & -2 & 1800 \\ -2 & 8 & -3 & 2000 \\ -4 & -2 & 7 & 3000 \end{bmatrix}$$, row echelon form: $$\begin{bmatrix} 1 & 0 & 0 & 1075 \\ 0 & 1 & 0 & 1019 \\ 0 & 0 & 1 & 1334 \end{bmatrix}$$

$x_1 = 1075M\$, x_2 = 1019M\$, x_3 = 1334M\$$

Alternate Method:

$I - M$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} .30 & .30 & .20 \\ .20 & .20 & .30 \\ .40 & .20 & .30 \end{bmatrix} = \begin{bmatrix} .7 & -.3 & -.2 \\ -.2 & .8 & -.3 \\ -.4 & -.2 & .7 \end{bmatrix}$$

$(I - M)^{-1}$

$$\begin{bmatrix} .7 & -.3 & -.2 \\ -.2 & .8 & -.3 \\ -.4 & -.2 & .7 \end{bmatrix}$$, inverse: $$\begin{bmatrix} 2.5 & 1.25 & 1.25 \\ 1.3 & 2.05 & 1.25 \\ 1.8 & 1.3 & 2.5 \end{bmatrix}$$

$(I - M)^{-1}D = X$

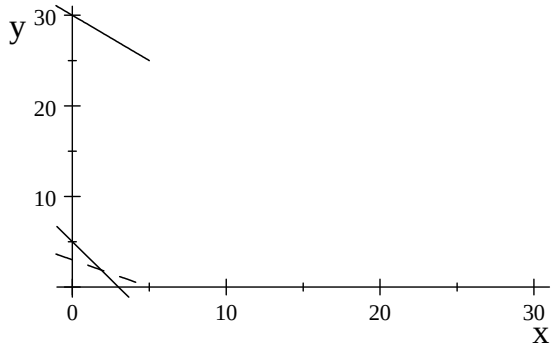
$$\begin{bmatrix} 2.5 & 1.25 & 1.25 \\ 1.3 & 2.05 & 1.25 \\ 1.8 & 1.3 & 2.5 \end{bmatrix} \begin{bmatrix} 180 \\ 200 \\ 300 \end{bmatrix} = \begin{bmatrix} 1075.0 \\ 1019.0 \\ 1334.0 \end{bmatrix}$$

$x_1 = 1075M\$, x_2 = 1019M\$, x_3 = 1334M\$$

Question 8

$x + y \leq 30 \quad 3x + 5y \geq 15 \quad 5x + 3y \geq 15 \quad x \geq 0 \quad y \geq 0$

$y = -x + 30 \quad y = -\frac{3}{5}x + 3 \quad y = -\frac{5}{3}x + 5 \quad x = 0 \quad y = 0$



Feasability region bounded by all three lines and the axes.

Intersection points (x,y)	$P(x,y) = 108x - 1202y$
$(0,30)$	-36060
$(30,0)$	3240
$(0,5)$	-6010
$(5,0)$	540
$(2,2)$	-2044

$Max : P(30,0) = 3240$
$Min : P(0,30) = -36060$

Question 9

A)

$2 \times 3 \times 2 = 12$ or ${}_2C_1 \times_3 C_1 \times_2 C_1 = 12$

B)

$12 \times 6 = 72$ or ${}_2C_1 \times_3 C_1 \times_2 C_1 \times_6 C_1 = 72$

Question 10

Total: 40. Good:34, Defective:6

A)

Total number of combinations : ${}_{40}C_7$
Number of good combination: ${}_{34}C_7$
$P_{\text{all good}} = \frac{{}_{34}C_7}{{}_{40}C_7} = 0.28855 = 28.855\%$

B)

$P_{\text{1 defective}} = P_{\text{not all good}} = 1 - P_{\text{good}} = 1 - 0.28855 = .71145 = 71.145\%$